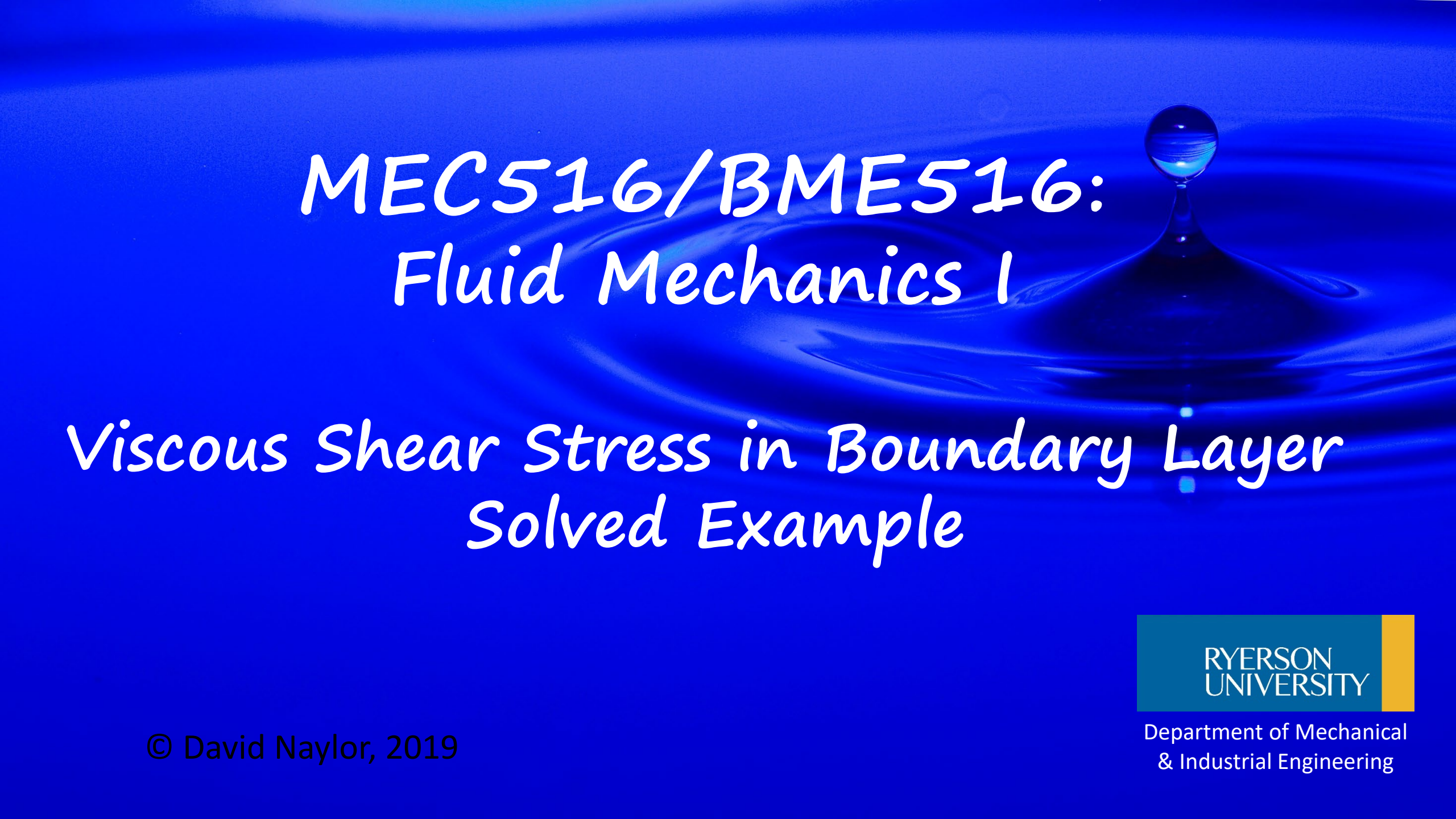

MEC516/BME516: Fluid Mechanics I

Viscous Shear Stress in Boundary Layer Solved Example

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Department of Mechanical & Industrial Engineering

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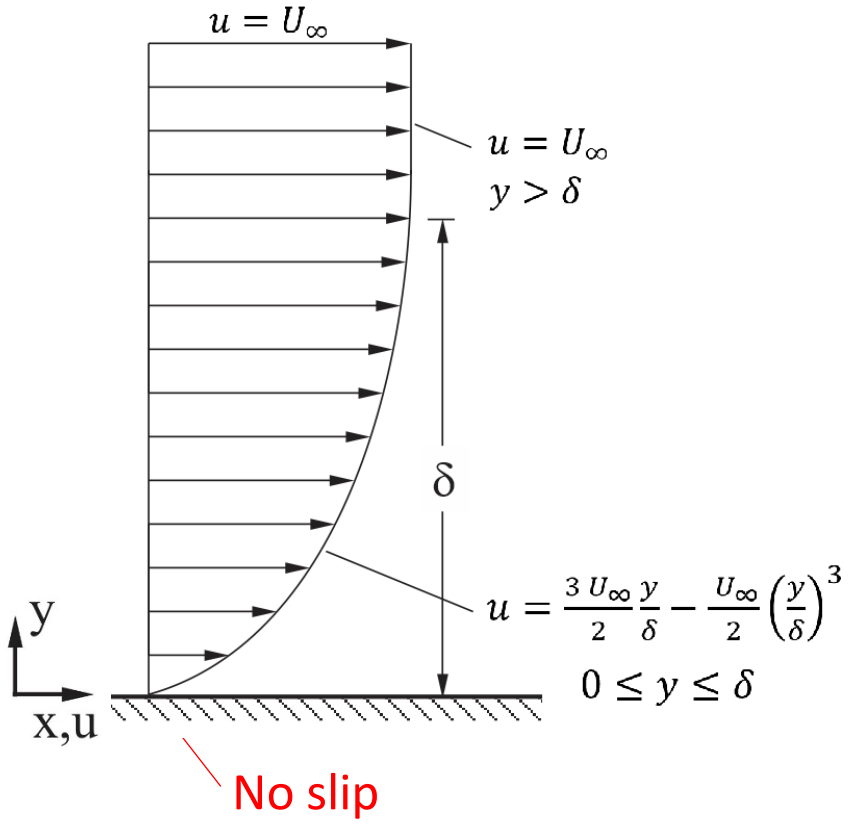
Example: Viscous Shear Stress in a Boundary Layer

When a fluid flows over a surface, the velocity is reduced near the surface due to the action of viscosity. The velocity profile is:

$$u = \frac{3\,U_\infty}{2}\frac{y}{\delta} - \frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3 \qquad 0 \le y \le \delta$$

$$u = U_\infty \qquad y > \delta$$

U_∞ is the freestream velocity
δ is the boundary layer thickness



Calculate:

(a) The magnitude and direction of fluid shear stress **<u>on</u>** the surface.
(b) The fluid shear stress at edge of the boundary layer (at $y = \delta$).
(c) For 20°C air at $U_\infty = 11\ m/s$, calculate the total shear force on a surface with area (one side) A=15 m^2. Assume the boundary layer thickness is δ=5.0 mm and constant over the entire surface

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Viscous Shear Stress in a Boundary Layer

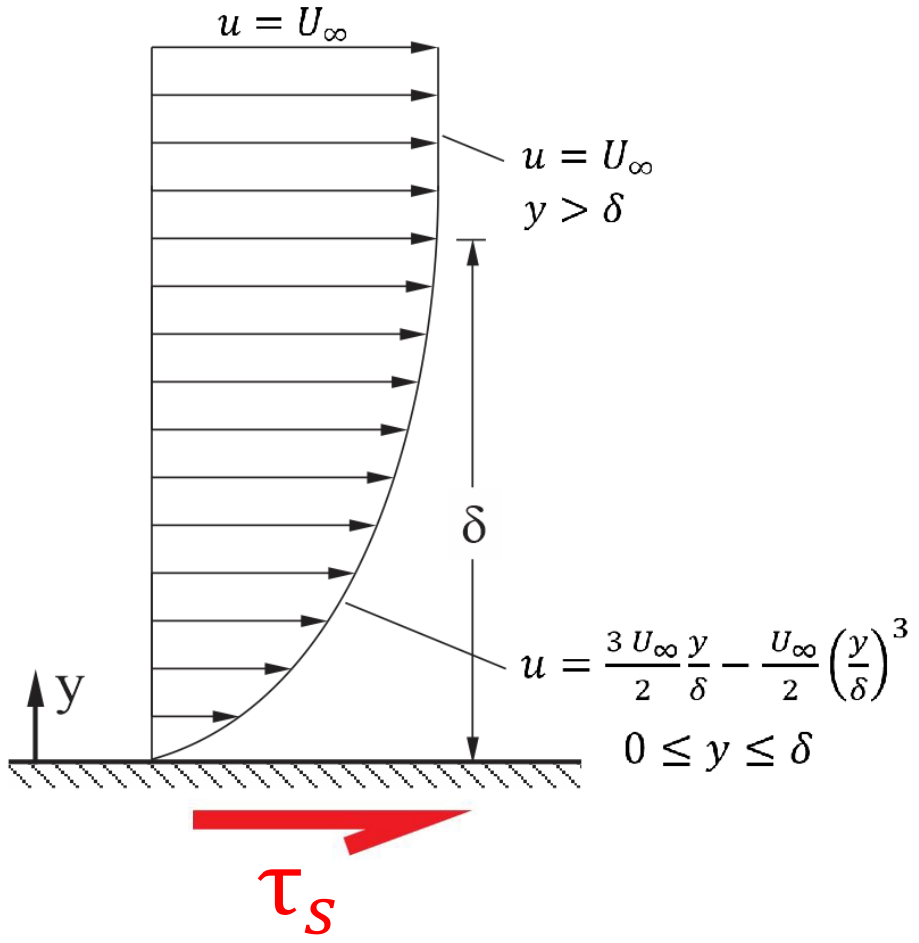
Solution

(a) Direction of the viscous shear stress (τ) on the surface?

- Intuitively, what is the direction of the force *on the surface*?

- The shear force exerted by the fluid ***on the surface*** will be in the flow direction

- The force *on the fluid* opposes fluid motion. This is analogous to friction



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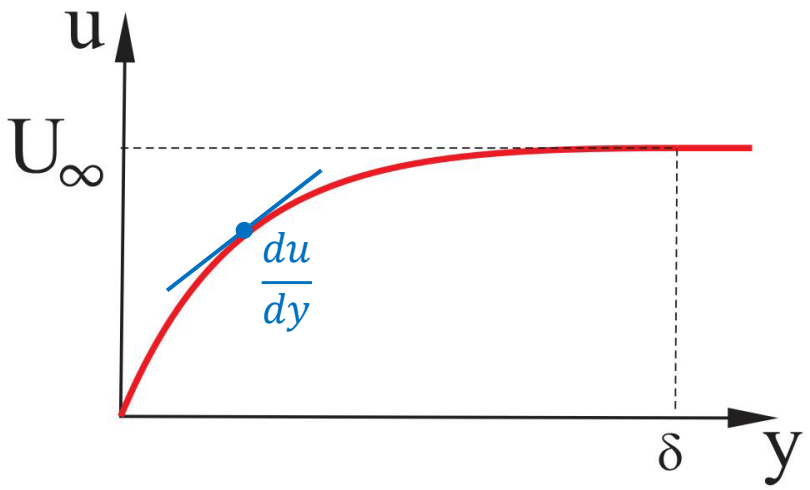
Viscous Shear Stress in a Boundary Layer

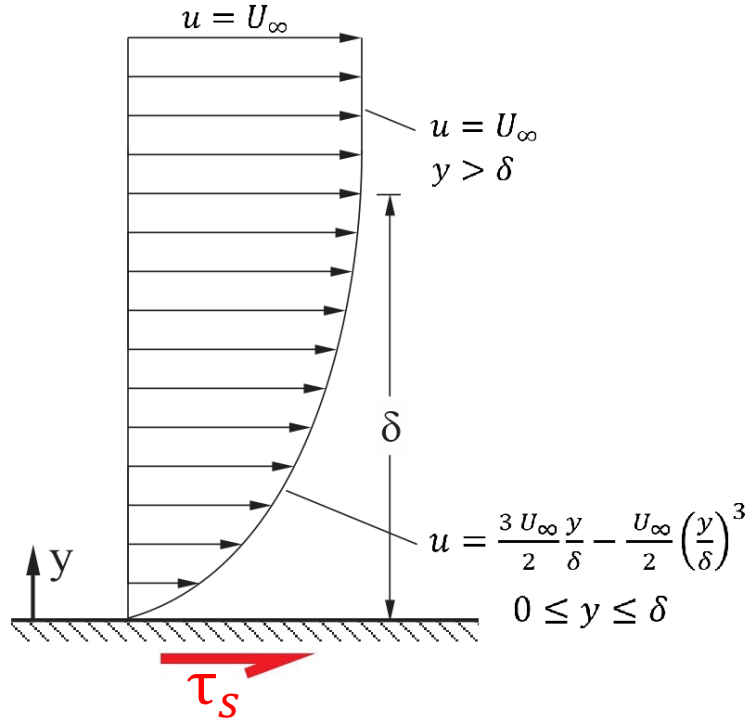
(a) Magnitude the viscous shear stress (τ) on the surface?

Local fluid shear stress: $\tau = \mu \frac{du}{dy}$

- Applies everywhere in the flow



Velocity Profile



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Viscous Shear Stress in a Boundary Layer

- We want the shear stress at the surface. i.e. at $y = 0$

$$\tau_s = \tau|_{y=0} = \mu \left.\frac{du}{dy}\right|_{y=0}$$

- Velocity profile is given as:

$$u = \frac{3\,U_\infty}{2}\frac{y}{\delta} - \frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3 \qquad 0 \le y \le \delta$$

- Differentiating:

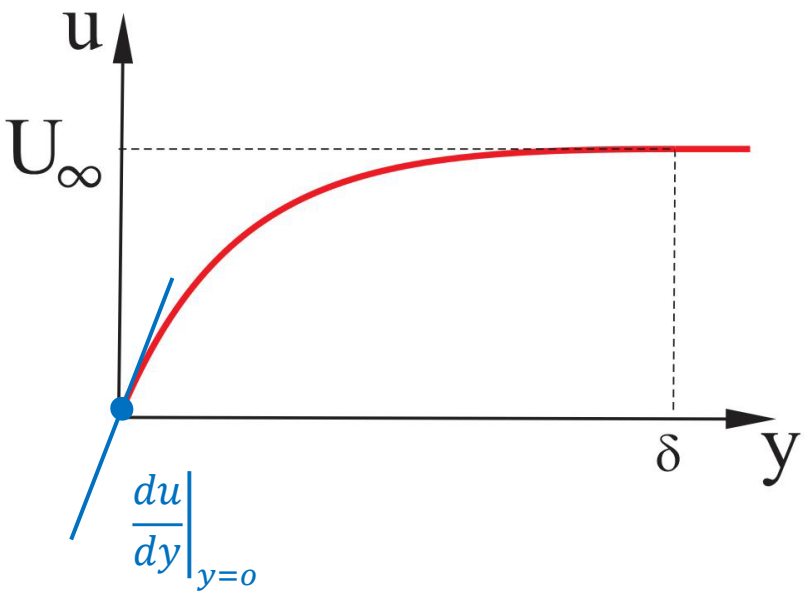
$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3}y^2 \qquad = \frac{3\,U_\infty}{2\delta}$$

y=0

- Thus, the shear stress on the surface:

$$\tau_s = \mu \left.\frac{du}{dy}\right|_{y=0} = \frac{3}{2}\frac{\mu\,U_\infty}{\delta}$$

Ans. (a)



Viscous Shear Stress in a Boundary Layer

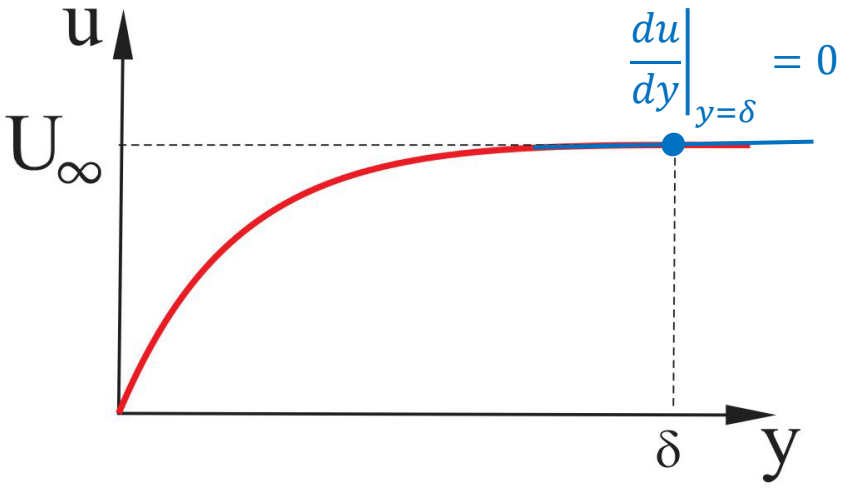
(b) Magnitude the viscous shear stress (τ) at y= δ?

$$\tau|_{y=\delta} = \mu \left.\frac{du}{dy}\right|_{y=\delta}$$

- Differentiation gave:

$$\frac{du}{dy} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3} y^2 \qquad y=\delta$$

$$\left.\frac{du}{dy}\right|_{y=\delta} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3}\delta^2 = 0$$



- Thus, the shear stress at the edge of the boundary layer

$$\tau|_{y=\delta} = \mu \left.\frac{du}{dy}\right|_{y=\delta} = 0$$ Ans. (b)

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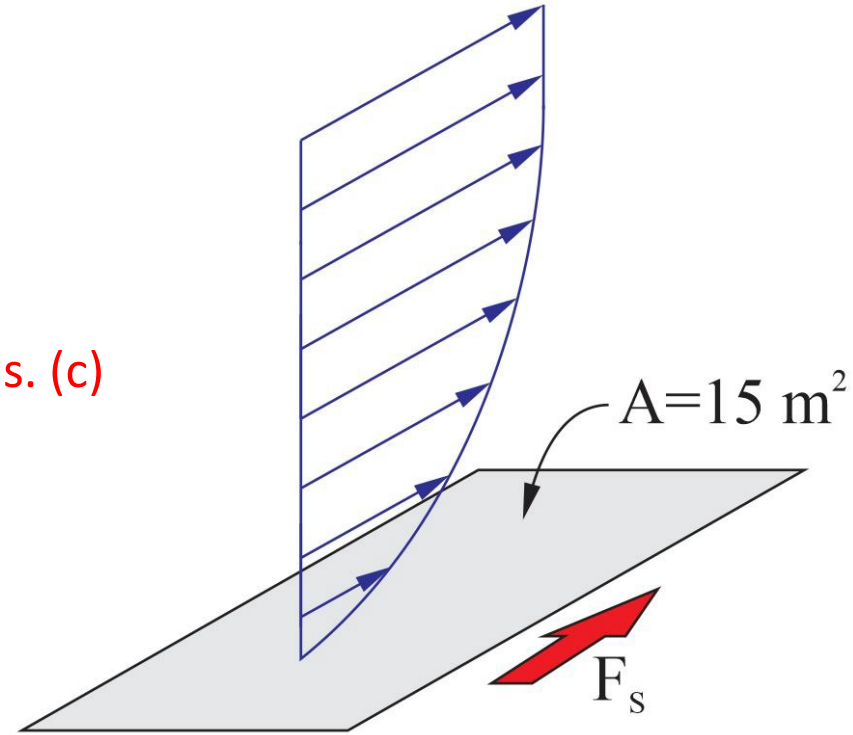
Viscous Shear Stress in a Boundary Layer

(c) For 20°C air at $U_\infty = 11\ m/s$, calculate the total shear force on the surface. Area A=15 m^2. Boundary layer thickness: δ=5.0 mm

From part (a): $\tau_s = \frac{3}{2}\frac{\mu\, U_\infty}{\delta}$

- Shear Force will be: $F_s = \tau_s A$ where $A = 15\ m^2$ (plan area)
- Dynamic viscosity of air at 20°C (Table A.2): $\mu = 1.80x10^{-5}\ \frac{Ns}{m^2}$

$$F_s = \tau_s A = \frac{3}{2}\frac{\mu\, U_\infty A}{\delta} = \frac{3}{2}\left(1.80x10^{-5}\ \frac{Ns}{m^2}\right) 11\,\frac{m}{s}\left(\frac{15\ m^2}{0.005m}\right) = 0.891\ N$$ Ans. (c)



- This viscous shear force is sometimes called *Skin Friction*
- *Skin friction* is one source of drag on your car. It is the drag force caused by the viscosity of the air

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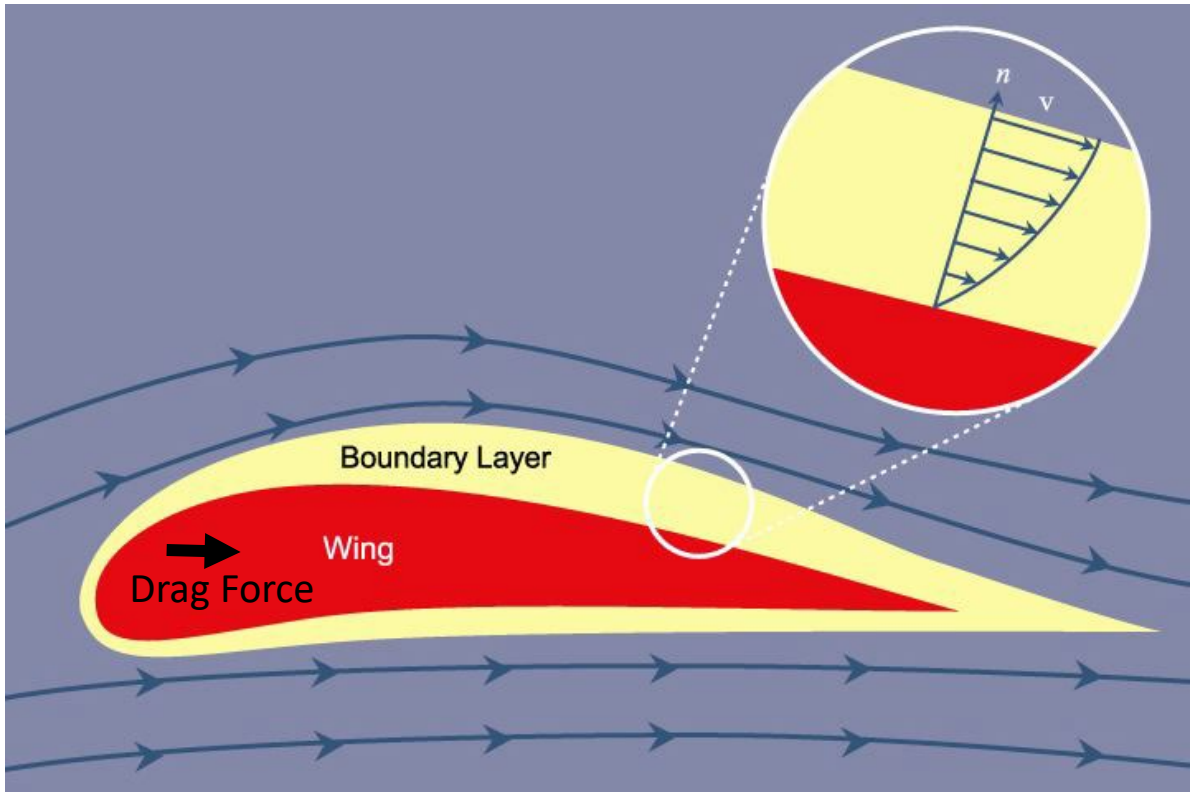

Skin friction can be the main source of drag on highly streamlined bodies, such as an airplane wing (Image credit: https://howthingsfly.si.edu/aerodynamics/friction-drag)

END NOTES

Presentation prepared and delivered by Dr. David Naylor

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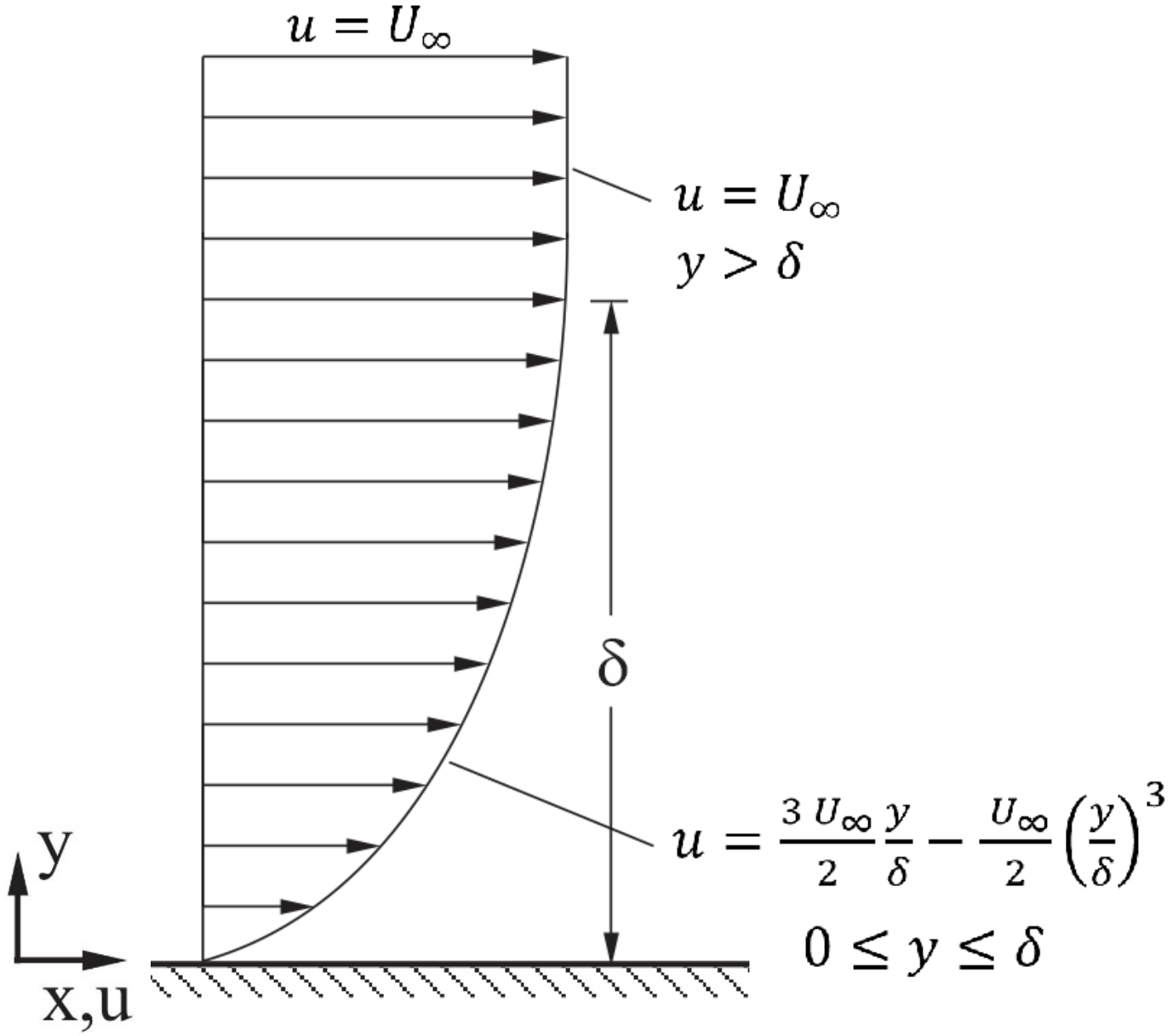
Toronto Metropolitan University

Solved Example:

Viscous Shear Stress in a Boundary Layer