MEC516/BME516: Fluid Mechanics I

Viscous Shear Stress in Boundary Layer Solved Example

Department of Mechanical & Industrial Engineering

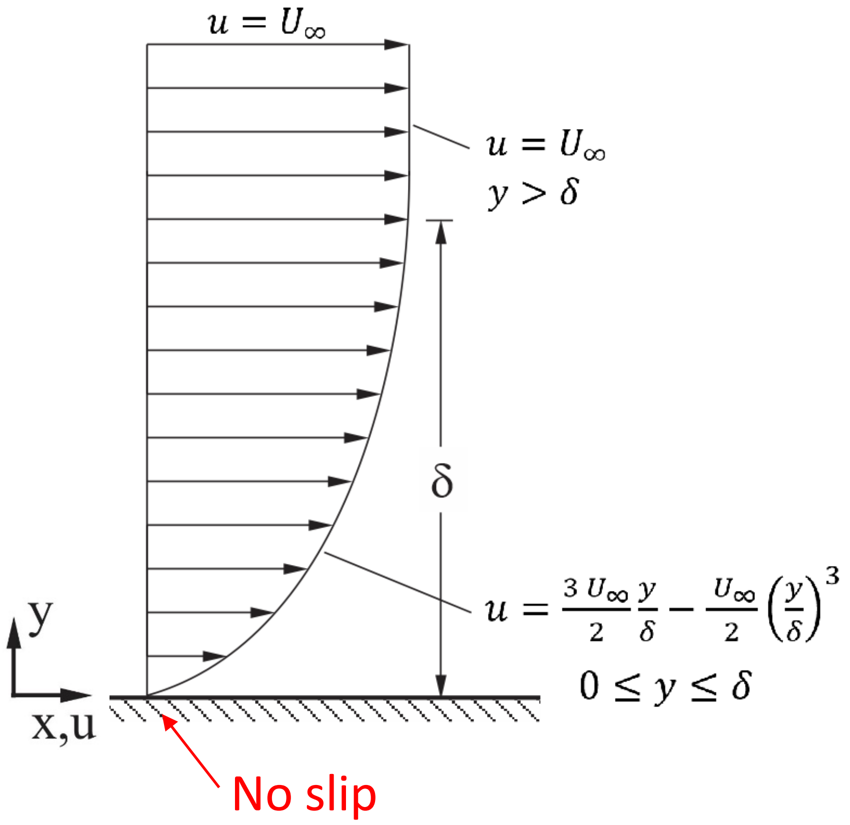
Example: Viscous Shear Stress in a Boundary Layer

When a fluid flows over a surface, the velocity is reduced near the surface due to the action of viscosity. The velocity profile is:

$$u = \frac{3}{2}\frac{U_\infty}{} \frac{y}{\delta} - \frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3 \qquad 0 \le y \le \delta$$

$$u = U_\infty \qquad y > \delta$$

U_∞ is the freestream velocity
δ is the boundary layer thickness



Calculate:

(a) The magnitude and direction of fluid shear stress **<u>on</u>** the surface.
(b) The fluid shear stress at edge of the boundary layer (at $y = \delta$).
(c) For 20°C air at $U_\infty = 11\ m/s$, calculate the total shear force on a surface with area (one side) A=15 m^2. Assume the boundary layer thickness is δ=5.0 mm and constant over the entire surface

Viscous Shear Stress in a Boundary Layer

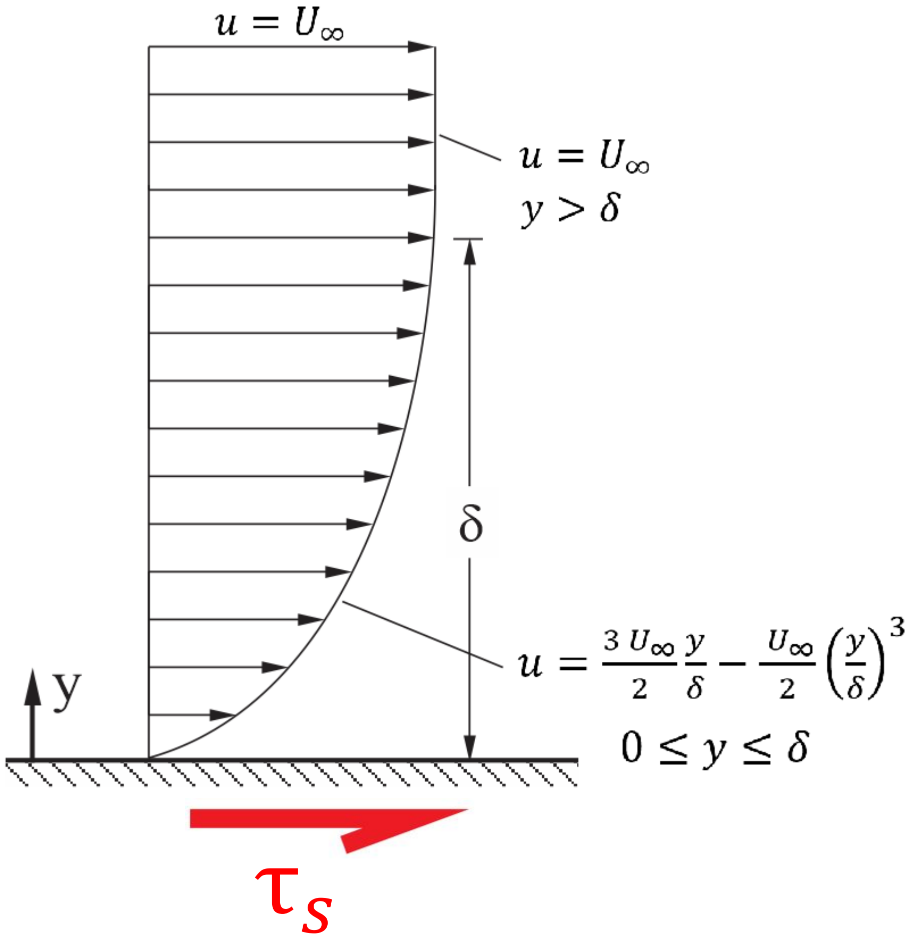
Solution

(a) Direction of the viscous shear stress (τ) on the surface?

- Intuitively, what is the direction of the force *on the surface*?

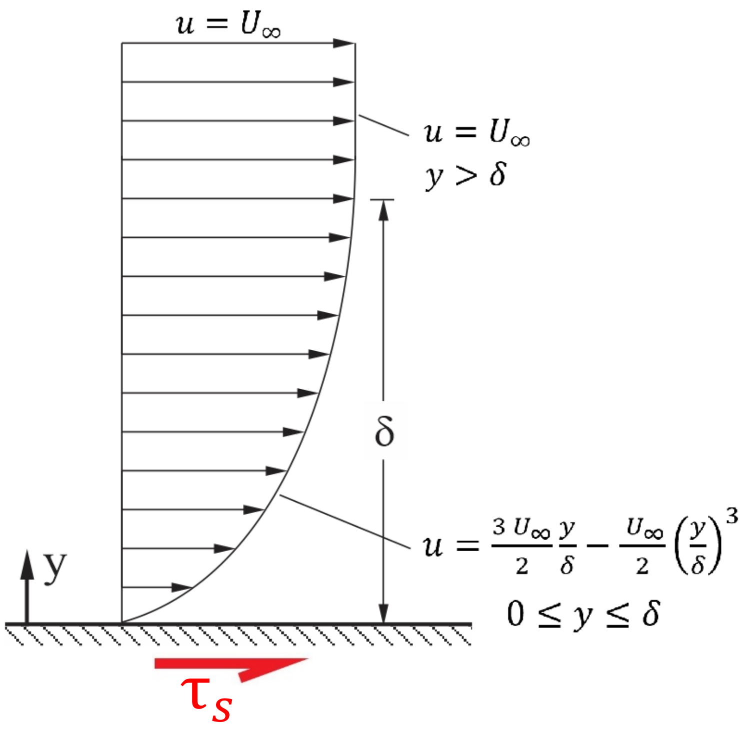
- The shear force exerted by the fluid ***on the surface*** will be in the flow direction

- The force *on the fluid* opposes fluid motion. This is analogous to friction



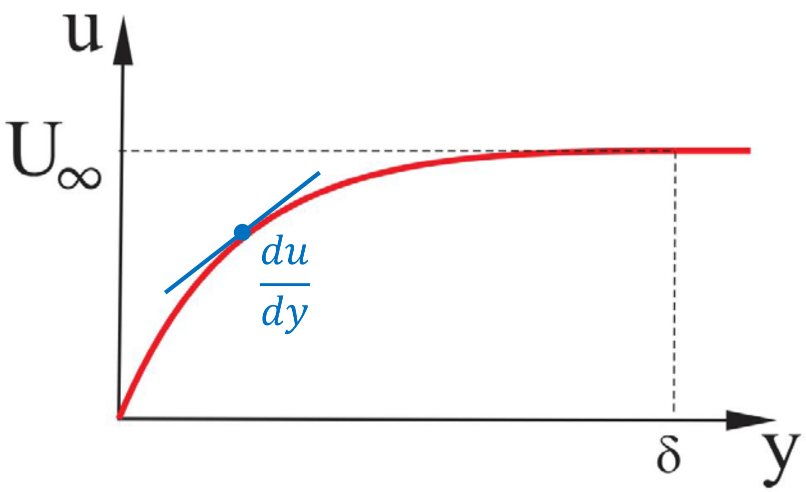
Viscous Shear Stress in a Boundary Layer

(a) Magnitude the viscous shear stress (τ) on the surface?



Local fluid shear stress: $\tau = \mu \dfrac{du}{dy}$

- Applies everywhere in the flow



Velocity Profile

Viscous Shear Stress in a Boundary Layer

- We want the shear stress at the surface. i.e. at $y = 0$

$$\tau_s = \tau|_{y=0} = \mu \left.\frac{du}{dy}\right|_{y=0}$$

- Velocity profile is given as:

$$u = \frac{3\,U_\infty}{2}\frac{y}{\delta} - \frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3 \qquad 0 \le y \le \delta$$

- Differentiating:

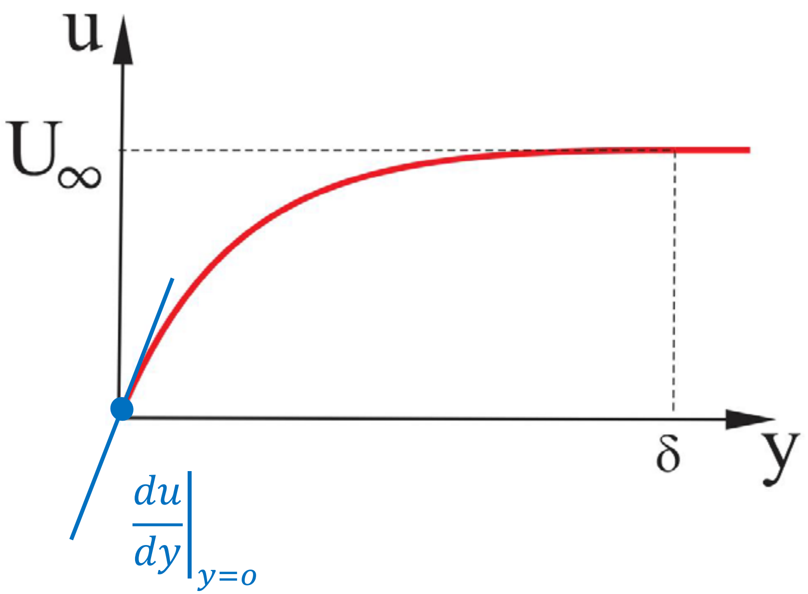
y=0

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3}y^2 \qquad = \frac{3\,U_\infty}{2\delta}$$

- Thus, the shear stress on the surface:

$$\tau_s = \mu \left.\frac{du}{dy}\right|_{y=0} = \frac{3}{2}\frac{\mu\,U_\infty}{\delta}$$

Ans. (a)



Viscous Shear Stress in a Boundary Layer

(b) Magnitude the viscous shear stress (τ) at y= δ?

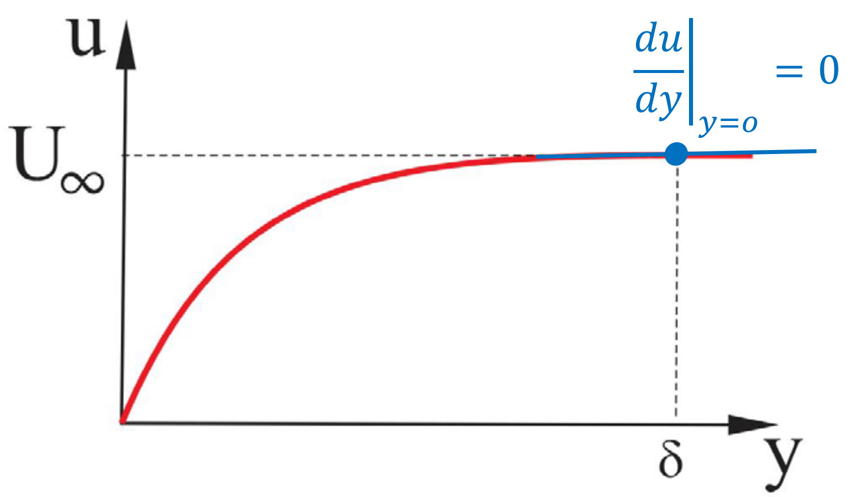
$$\tau|_{y=\delta} = \mu \left.\frac{du}{dy}\right|_{y=\delta}$$

- Differentiation gave:

y=δ

$$\frac{du}{dy} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3} y^2$$

$$\left.\frac{du}{dy}\right|_{y=\delta} = \frac{3\,U_\infty}{2\delta} - \frac{3\,U_\infty}{2\delta^3}\delta^2 = 0$$



- Thus, the shear stress at the edge of the boundary layer

$$\tau|_{y=\delta} = \mu \left.\frac{du}{dy}\right|_{y=\delta} = 0$$ Ans. (b)

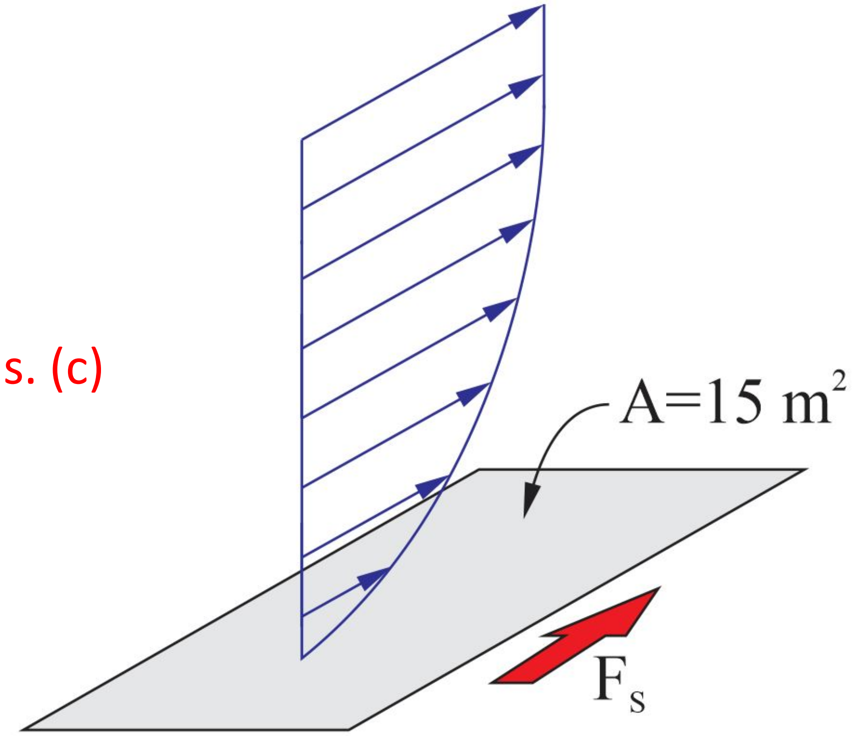
Viscous Shear Stress in a Boundary Layer

(c) For 20°C air at $U_\infty = 11\ m/s$, calculate the total shear force on the surface. Area A=15 m^2. Boundary layer thickness: δ=5.0 mm

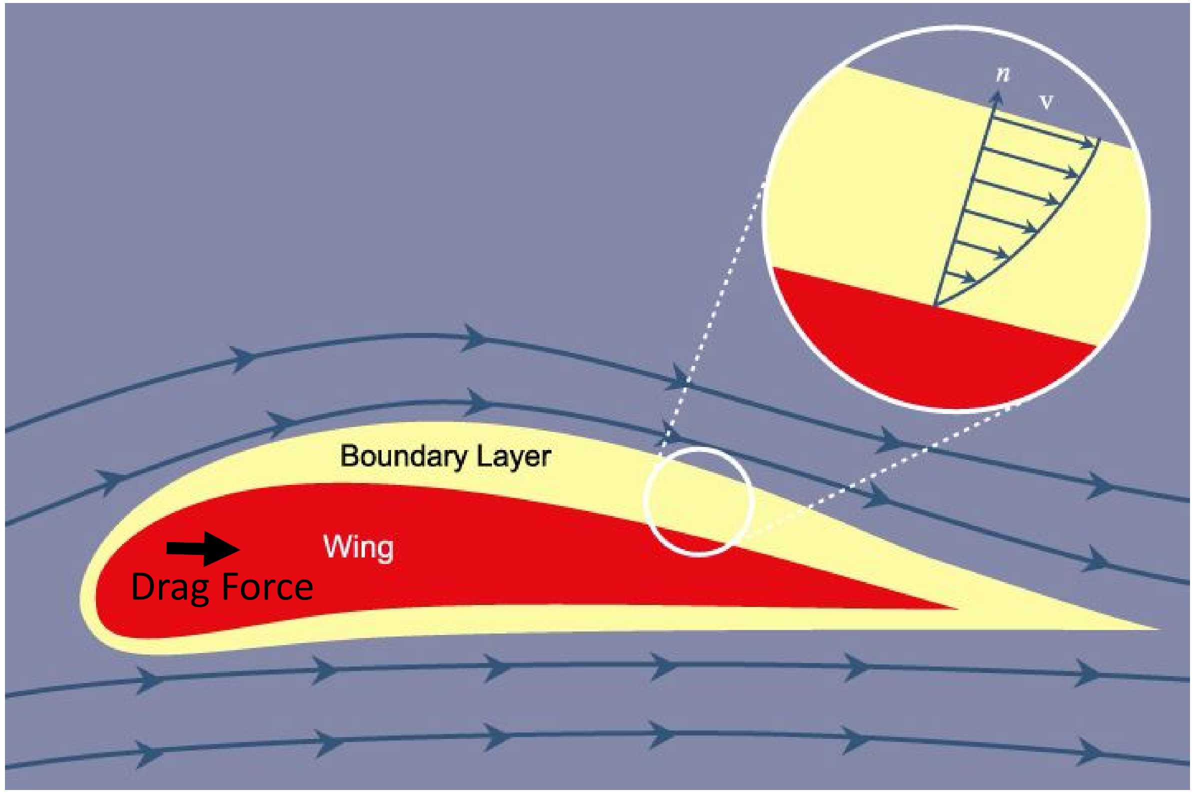
From part (a): $\tau_s = \frac{3}{2}\frac{\mu\, U_\infty}{\delta}$

- Shear Force will be: $F_s = \tau_s A$ where $A = 15\ m^2$ (plan area)
- Dynamic viscosity of air at 20°C (Table A.2): $\mu = 1.80x10^{-5}\ \frac{Ns}{m^2}$

$$F_s = \tau_s A = \frac{3}{2}\frac{\mu\, U_\infty A}{\delta} = \frac{3}{2}\left(1.80x10^{-5}\ \frac{Ns}{m^2}\right) 11\frac{m}{s}\left(\frac{15\ m^2}{0.005m}\right) = 0.891\ N$$ Ans. (c)



- This viscous shear force is sometimes called *Skin Friction*
- *Skin friction* is one source of drag on your car. It is the drag force caused by the viscosity of the air



Skin friction can be the main source of drag on highly streamlined bodies, such as an airplane wing (Image credit: https://howthingsfly.si.edu/aerodynamics/friction-drag)

END NOTES

Presentation prepared and delivered by Dr. David Naylor