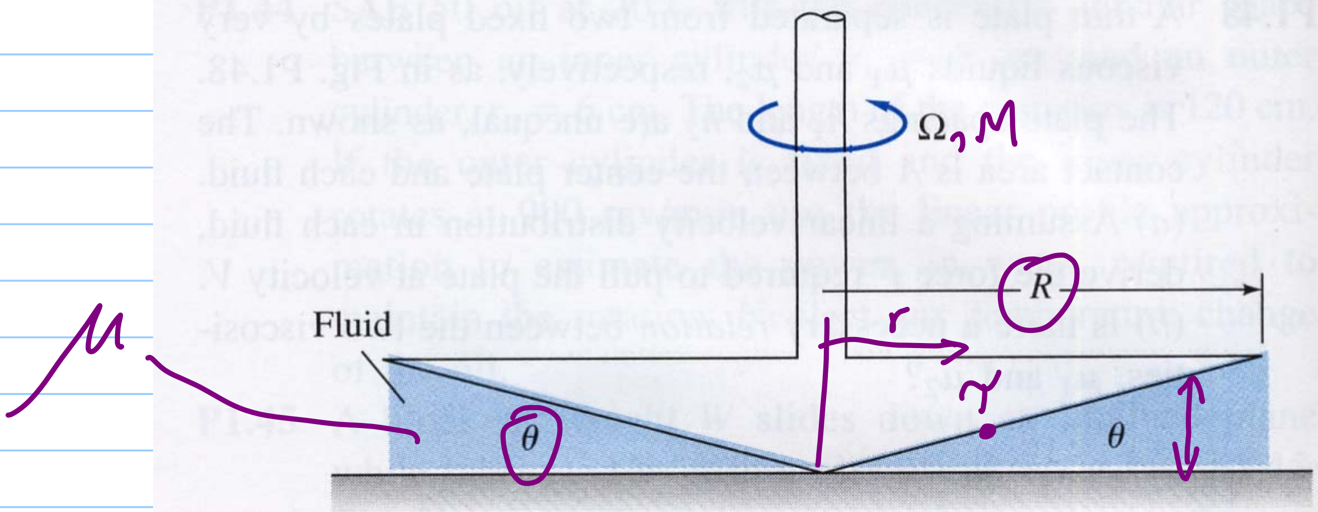
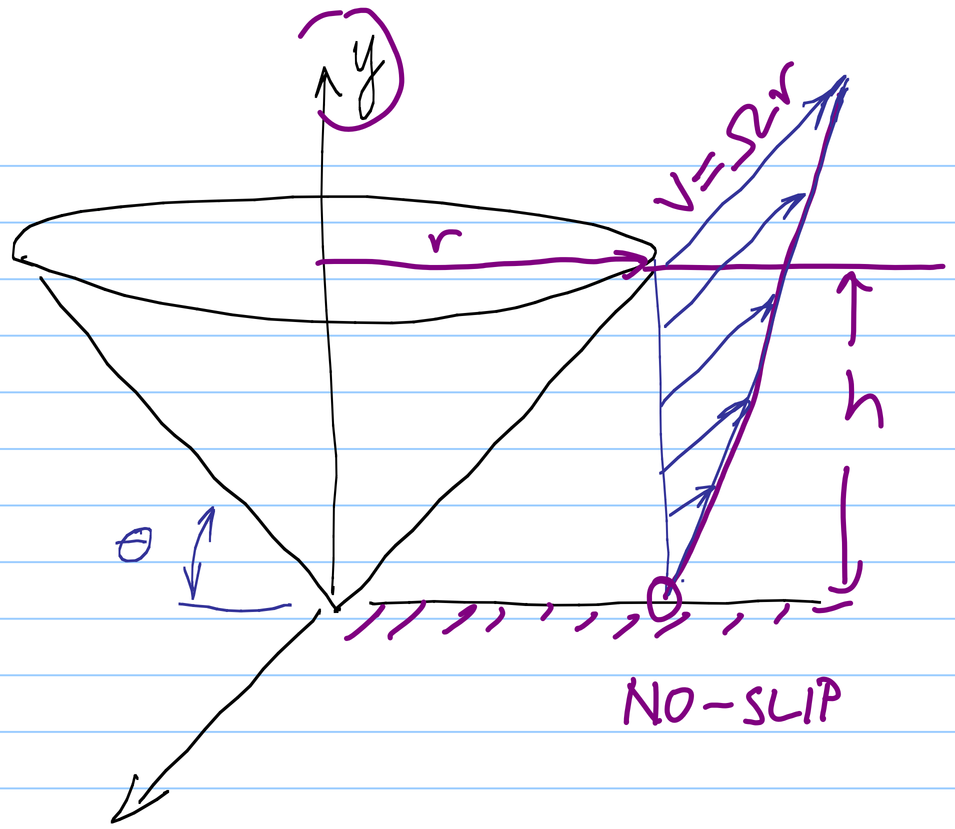
PROBLEM P1.56

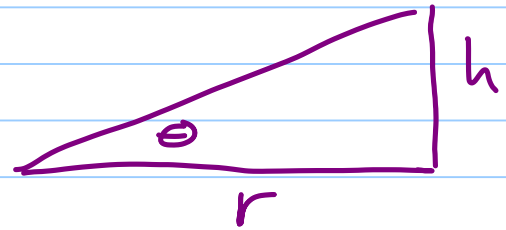
Note Title 18/05/2014

*P1.56 The device in Fig. P1.56 is called a *cone-plate viscometer* [29]. The angle of the cone is very small, so that $\sin \theta \approx \theta$, and the gap is filled with the test liquid. The torque M to rotate the cone at a rate Ω is measured. Assuming a linear velocity profile in the fluid film, derive an expression for fluid viscosity μ as a function of (M, R, Ω, θ).





$$\frac{du}{dy} = \frac{V}{h} = \frac{\Omega r}{h}$$

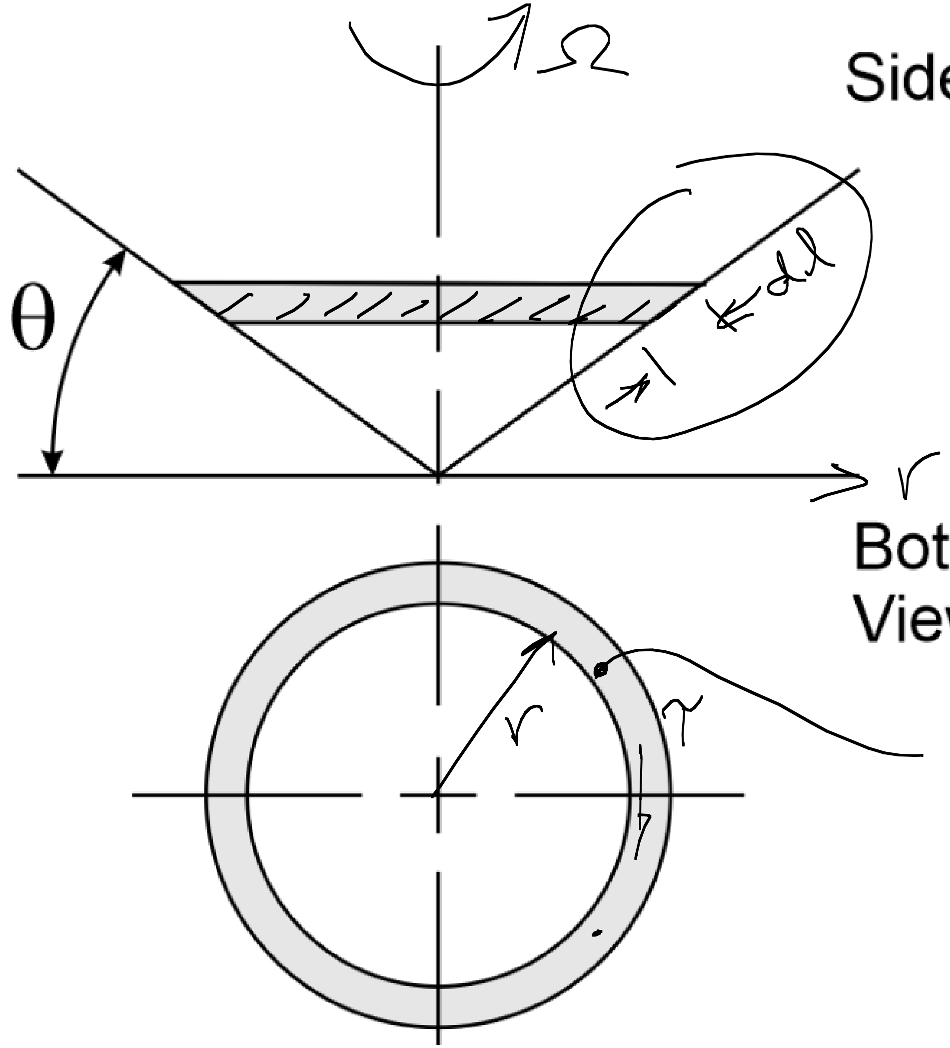


$$\tan\theta = \frac{h}{r} \qquad h = r\tan\theta$$

LOCAL SHEAR STRESS τ

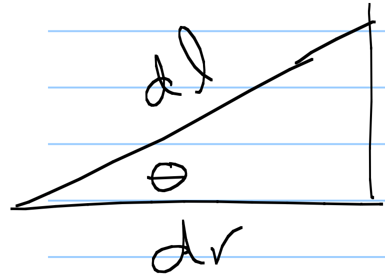
$$\frac{du}{dy} = \frac{\Omega \cancel{r}}{\cancel{r}\tan\theta} = \frac{\Omega}{\tan\theta}$$

$$\tau = \mu\frac{du}{dy} = \frac{\mu\,\Omega}{\tan\theta}$$



Side View

Bottom View



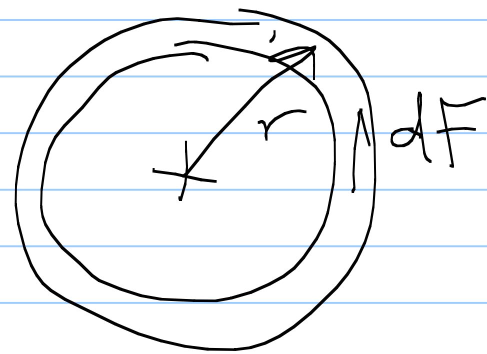
$$dr = dl\cos\theta$$

$$dA = 2\pi r\,dl = \frac{2\pi r\,dr}{\cos\theta}$$

SHEAR FORCE $dF = \tau \, dA = \frac{\mu \Omega}{\tan\theta} \frac{2\pi r \, dr}{\cos\theta}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$dF = \frac{2\pi \mu \Omega r \, dr}{\sin\theta}$$



SHEAR MOMENT $dM = dF \cdot r$

$$dM = \frac{2\pi \mu \Omega r^2 \, dr}{\sin\theta}$$

TOTAL MOMENT

$$M = \int_0^R dM$$

$$M = \frac{2\pi\mu\Omega}{\sin\theta}\int_0^R r^2\,dr = \frac{2\pi\mu\Omega}{\sin\theta}\left[\frac{r^3}{3}\right]_0^R$$

$$M = \frac{2\pi\mu\Omega R^3}{3\sin\theta}$$

SOLVE FOR μ: $\quad \mu = \frac{3M\sin\theta}{2\pi\Omega R^3}$

SMALL θ $\quad \sin\theta \cong \theta$ $\quad \boxed{\mu = \frac{3M\theta}{2\pi\Omega R^3}}$ ANS.