MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis

Draining Tank Puzzle

Toronto Metropolitan University

Fluid Mechanics Puzzle:

Consider two draining tanks:

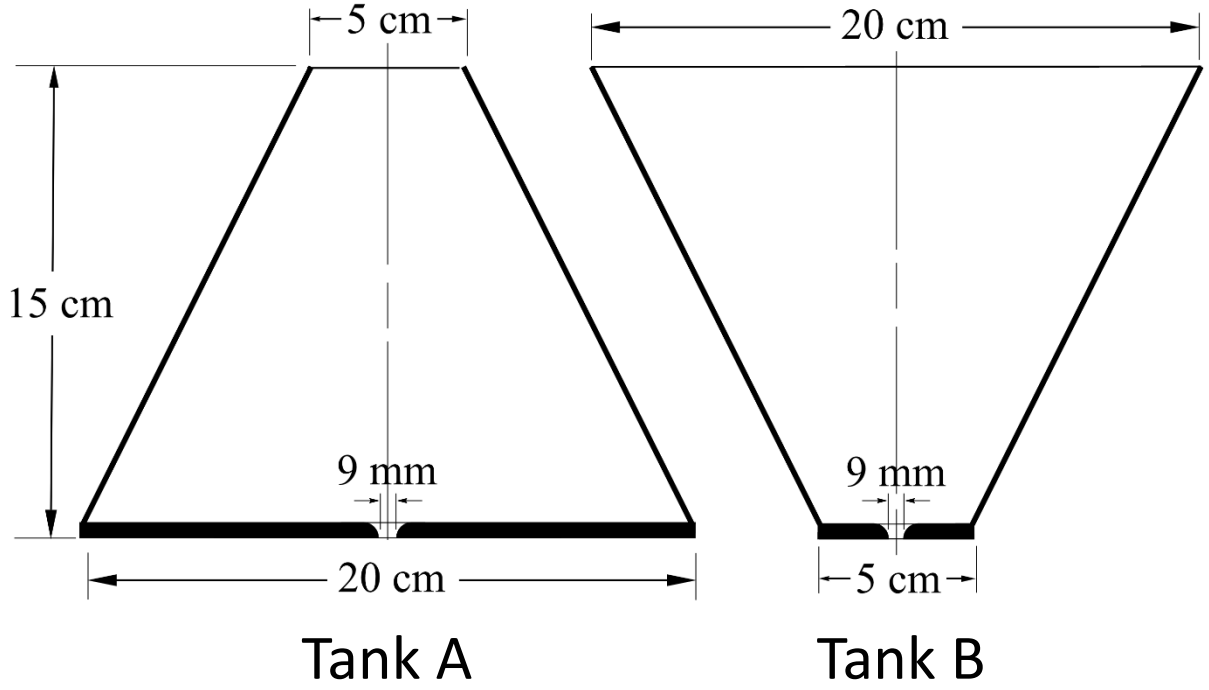
- Same total volume of water
- Same starting depth
- ***Only*** difference is the taper

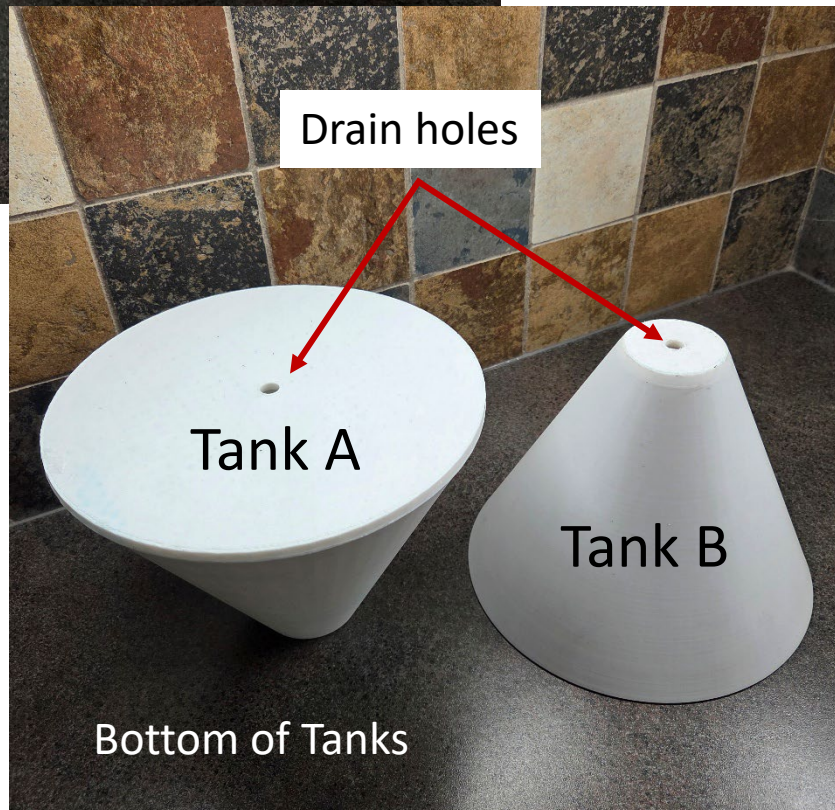
Which Tank Will Empty First?

- ☐ Tank A
- ☐ Tank B
- ☐ Both Same

Draining Tank Puzzle

- Demonstration with two 3-D printed tanks





Unsteady Discharge Rate

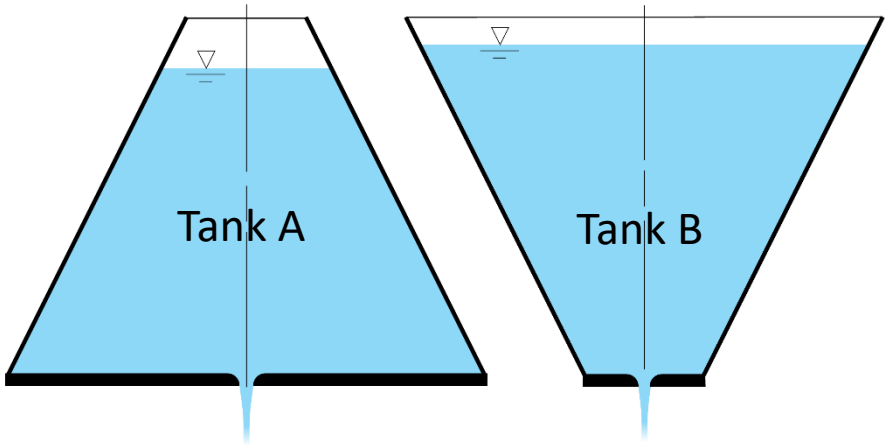
- Modified Torricelli's law (1644):

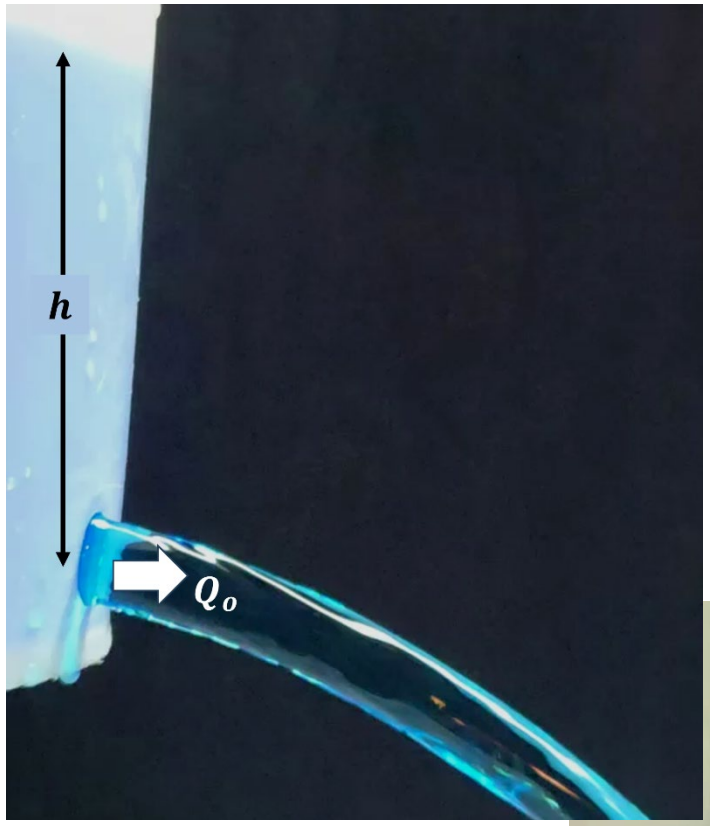
Discharge coefficient (viscous flow)

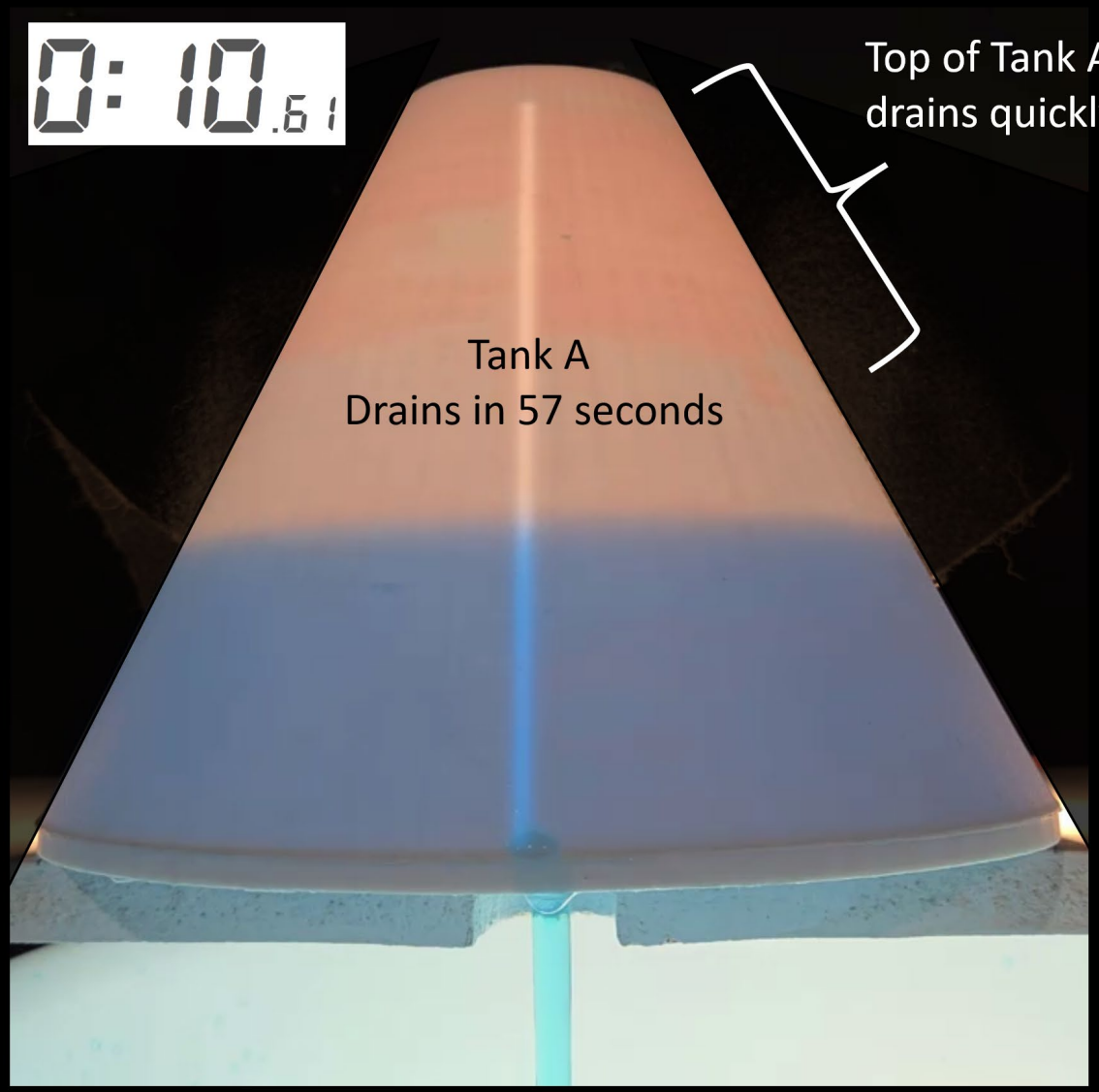
$$V = C_D\sqrt{2gh}$$

- Unsteady discharge volume flow rate:

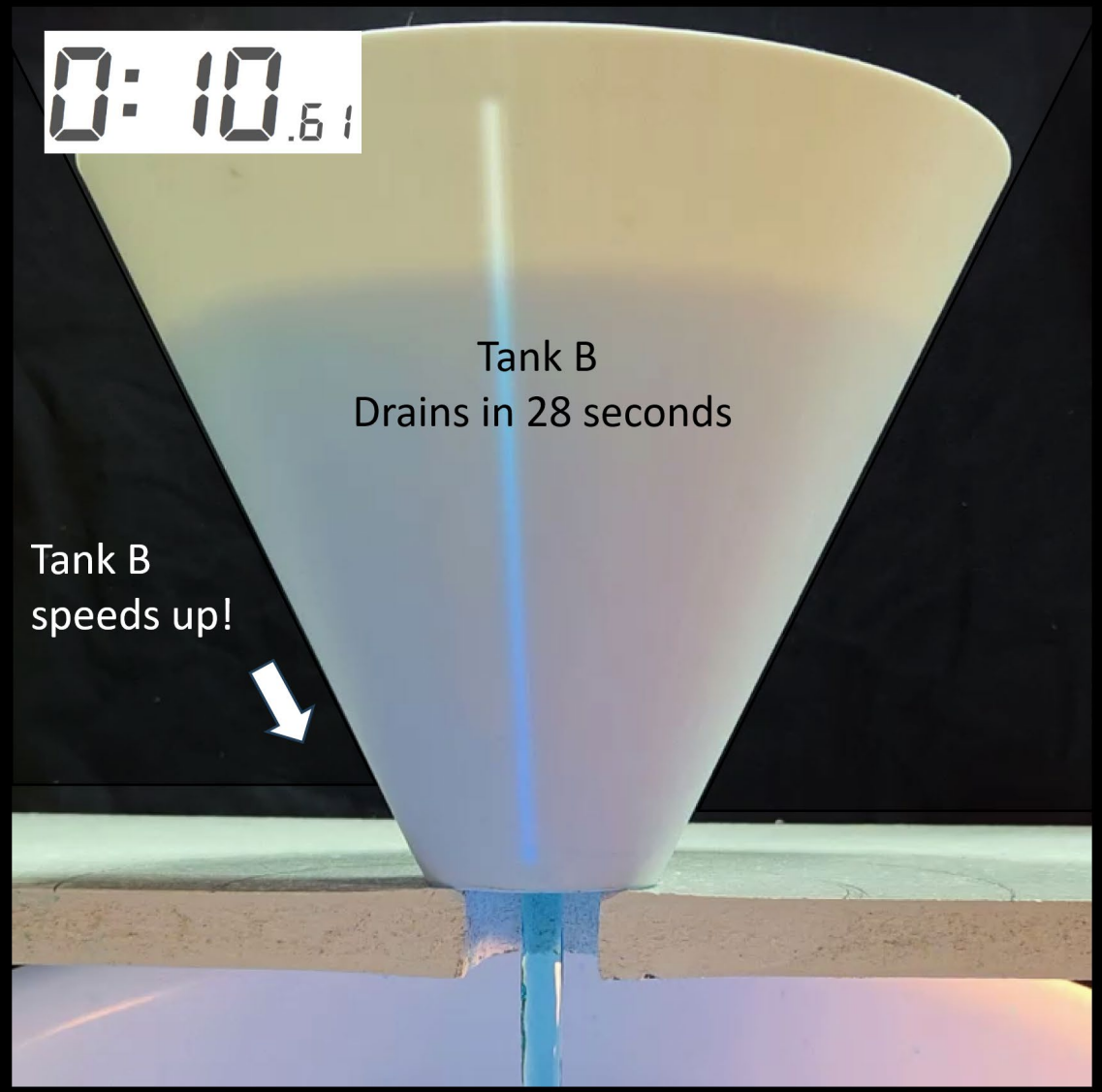
$$Q = VA_o \propto h^{1/2}$$



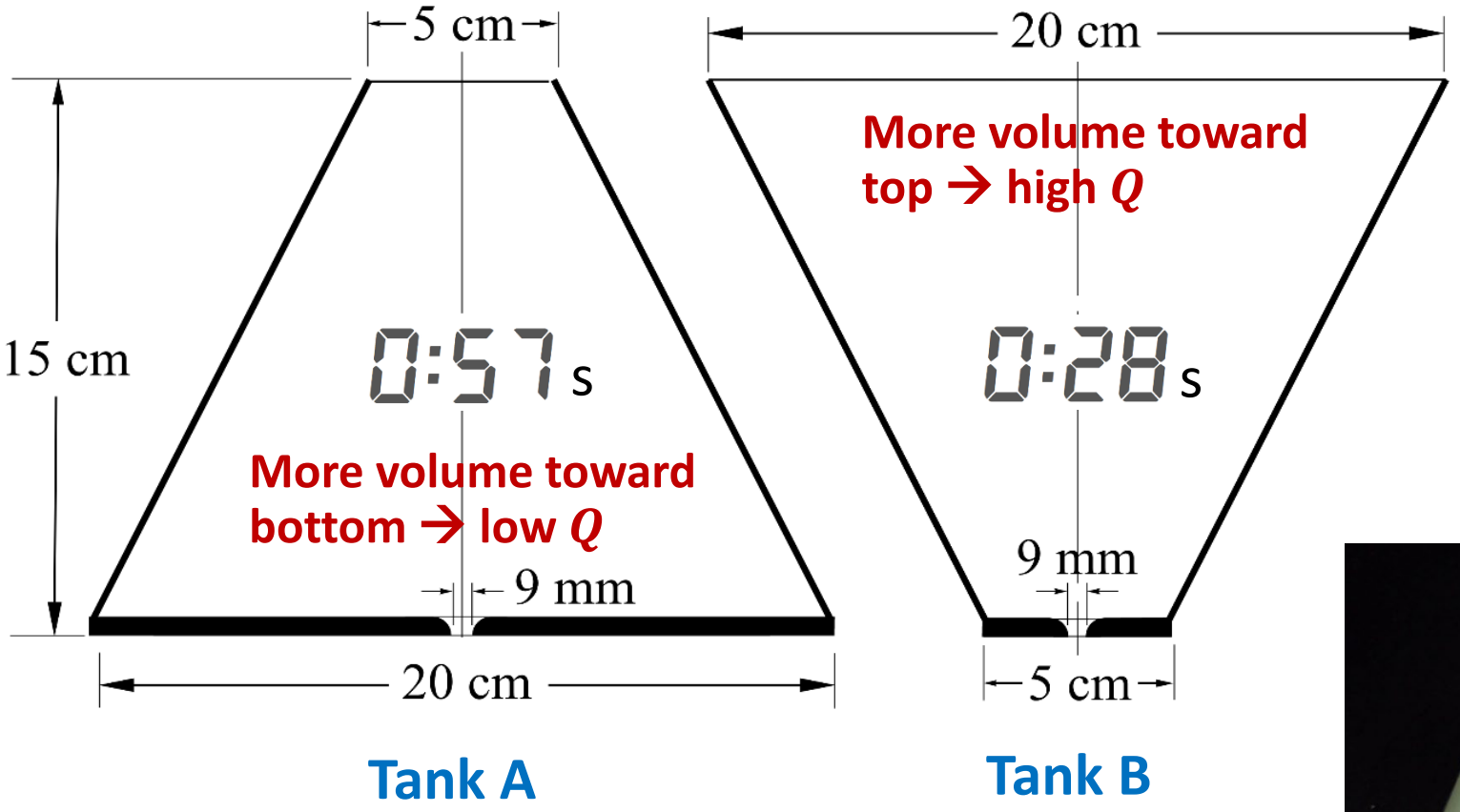


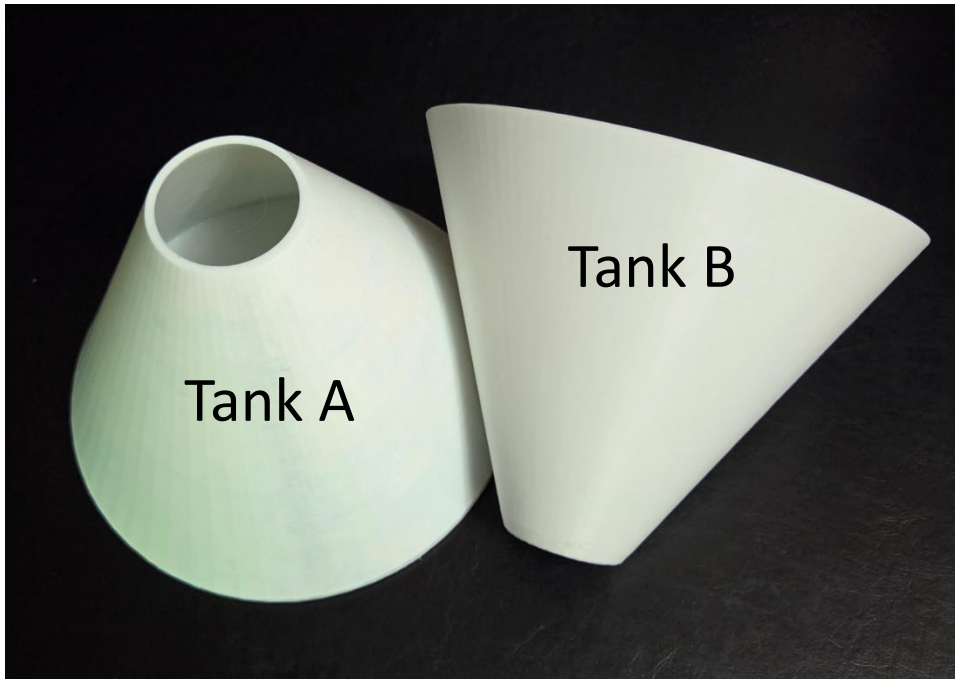

0: 10.61
Top of Tank A
drains quickly!
Tank A
Drains in 57 seconds

Tank A


0: 10.61
Tank B
Drains in 28 seconds
Tank B
speeds up!

Tank B





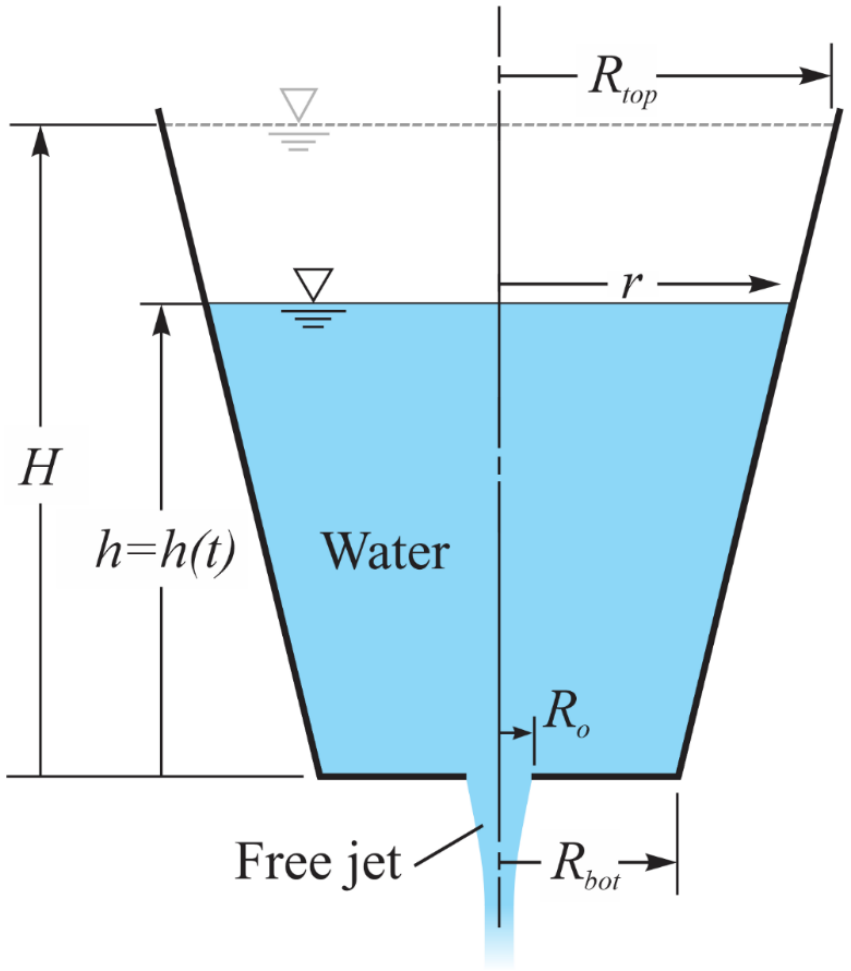
- Measured Time Ratio: $\frac{t_{Tank\,A}}{t_{Tank\,B}} = \frac{57s}{28s} = 2.0 \pm 0.2$

- Closed-form solution for time to fully drain:

$$t = \frac{1}{R_o^2 C_D \sqrt{2g}} \left(2R_{bot}^2 H^{\frac{1}{2}} + \frac{4}{3} \left(\frac{R_{top} - R_{bot}}{H} \right) R_{bot} H^{\frac{3}{2}} + \frac{2}{5} \left(\frac{R_{top} - R_{bot}}{H} \right)^2 H^{\frac{5}{2}} \right)$$

- Theoretical Time Ratio: $\frac{t_{Tank\,A}}{t_{Tank\,B}} = 2.04$

- Next video presents the mathematical analysis…



END NOTES

Presentation prepared and delivered by Professor David Naylor

- *Many thanks to Jeremy Catania for 3-D printing*

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Fluid Mechanics Puzzle:

Which Tank Will Empty First?

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