Two Solved Examples

Calculation of the *form drag*

- Example 1
  Drag force on a sphere in water

- Example 2
  Drag force on a sports car

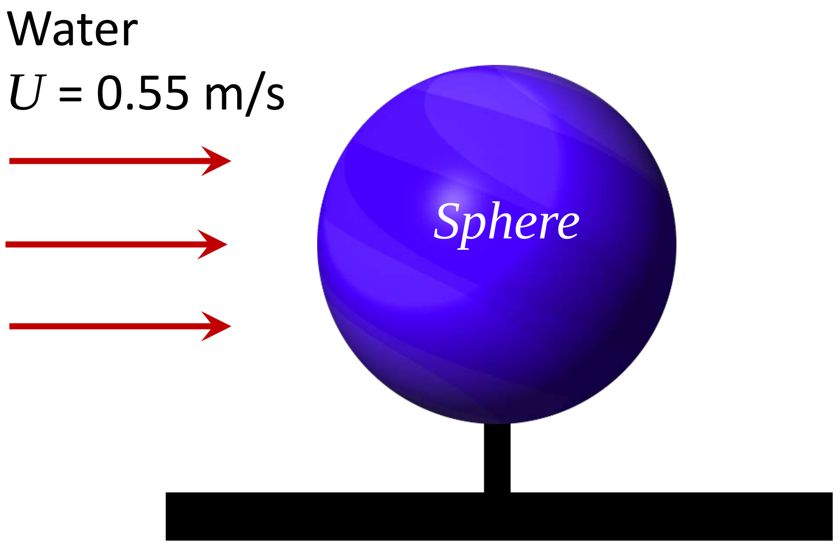


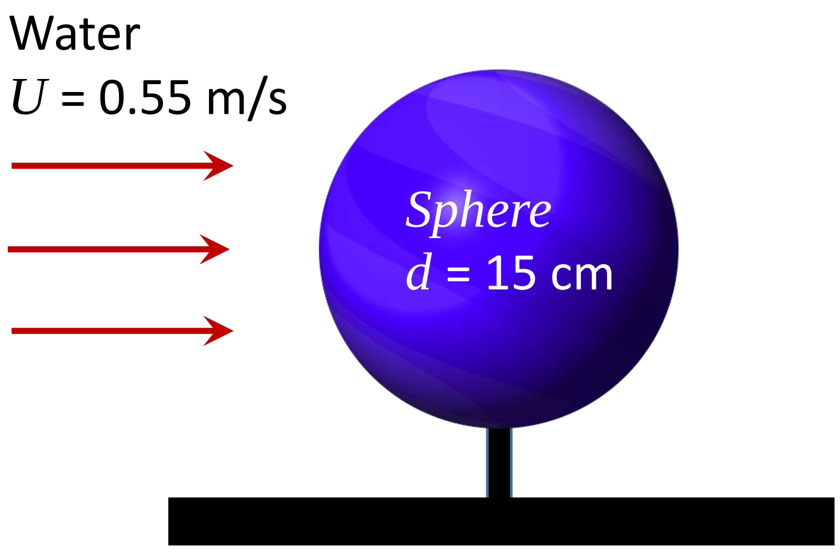
photo credit: www.porche.com

Example 1

Water at 20°C flows at U=0.55 m/s over a fixed sphere with an outside diameter of d=15.0 cm. Calculate:

(a) the drag coefficient (C_D).

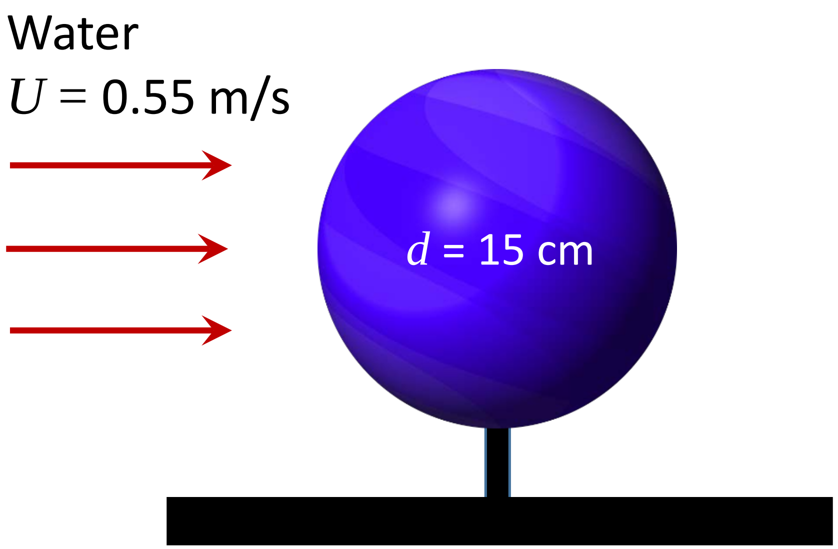
(b) the drag force on the sphere.



Example 1

(a) The drag coefficient is a dimensionless parameter:

$$C_D = \frac{F_D}{\underbrace{\frac{1}{2}\rho U^2 A_{proj}}_{\text{stagnation pressure force}}}$$



where F_D is the drag force

A_{proj} is the projected frontal area, taken in the direction of the flow

Drag force F_D is called the *form drag* or *pressure drag*.

This force does not include the fluid shear stress at the surface (skin friction, $\tau_w = \mu du/dy$). Skin friction is usually small, except on highly streamlined objects (like airfoils).

Example 1

From dimensional analysis, for a fixed geometry:

$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A_{proj}} = f(Re)$$

C_D is only a function of Reynolds number.

So, we start by calculating the Reynolds number.

$$Re = \frac{\rho U d}{\mu}$$

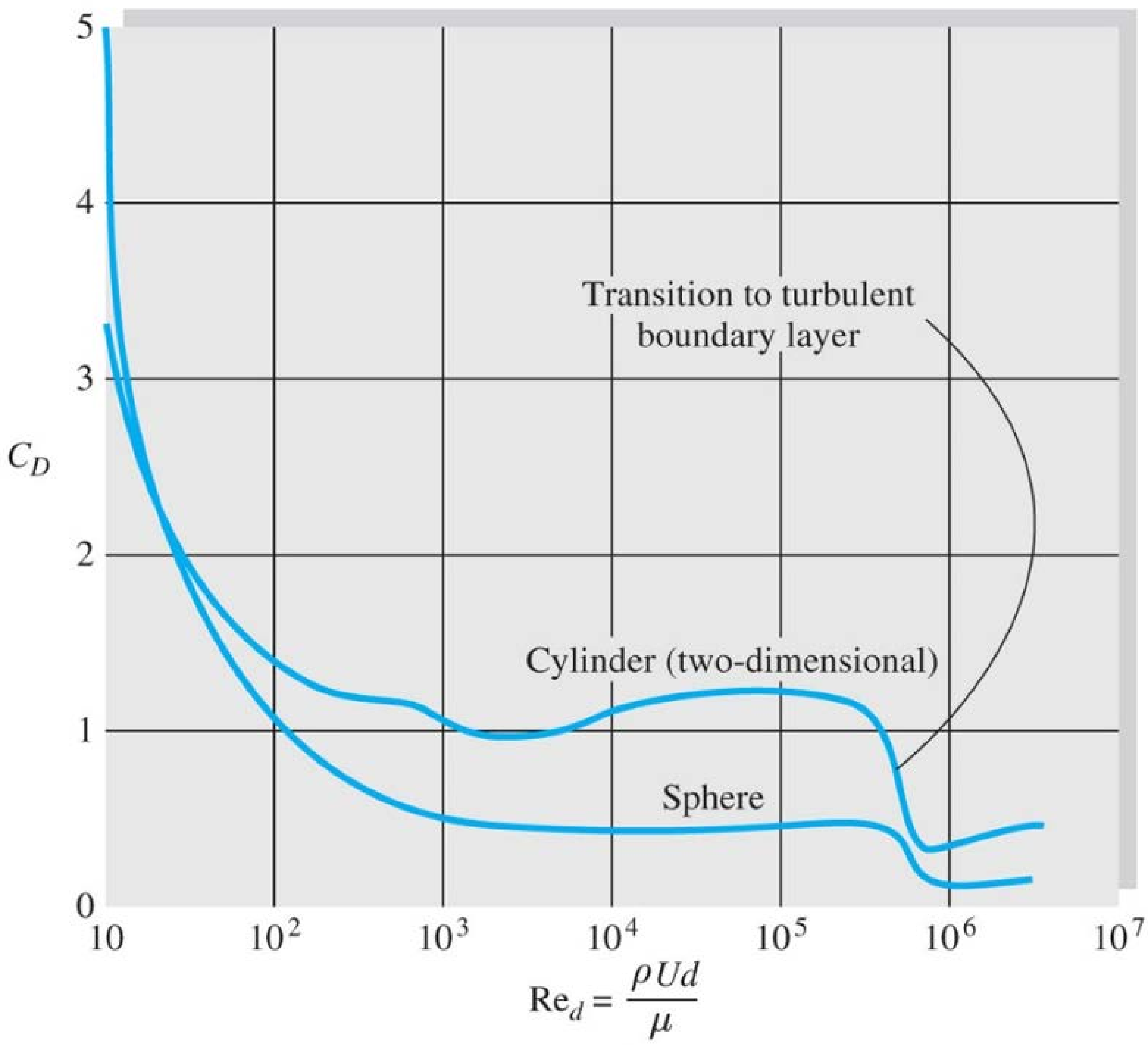


Figure: Curves from experimental measurements

Example 1

Reynolds number: $Re = \frac{\rho U d}{\mu}$

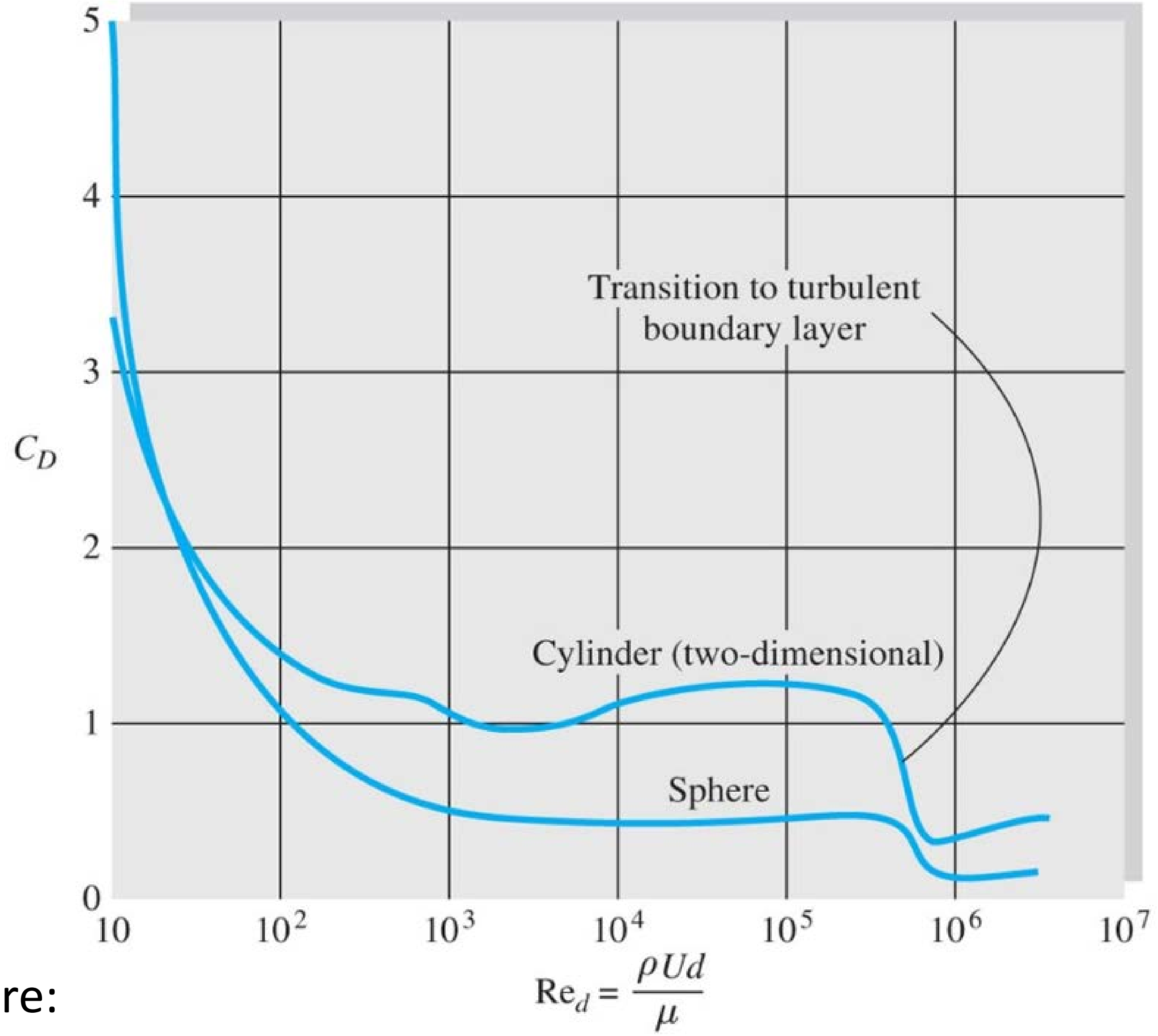
Water at 20°C: ρ = 998 kg/m^3

μ = 1.003x10^{-3} kg/(ms)

$$Re = \frac{998\,\frac{kg}{m^3}\left(0.55\,\frac{m}{s}\right)0.15m}{1.003x10^{-3}\;\frac{kg}{ms}} = 8.2\;x10^4$$

From the experimental measurements for a sphere:

$C_D \approx 0.45$ Ans. (a)

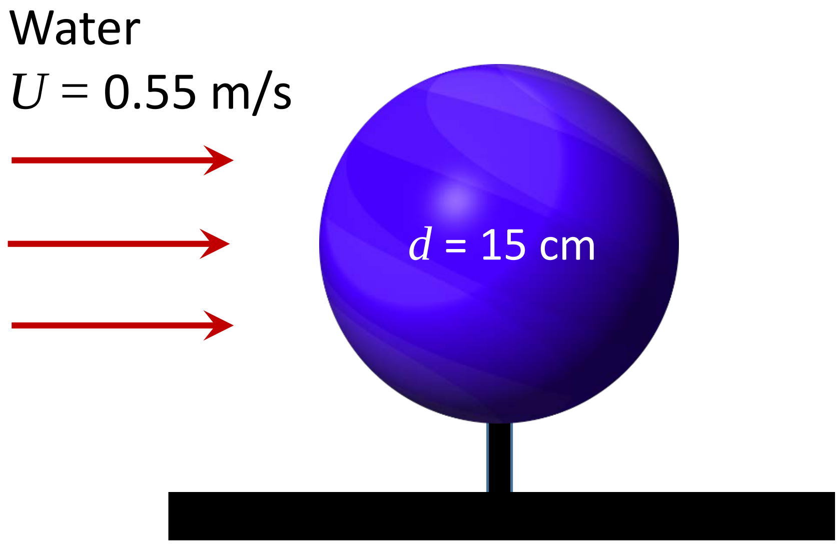


Example 1

(b) Drag coefficient:

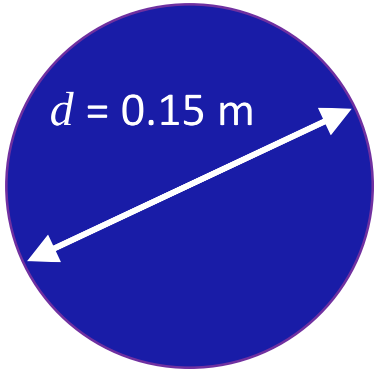
$$C_D = \frac{F_D}{\frac{1}{2}\rho U^2 A_{proj}}$$

$$F_D = C_D\,\frac{1}{2}\rho U^2 A_{proj}$$



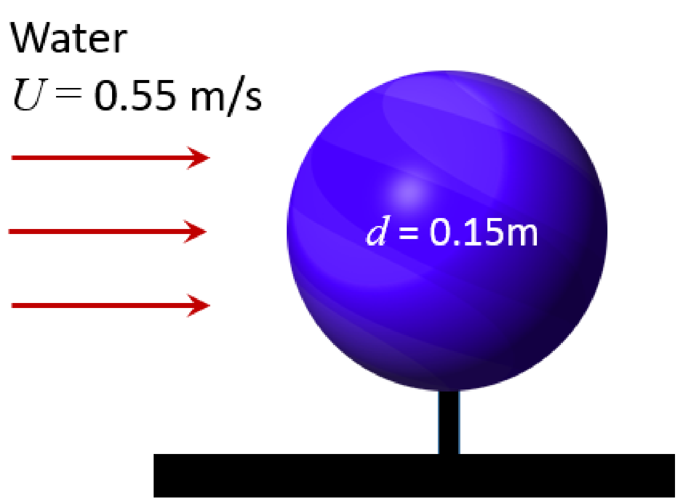
The projected frontal area for a sphere is the area of a circle.

$$A_{proj} = \frac{\pi d^2}{4} = \frac{\pi(0.15)^2}{4} = 1.77x10^{-2}\ m^2$$



This is the area normal to the oncoming flow -- the area the flow “sees” as an obstruction. So, the projected frontal area is a circle, NOT the surface are of the sphere!

Example 1



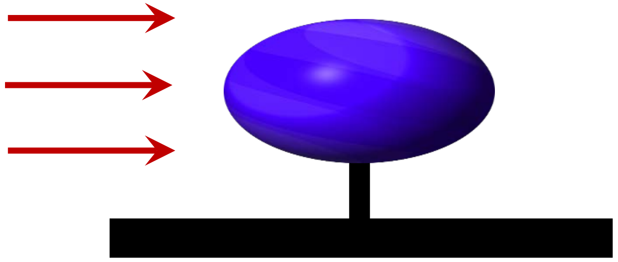
Drag Force: $$F_D = C_D \frac{1}{2}\rho U^2 A_{proj}$$

$$F_D = (0.45)\frac{1}{2}\left(998\ \frac{kg}{m^3}\right)(0.55\frac{m}{s})^2(1.77x10^{-2}\ m^2)$$

$$F_D = 1.20\ \frac{kg\ m}{s^2} = 1.20\ N$$ Ans. (b)

Comments

- Drag force caused by the pressure distribution over the sphere.
- The drag force can be reduced by making the shape more "streamlined". Two effects:
    - reduced frontal area, A_{proj}
    - more favorable pressure distribution, delayed flow separation and reduced size of wake region.



Example 2

A *Porche 911 Turbo*^{TM} has a drag coefficient of C_D=0.31 at top speed. The effective projected frontal area is about 2.2 m^2.

(a) Calculate the drag force in air at 20^{o}C (1 atm) at its top track speed of 315 km/h.

(b) Estimate the horse power required to overcome form drag at this speed.

Air

U= 315 km/h

photo credits: www.porche.com

Example 2

Air, 20°C
U = 315 km/h
C_D = 0.31
A_{proj} = 2.2 m^2

(a) Form drag: $F_D = C_D \underbrace{\frac{1}{2}\rho U^2 A_{proj}}_{}$ (stagnation pressure force)

Property tables, air at 20°C: $\rho = 1.20\ kg/m^3$

Speed conversion: $U = 315\ \frac{km}{h}\ (1000\ \frac{m}{km})\ \frac{1}{3600}\frac{h}{s} = 87.5\ \frac{m}{s}$

Form drag force at top speed:

$$F_D = (0.31)\frac{1}{2}\left(1.20\frac{kg}{m^3}\right)(87.5\frac{m}{s})^2(2.2\ m^2) = 3133\ N = 3.13\ kN$$ Ans. (a)

Note: $F_D \sim U^2$

Example 2

Air, 20°C

U = 87.5 m/s

C_D = 0.31

A_{proj} = 2.2 m^2

(b) How much power is required to overcome form drag at this speed?

Form drag: $F_D = 3.13\ kN$

$$Power = force\ x\ \frac{distance}{time} = force\ x\ speed$$

Power consumed by form drag force at top speed:

$$P = F_D\,U = 3.13x10^3\ N\left(87.5\frac{m}{s}\right) = 2.74\ x\ 10^5\ W$$

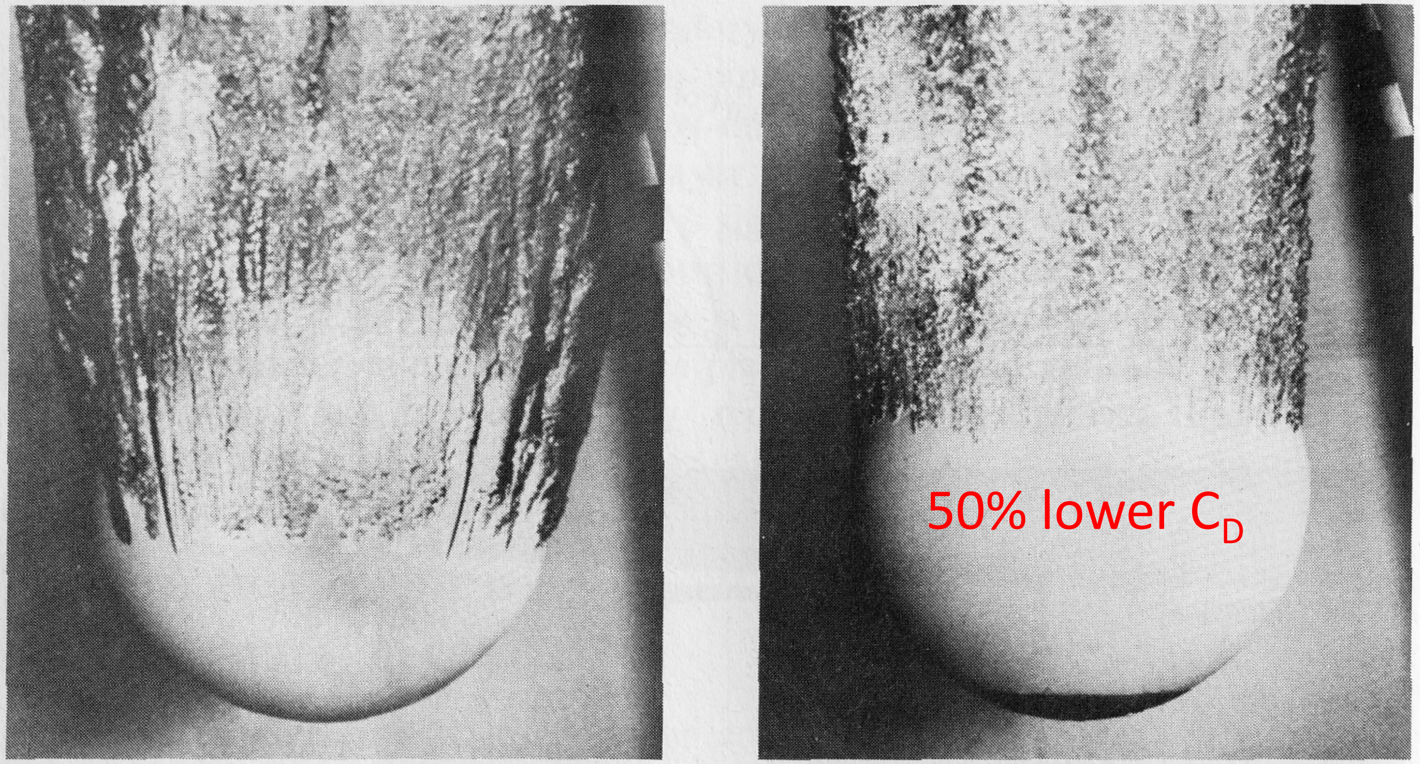
$$P = 2.74\ x\ 10^5\ W\ \left(\frac{1hp}{745.7\ W}\right) = 368\ hp$$

Ans. (b)

Effect of Surface Roughness

- Two bowling balls dropped into water at 25 ft/s.
- The left ball is smooth.
- The right ball is smooth, except for a small patch of sand glued to the nose.
- Turbulence in the boundary layer causes delayed flow separation. Smaller wake region.

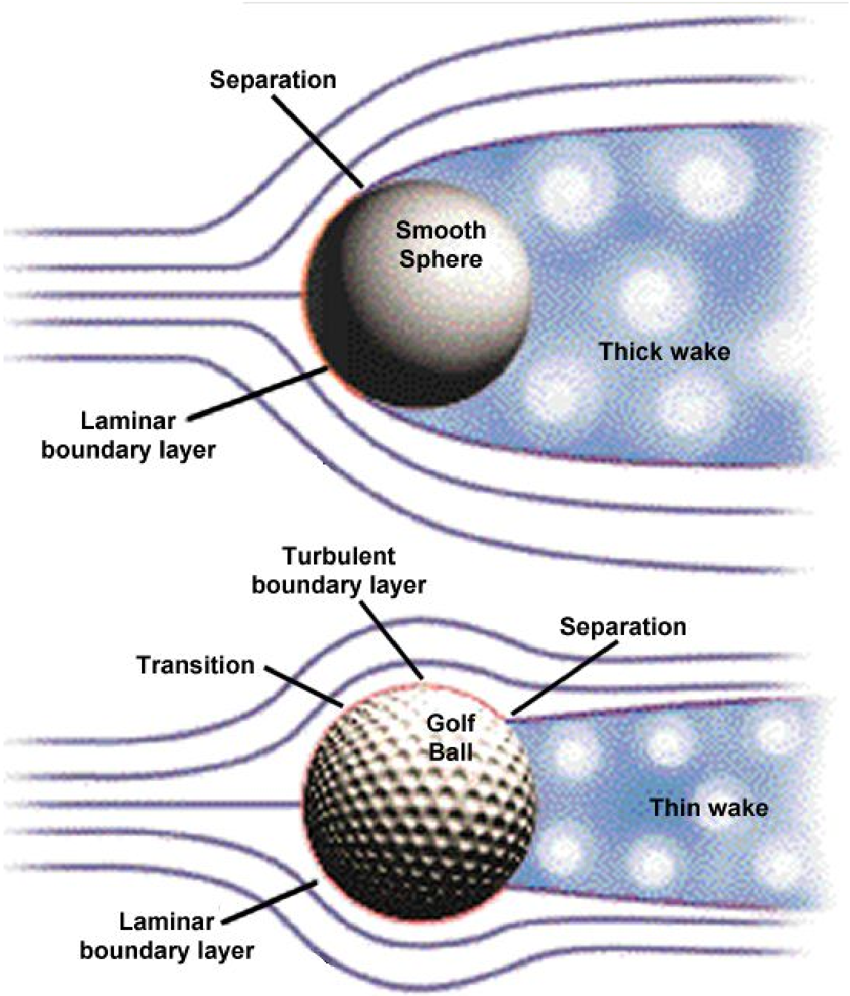
- Result: More than 50% reduction in drag by roughening the nose.



(*a*) (*b*)

Figure 5.20 Shift in separation point due to induced turbulence: (*a*) 8.5-in bowling ball, smooth surface, 25 ft/s entry velocity into water; (*b*) same except for 4-in-diameter patch of sand on nose. (*Official U.S. Navy photograph made at Navy Ordnance Test Station, Pasadena Annex.*)

Photo Credit: Fluid Mechanics, Streeter and Wiley, McGraw-Hill, 1979.



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END NOTES

Presentation prepared and delivered by Dr. David Naylor.