MEC516/BME516: Fluid Mechanics I

Viscous Shear Stress

Solved Midterm Example

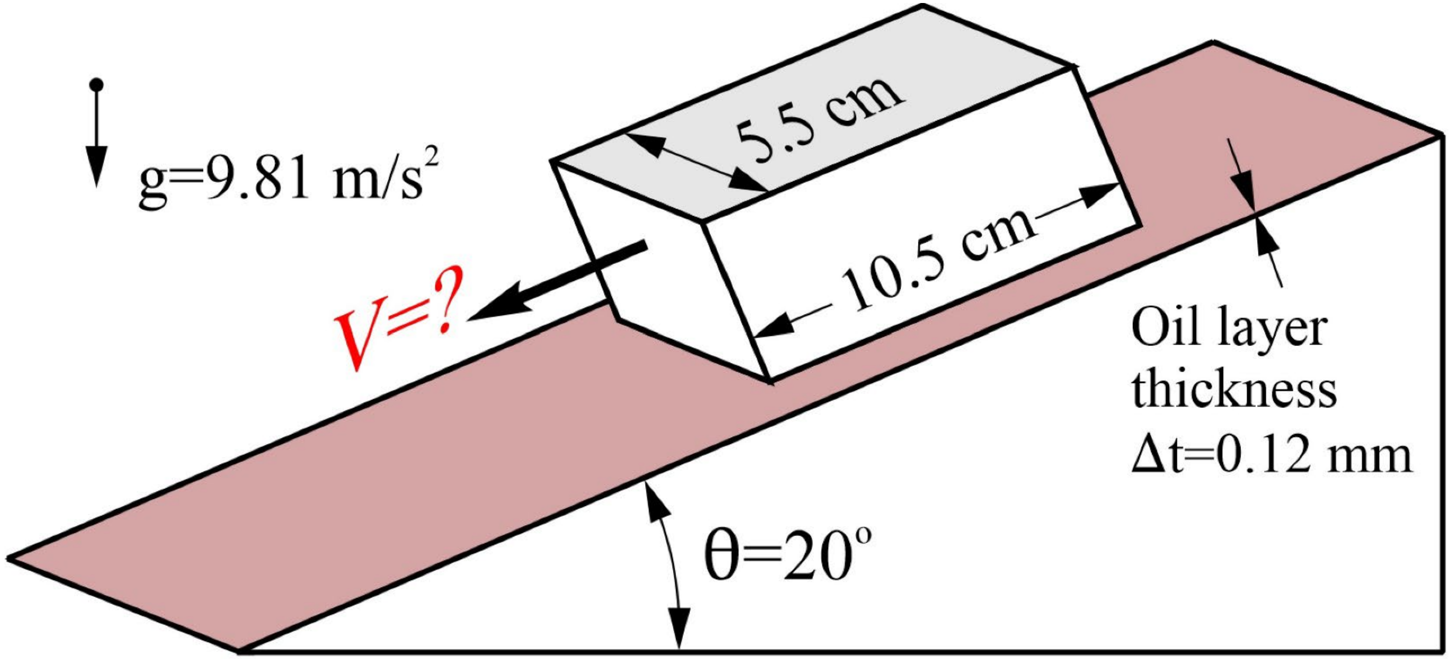
Toronto Metropolitan University

Department of Mechanical Industrial & Mechatronics Engineering

Midterm Question Fall 2023

A block with mass of 50g slides down an inclined plane at θ=20°. The lower surface of the block (5.5cm x 10.5cm) is lubricated with a thin layer of oil (μ=0.874 Ns/m^2) with constant thickness Δt=0.12 mm. The drag of the air is negligible compared to the viscous shear force of the oil.

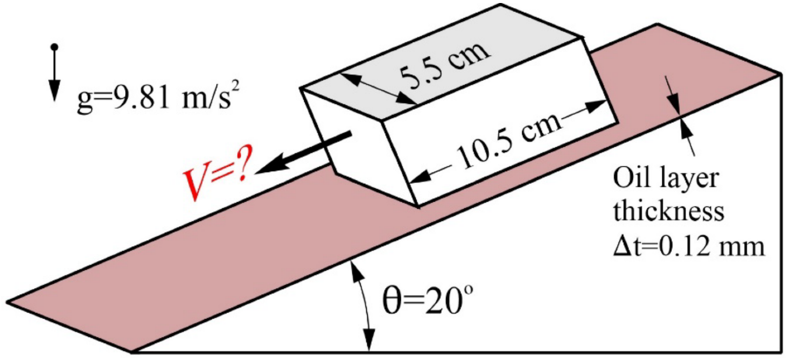
Assuming a linear velocity profile in the oil layer, calculate the **steady-state** speed (V) of the block.

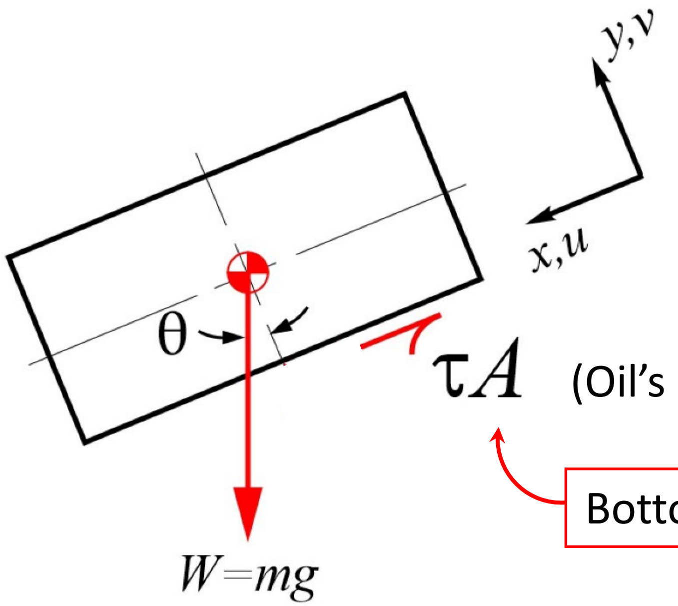


Hard copy (pdf) can be downloaded: www.drdavidnaylor.net

Solution

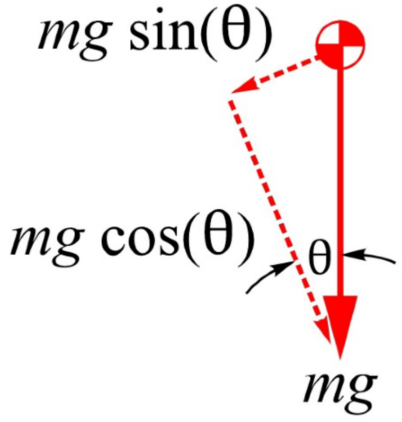
- Draw a ***free body diagram*** of the block:





(Oil's viscous shear **force**)

Bottom surface area of block



- Block is not accelerating (V is a steady-state):

$$\sum F_x = m\cancel{a_x}^{\,0} = 0$$

$$mg\sin\theta - \tau A = 0 \quad \rightarrow \quad mg\sin\theta = \tau A$$

Solution

τ depends on V

$$mg\sin\theta = \tau A \qquad (1)$$

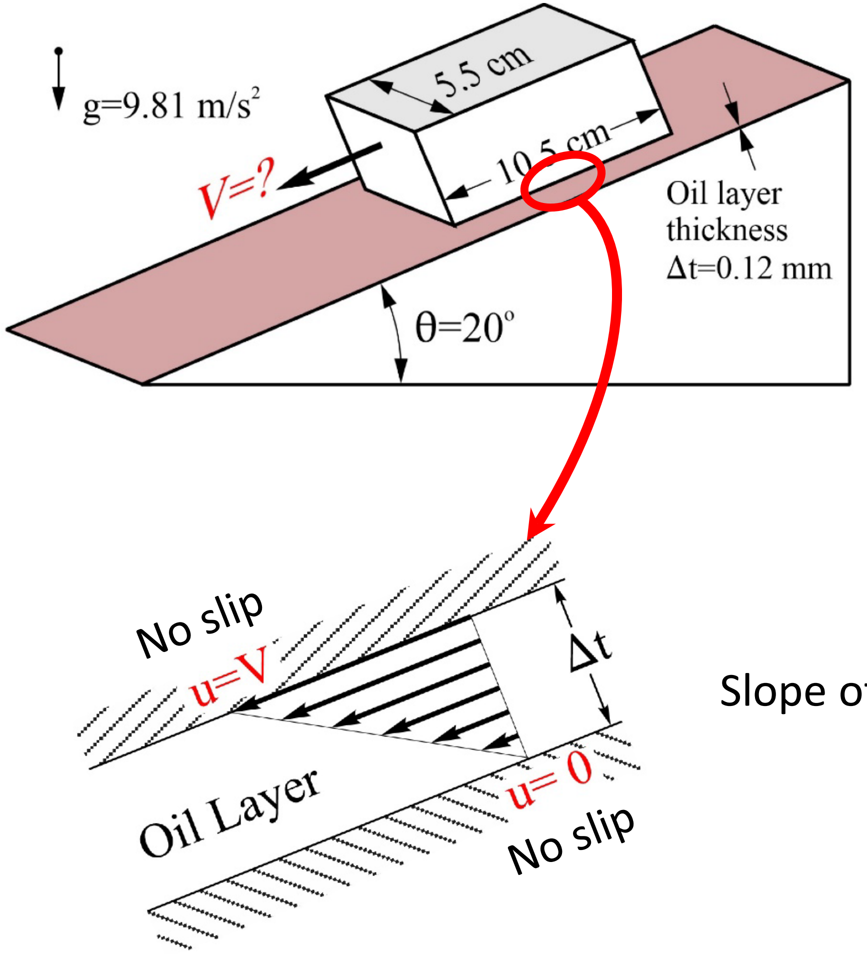
- Newton's Law of Viscosity

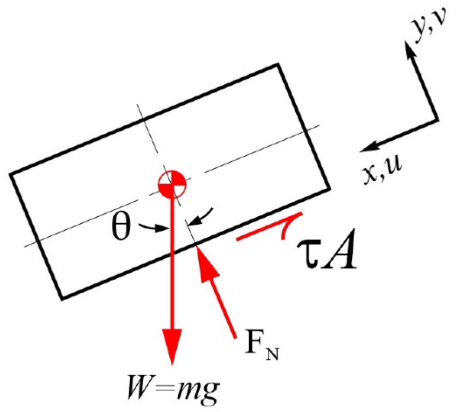
$$\tau = \mu\frac{du}{dy} = \mu\frac{V}{\Delta t} \qquad (2)$$

- Substitute (2) into (1):

$$mg\sin\theta = \mu\frac{V}{\Delta t}A$$

- Solve for block speed: $V = \dfrac{mg\sin\theta\ \Delta t}{\mu A}$



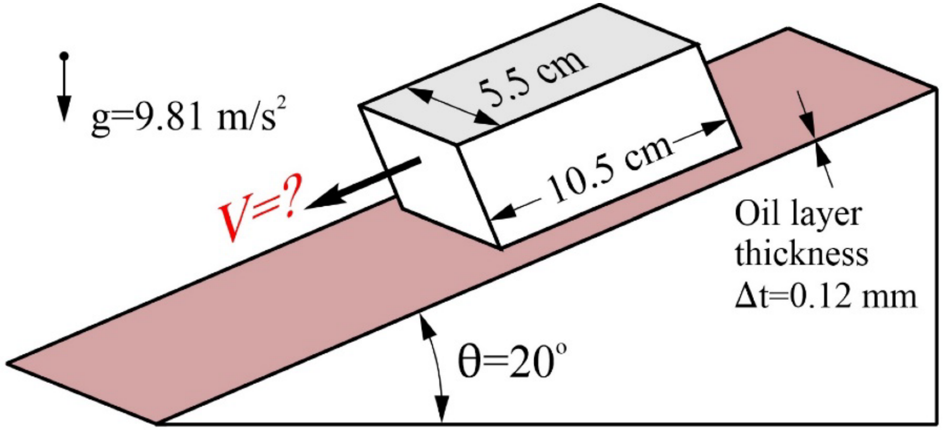


Slope of the **linear** velocity profile:

$$\frac{du}{dy} = \frac{V-0}{\Delta t} = \frac{V}{\Delta t}$$

Solution

$$\boxed{V = \frac{mg\sin\theta\ \Delta t}{\mu\, A}}$$



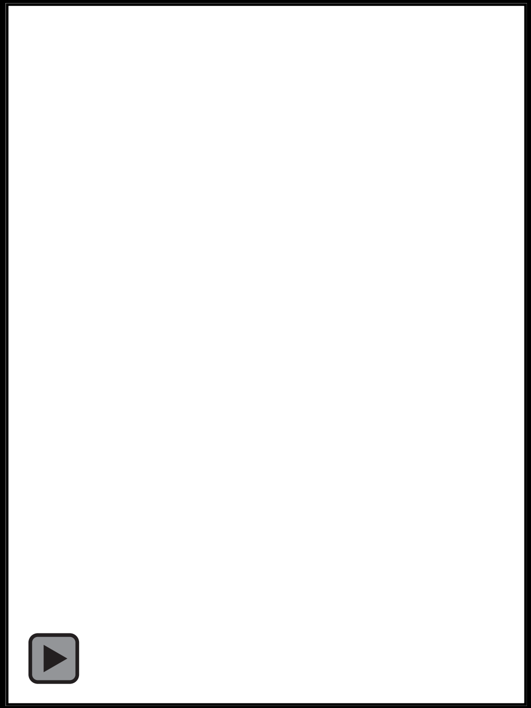
Block: $m = 50g = 0.05\ kg$
Oil: $\mu = 0.874\ Ns/m^2$

- Making the numerical substitutions:

$$\cancel{N}$$

$$V = \frac{0.050\,kg\left(9.81\frac{m}{s^2}\right)\sin(20^o)\ (0.00012\ m)}{0.874\,\frac{\cancel{N}s}{\cancel{m^2}}\,(0.105\cancel{m})(0.055\cancel{m})} = 3.99\ x10^{-3}\,\frac{m}{s}$$

$$V = 3.99\,\frac{mm}{s}$$ Answer



Video credit: Patrick Herd
https://youtu.be/4rfr_vLJnJw?si=-0EhyQN2uZ47C1X-

“Liquid Rope Coiling” of honey, a highly viscous fluid.

END NOTES

Prepared and delivered by Professor David Naylor

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