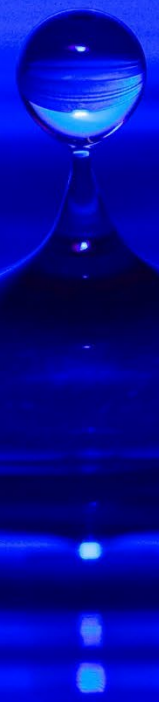
MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis



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www.drdavidnaylor.net

Toronto Metropolitan University

Department of Mechanical & Industrial Engineering

Concept Problem: Conservation of Linear Momentum

Oil flows at a steady mass flow rate $\dot{m}$ through a 90° elbow with at constant speed V. The pipe has cross sectional area A. The gauge pressures at the inlet and outlet are p_1, p_2. The weight of the elbow and oil is W.

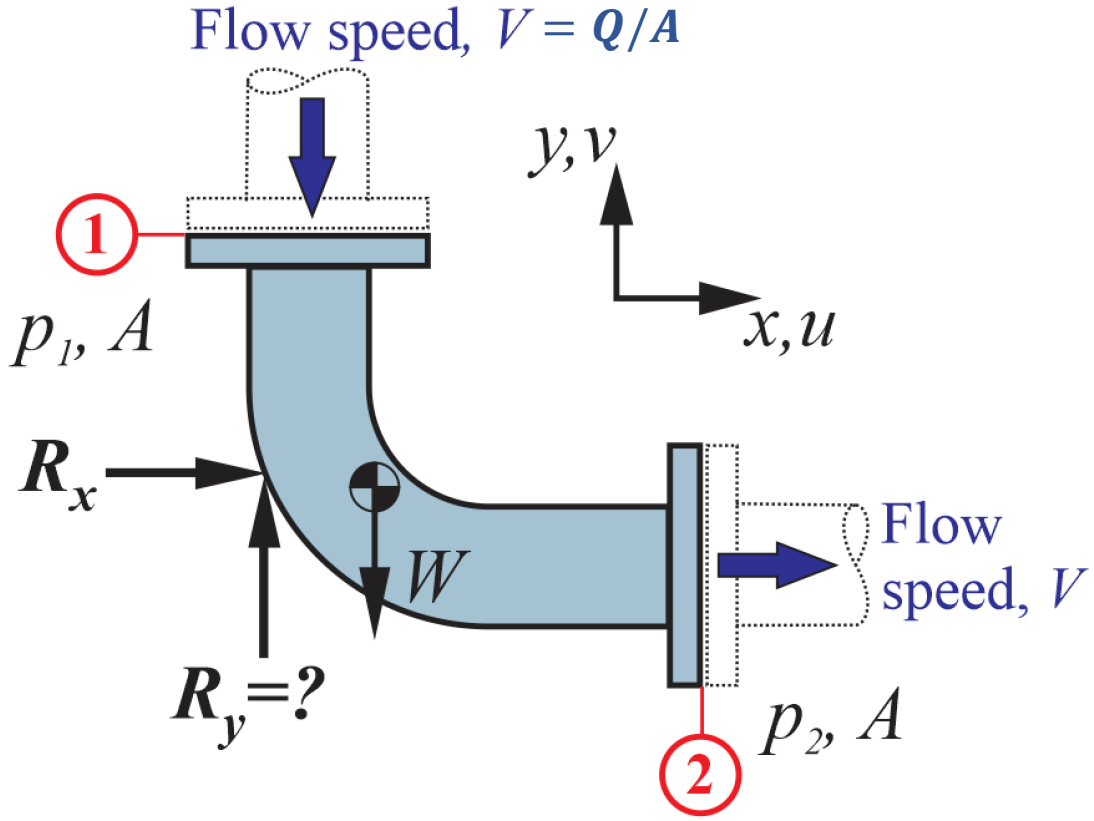
What force in the y-direction (R_y) is required to hold the elbow in place?

a) $R_y = \dot{m}V + W + p_1 A$

b) $R_y = \dot{m}V + W - p_1 A$

c) $R_y = -\dot{m}V + W + p_1 A$

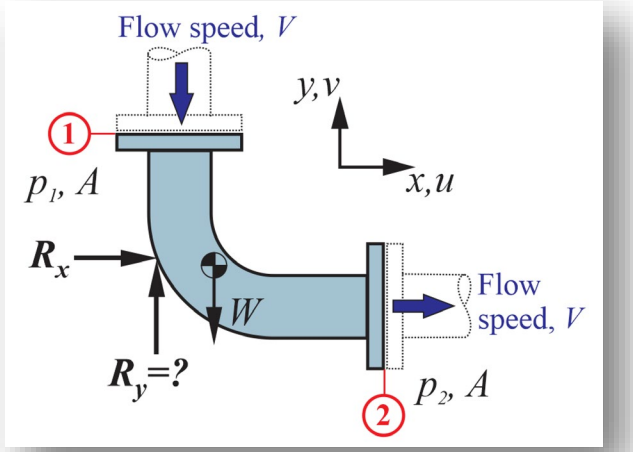
d) $R_y = -\dot{m}V - W - p_1 A$

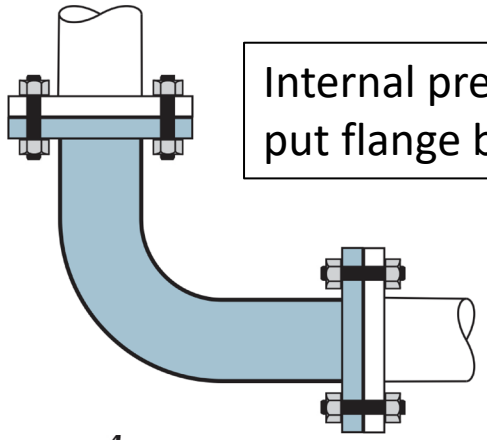


(I'll also consider R_x, the flange bolt forces in the x-direction.)

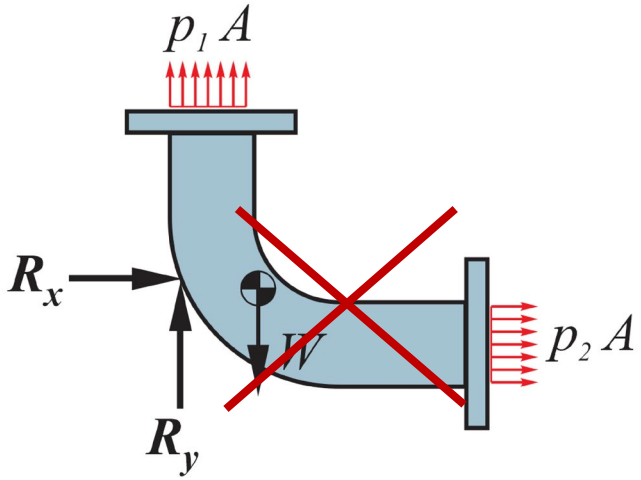
Conservation of Linear Momentum

Free Body Diagram

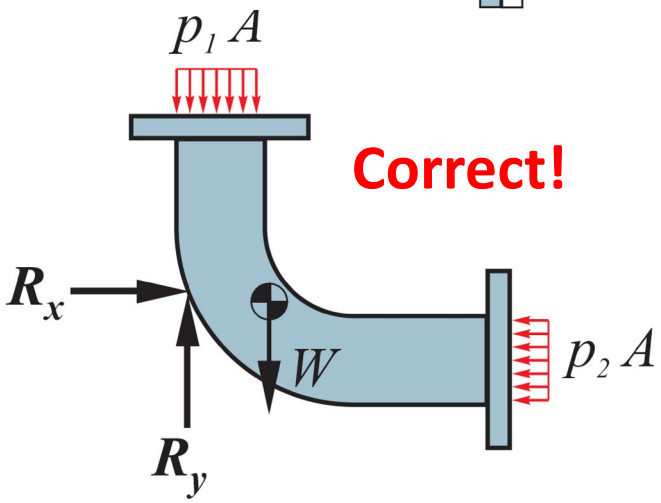




Internal pressure will put flange bolts in tension!

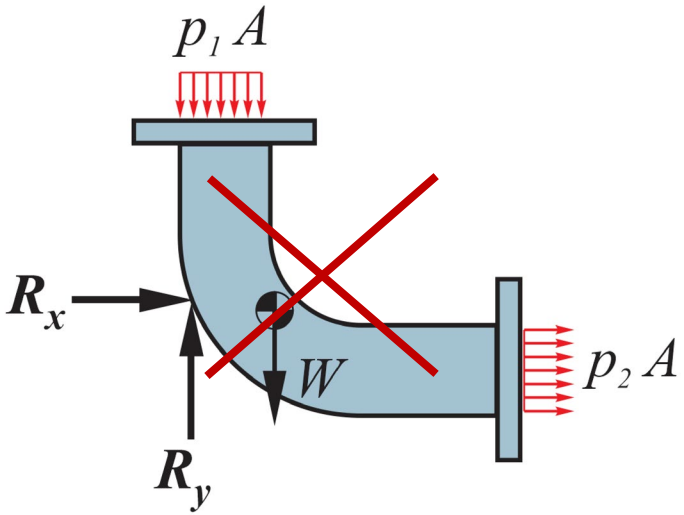


Pressure Acts Outward?



Correct!

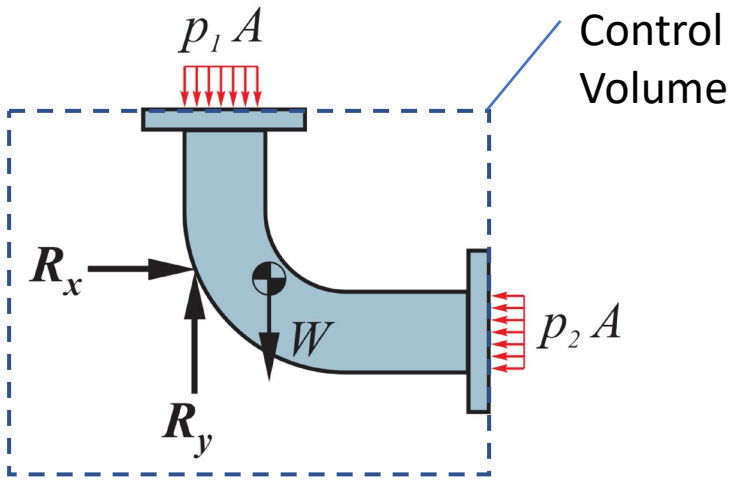
Pressure Acts Inward ?

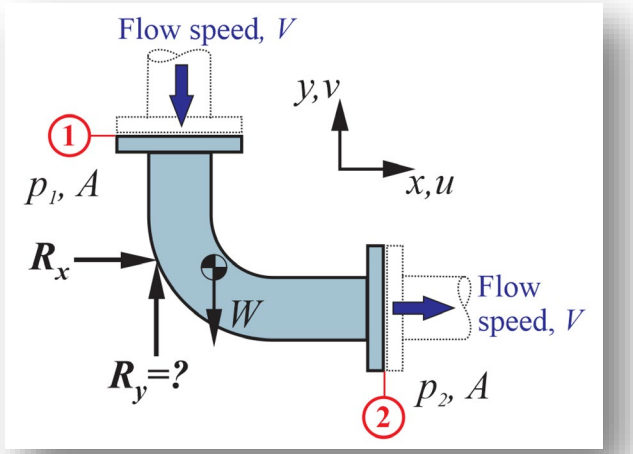


Pressure Acts in Flow Direction?

Conservation of Linear Momentum

- All forces applied to C.V.
  - Unknown forces R_y, R_x
  - Weight & pressure forces
  - **Must** use *gauge* pressures
    (Why? See link in description)





- Conservation of momentum in y-direction. Steady flow, one inlet and one outlet:

$$\sum F_y = \dot{m}\, v_{out} - \dot{m}\, v_{in} = \dot{m}\,(\cancel{v_2}^{\,0} - \cancel{v_1}^{\,-V}) = \dot{m}\, V$$

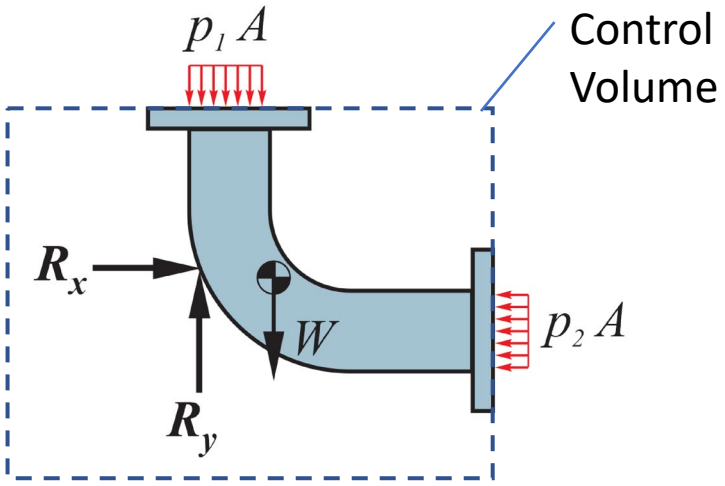
All forces on C.V. in y-direction

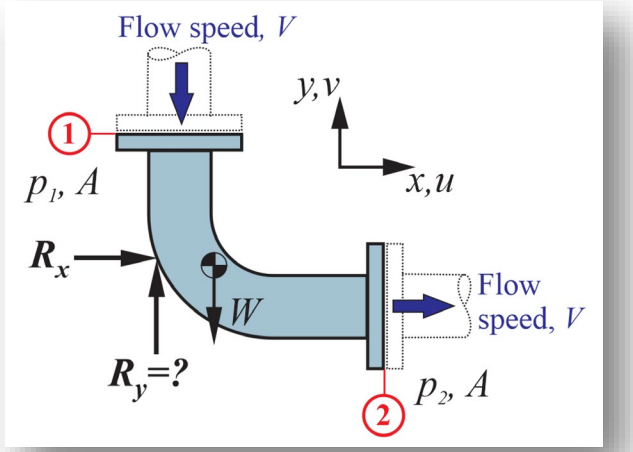
$$\uparrow^{+} \quad \sum F_y = R_y - p_1 A - W = \dot{m}\, V$$

$$a) \quad R_y = p_1 A + W + \dot{m}\, V$$ Answer

Conservation of Linear Momentum

Although $\boldsymbol{R_x}$ was not required…





- Conservation of momentum in x-direction:

$$\sum F_x = \dot{m}\, u_{out} - \dot{m}\, u_{in} = \dot{m}\,(\overset{V}{\cancel{u_2}} - \overset{0}{\cancel{u_1}}) = \dot{m}\, V$$

$$\xrightarrow[+]{} \quad \sum F_x = R_x - p_2 A = \dot{m}\, V$$

$$R_x = p_2 A + \dot{m}\, V \qquad \text{Answer}$$

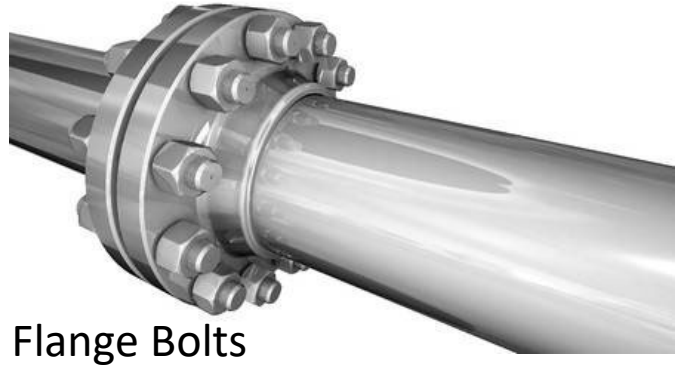
Conservation of Linear Momentum

- R_x and R_y are the reaction forces required to hold the elbow place i.e., the bolt forces at the two flanges

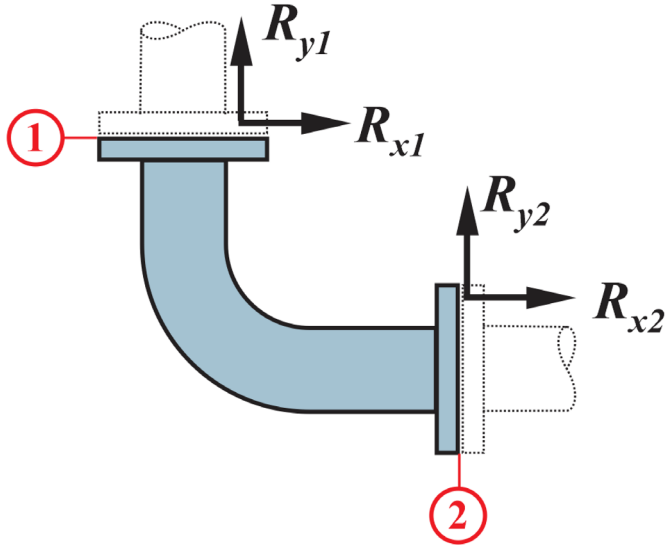
- Linear momentum analysis can only tell you the *total* force in each direction:

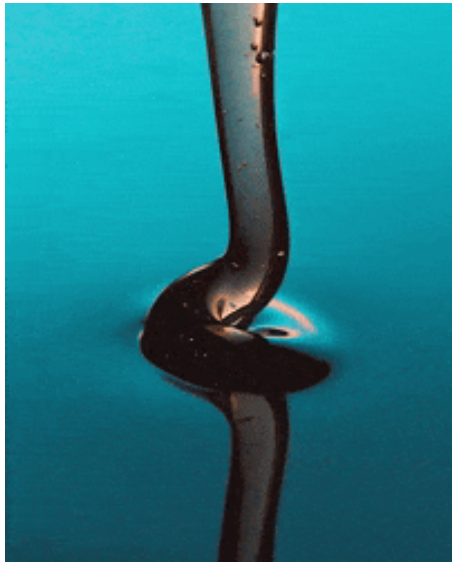
$$R_x = R_{x1} + R_{x2}$$

$$R_y = R_{y1} + R_{y2}$$



Flange Bolts



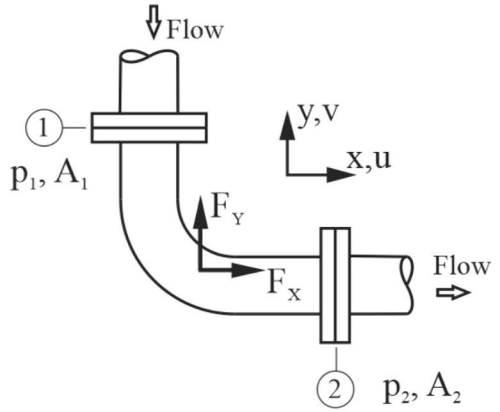


Liquid Rope Coil Effect

Credit: www.gifer.com

Random Question 4

Consider flow through a 90 degree pipe elbow, shown in the sketch. The sketch shows the gauge pressures and cross sectional areas at points 1 and 2 The weight of the elbow is negligible. The magnitude of the constant fluid velocity (i.e. speed of the flow) in the pipe is *V*. What is the force in the y-direction (F_y) required to hold a pipe elbow in place? (Note that F_y is positive upward.)



- $F_y = -\dot{m}\, v_1 + p_1 A_1 = \dot{m} V + p_1 A_1$ — 8 (36.36 %)
- $F_y = -\dot{m}\, v_1 - p_1 A_1 = \dot{m} V - p_1 A_1$ — 3 (13.64 %)
- $F_y = \dot{m}\, v_1 + p_1 A_1 = -\dot{m} V + p_1 A_1$ — 9 (40.91 %)
- $F_y = \dot{m}\, v_1 - p_1 A_1 = -\dot{m} V - p_1 A_1$ — 2 (9.09 %)

⬅ Ouch!

Average Grade 36.4%

END NOTES

- All the videos (and pdf downloads) for this course available at: www.drdavidnaylor.net
- Presentation prepared and delivered by Professor David Naylor

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