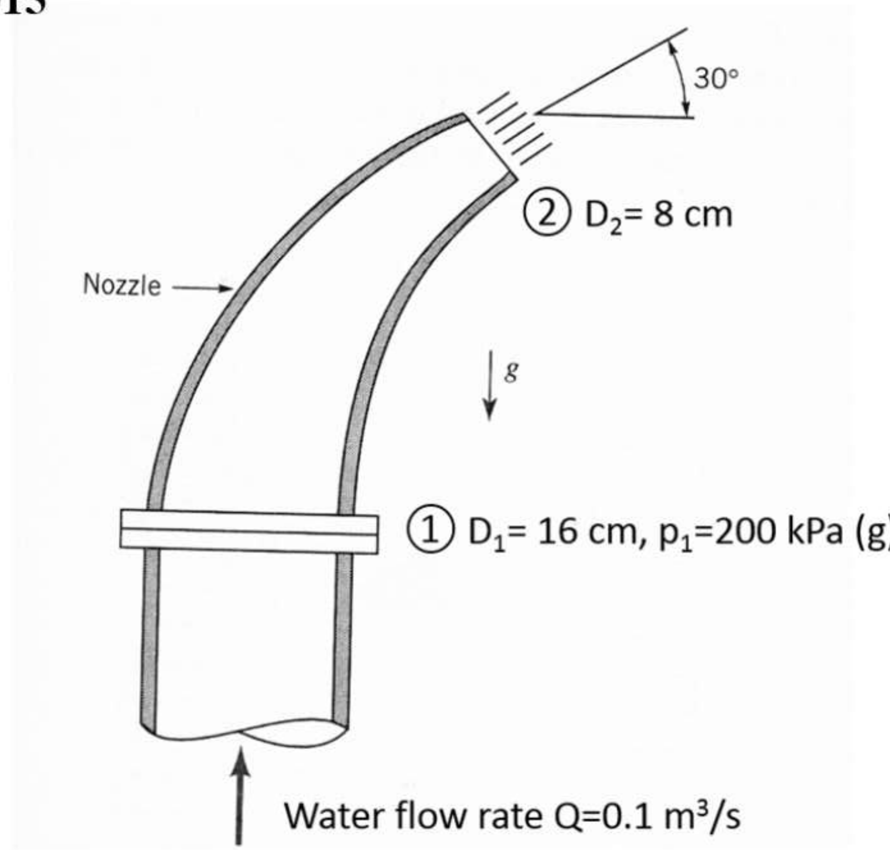
Final Exam Fall 2015

A curved nozzle is attached to a vertical pipe. At the flange the pipe has an inside diameter of D_1=16.0 cm and the gauge pressure is p_1=200 kPa. The end of the nozzle has an inside diameter of D_2=8.0 cm and discharges a jet of water to atmosphere at an angle of 30° from horizontal. The flow rate of the water (ρ=998 kg/m^3) is Q=0.1m^3/s. The nozzle has a mass of 20 kg and the volume of water contained inside the nozzle is 15 litres. The atmospheric pressure is 100 kPa.

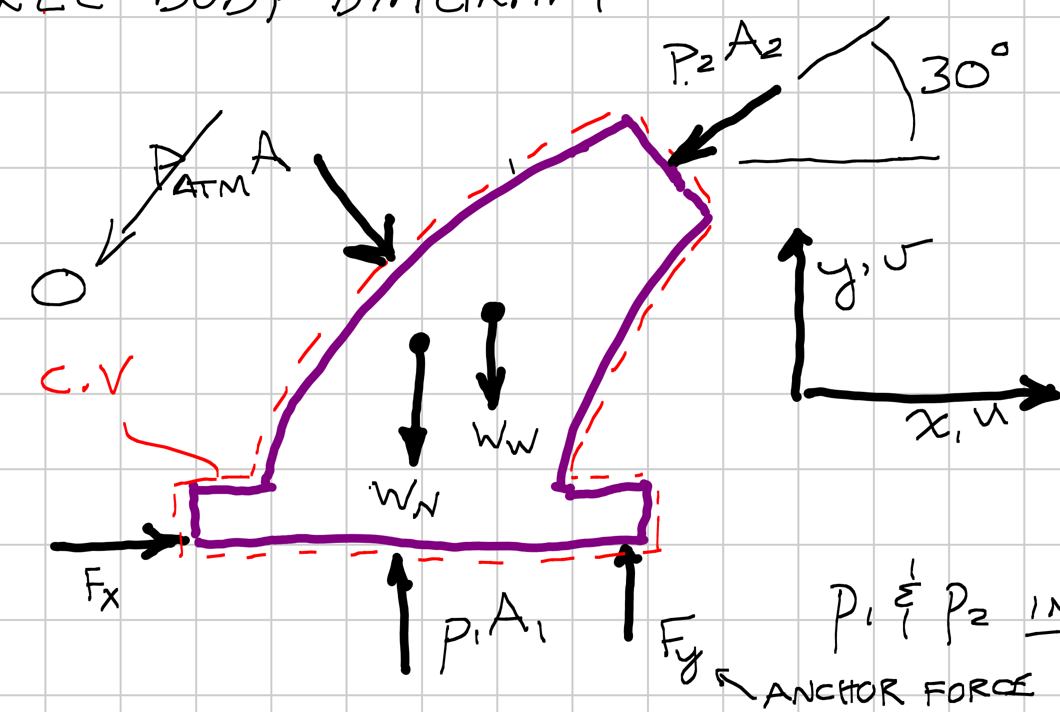
(a) Draw a free body diagram, showing all the forces on the nozzle.

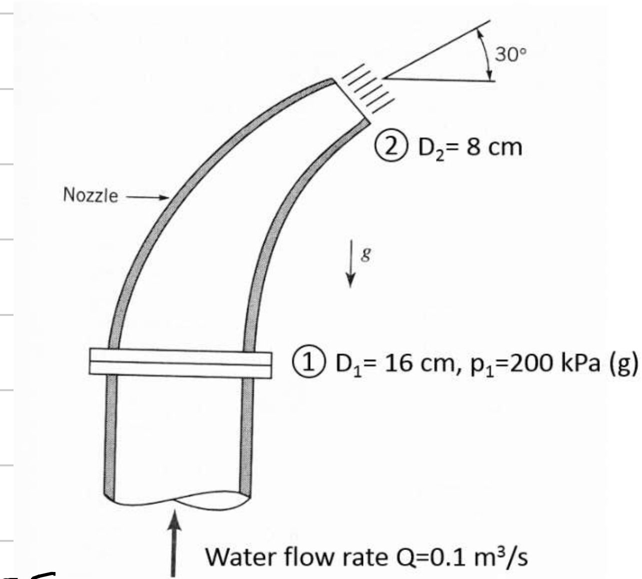
(b) Calculate the **vertical component** of the force at the flange (location ①) required to hold the nozzle in place. Clearly indicate the direction of this force.



- CONSERVATION OF LINEAR MOMENTUM FOR A CONTROL VOLUME (C.V.)

FREE BODY DIAGRAM



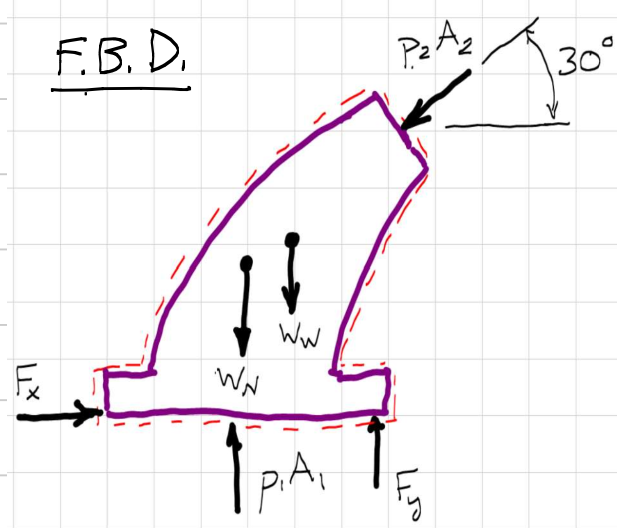


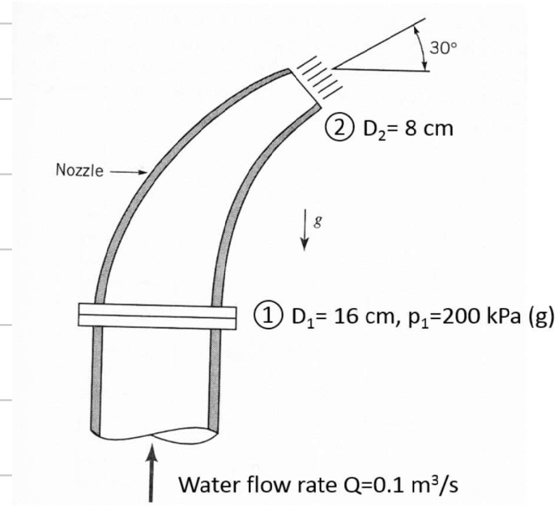
P_1 & P_2 IN GAUGE

$$\sum \vec{F}_{CV} = \sum \dot{m}_{OUT} \vec{V}_{OUT} - \sum \dot{m}_{in} \vec{V}_{in}$$ ($\vec{F} = m\vec{a}$ FOR A C.V.)

$$\sum_{CV} F_y = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}(v_2 - v_1)$$ $\dot{m}_1 = \dot{m}_2 = \dot{m}$

$$\sum_{CV} F_y = \dot{m}(v_2 - v_1)$$





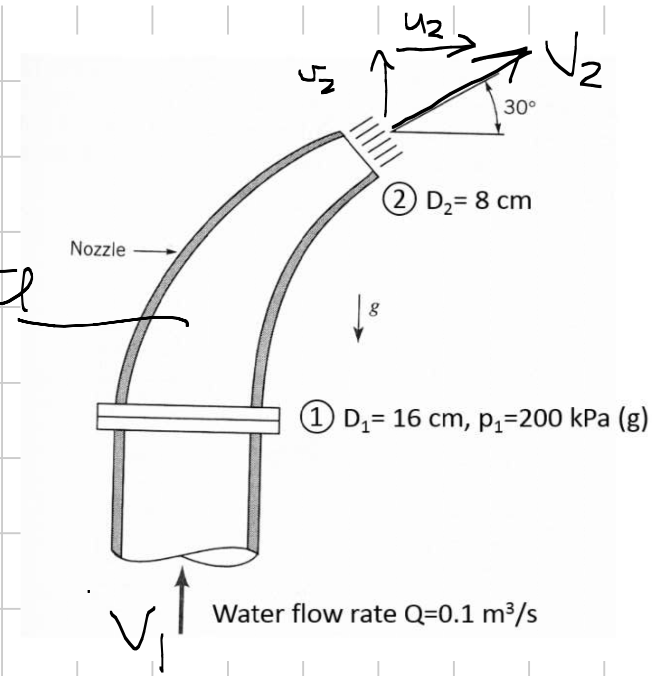
VERTICAL ANCHOR FORCE

$$F_y + p_1 A_1 - p_2 A_2 \sin 30° - W_N - W_W = \dot{m}(v_2 - v_1)$$

$$F_y = \dot{m}(v_2 - v_1) - p_1 A_1 + p_2 A_2 \sin 30° + W_N + W_W$$

CHECK THAT SIGNS MAKE SENSE ✓

$$F_y = \dot{m}(v_2 - v_1) - p_1 A_1 + p_2 A_2 \sin 30^\circ + W_N + W_W$$



CALCULATIONS

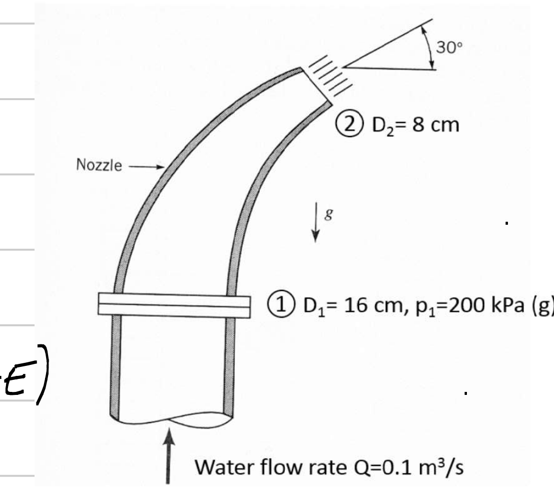
$$\dot{m} = \rho Q = 998\ \mathrm{kg/m^3}\,(0.1\ \mathrm{m^3/s}) = 99.8\ \mathrm{kg/s}$$

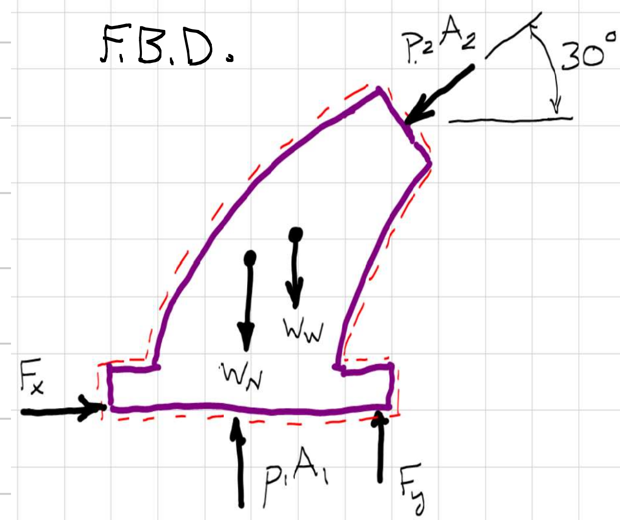
$$Q = V_1 A_1 = V_2 A_2$$

$$V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2} = \frac{4(0.1\ \mathrm{m^3/s})}{\pi\,(0.16\ \mathrm{m})^2} = 4.974\ \mathrm{m/s} \qquad v_1 = V_1$$

$$V_2 = \frac{Q}{A_2} = \frac{V_1 A_1}{A_2} = V_1\left(\frac{D_1}{D_2}\right)^2 = 4V_1 = 19.89\ \mathrm{m/s} \qquad v_2 = V_2 \sin 30^\circ = 9.947\ \mathrm{m/s}$$

$$W_N = m_N g = 20\ \mathrm{kg}\,(9.8\ \mathrm{m/s^2}) = 196.2\ \mathrm{N} \qquad W_W = \gamma_W \forall = 9790\ \mathrm{N/m^3}\,(15\times10^{-3}\ \mathrm{m^3}) = 146.8\ \mathrm{N}$$





$p_2 = 0$ (GAUGE)

$$F_y = \dot{m}(v_2 - v_1) - p_1 A_1 + p_2 A_2 \sin 30° + W_N + W_W$$

$$F_y = 99.8 \frac{kg}{m^3} (9.947 - 4.974) \frac{m}{s} - 200 \times 10^3 \frac{N}{m^2} \left(\frac{\pi\, 0.16^2}{4}\right) m^2 + 0 + 196.2 N + 146.8 N$$

$$F_y = 496.3 N - 4021.2 N + 196.2 N + 146.8 N = -3180 N$$

i.e. $F_y = 3180 N \downarrow$ ANS.

BONUS

$F_x = ?$

$\sum_{CV} F_x = \dot{m}(u_2 - u_1) = \dot{m} u_2$ ($u_1 = 0$)

$F_x - p_2 A_2 \cos 30° = \dot{m} u_2$ ($p_2 = 0$ (GAUGE))

$u_2 = V_2 \cos 30°$ $V_2 = 19.89\ m/s$

$F_x = \dot{m} V_2 \cos 30° = 99.8\ \frac{kg}{s}\left(19.89\ \frac{m}{s}\right)\cos 30°$

$F_x = {}^{+}1720\ N \rightarrow$ ANS.