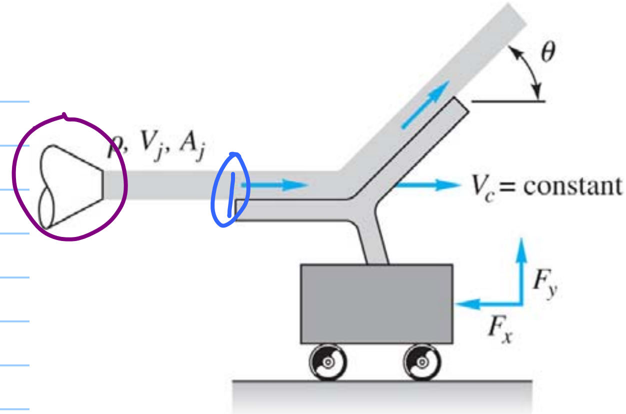
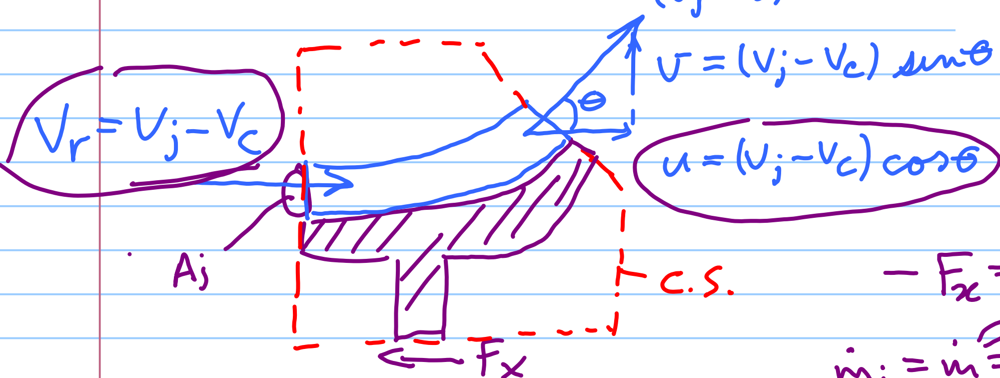
In Fig. P3.55 the jet strikes a vane that moves to the right at constant velocity V_c on a frictionless cart. Compute (*a*) the force F_x required to restrain the cart and (*b*) the power P delivered to the cart. Also find the cart velocity for which (*c*) the force F_x is a maximum and (*d*) the power P is a maximum.



$$\boxed{\sum \vec{F} = \dot{m}_o \vec{V}_{r,o} - \dot{m}_i \vec{V}_{r,i}}$$



$$-F_x = \dot{m}_o (V_j - V_c)\cos\theta - \dot{m}_i (V_j - V_c)$$

$$\dot{m}_i = \dot{m} = \rho A_j (V_j - V_c)$$

WHEN $V_c = V_j$ $\quad V_r = 0$

$V_c = 0 \quad V_r = V_j$

$$F_x = \rho A_j (V_j - V_c)^2 - \rho A_j (V_j - V_c)^2 \cos\theta$$

$$\Rightarrow F_x = \rho A_j (V_j - V_c)^2 (1 - \cos\theta) \qquad \text{ANS/ (a)}$$

(b) POWER (P) DELIVERED TO CART BY FLUID JET

WORK $\quad W = F_x x \qquad$ POWER $\quad P = \dfrac{dW}{dt} = F_x \dfrac{dx}{dt} = F_x V_c$

$$\boxed{P = F_x V_c = \rho A_j (V_j - V_c)^2 V_c (1 - \cos\theta)} \qquad \text{ans. (b)}$$

(c) FIND V_c FOR $F_{x,\max}$

$$F_x = \rho A_j (V_j - V_c)^2 (1 - \cos\theta) \qquad C = \rho A_j (1 - \cos\theta)$$

$$F_x = C(V_j - \underbrace{\cancel{V_c}}_{0})^2 \qquad \text{BY OBSERVATION} \quad V_c = 0 \rightarrow F_{x,\max}$$

$$\boxed{\therefore\ F_{x,max} = \rho A_j (1-\cos\theta) V_j^2}$$ ans. (c)

(d) FIND V_c FOR P_{MAX}.

$$P = \rho A_j (V_j - V_c)^2 V_c (1-\cos\theta)$$

$$\frac{dP}{dV_c} = 0 \qquad \text{Let } C = \rho A_j (1-\cos\theta)$$

$$\boxed{P = C(V_j - V_c)^2 V_c = C(V_c^3 - 2V_jV_c^2 + V_j^2 V_c)}$$

$$\frac{dP}{dV_c} = C(3V_c^2 - 4V_jV_c + V_j^2) = 0$$

$$\underset{a}{3}V_c^2 \underset{b}{-4V_j}V_c + \underset{c}{V_j^2} = 0 \qquad \text{QUADRATIC EQUATION}$$

$$V_c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4V_j \pm \sqrt{16V_j^2 - 12V_j^2}}{2(3)}$$

$$V_c = \frac{4V_j \pm 2V_j}{6} = \overset{P=0}{V_j},\ \frac{V_j}{3}$$

$$\boxed{P_{max} \rightarrow V_c = \frac{V_j}{3}}$$

ANS. (d)