RYERSON UNIVERSITY

Department of Mechanical and Industrial Engineering

FLUID MECHANICS I – BME516/MEC516

MIDTERM EXAM

DATE: Monday, October 21, 2019
TIME: 1:10 - 2:55pm
ROOM: ENGLG14

EXAMINER: Dr. D. Naylor
DURATION: 105 minutes

INSTRUCTIONS:

1. This is a CLOSED BOOK EXAM. Permitted aids are: one 8.5 inch × 11 inch personal equation (aid) sheet, both sides; non-communicating electronic calculator; and drawing and writing instruments (i.e., ruler, pens and pencils).
2. A table of centroids and second moments of area is included with this exam paper. A basic formula sheet is also included with this exam paper.
3. Prohibited items include: textbooks, class notes, cell-phones and other wireless devices, laptop computers, etc. **Possession of a communication device will trigger charges of academic misconduct.**
4. A valid student identification card must be presented when attendance is taken.
5. Answer all questions. Marks are indicated beside each question and in the table below.
6. To get full marks you must clearly show the formulas, methods and numbers used to solve the problem.
7. **You must use the symbols given in the problem statement.** Also, be sure to give the proper units on all intermediate results.
8. Marks will be deducted for incorrect or missing units.

Student Name (Please Print): SOLUTIONS

Student Number: ____________ Section Number: ____________

Please Check One: ☐ Online Course

☐ Face-to-Face Lecture Course

Question	Mark
A1-A5	/10
B1	/10
B2	/10
B3	/10
Total	/40

PART A - MULTIPLE CHOICE QUESTIONS

Each of the questions below is followed by several suggested answers. ***On the exam paper, circle the ONE that is best.*** There is no penalty for incorrect answers.

Questions *A1* to *A5* are worth **2 *marks*** each.

A1. Using the nomenclature of Chapter 1, what are the ***dimensions*** of surface tension (Υ)?

(a) $\{M^{-1}L^{-1}T^{-2}\}$
(b) $\{ML^{-1}T^{2}\}$
(c) $\{MT^{-2}\}$
(d) $\{ML^{-1}T^{-2}\}$

$\Upsilon \equiv N/m \equiv \frac{kg\,\cancel{m}}{\cancel{m}\,s^2} = \frac{kg}{s^2}$ $\quad \{\Upsilon\} = \left\{\frac{M}{T^2}\right\}$

A2. On the factory floor, you read a Bourdon pressure gauge on a compressed air tank. The Bourdon gauge reads 100 kPa. If the temperature of the tank is 30°C, estimate the density (ρ) of the air in the tank. The gas constant for air is R=287 J/kgK. $P_{atm} \approx 100\,kPa$

(a) 1.1 kg/m^3
(b) 2.3 kg/m^3
(c) 4.6 kg/m^3
(d) 11.6 kg/m^3

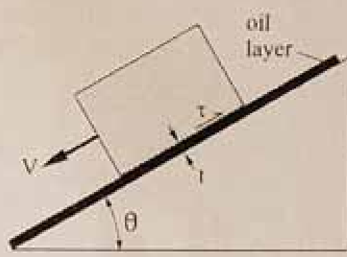
$\rho = \frac{P}{RT} = \frac{200\times10^3\, N/m^2}{287\frac{J}{kgK}(30+273)K} \cong 2.3\frac{kg}{m^3}$

A3. A block slides down an inclined plane on a layer of oil with thickness t=0.20 mm. The block has a speed of V=0.35 m/s. The oil has a kinematic viscosity of ν=3.4x10^{-4} m^2/s and a specific gravity of SG=0.85. Calculate the fluid shear stress (τ) on the block.

(a) 0.60 N/m^2
(b) 51.0 N/m^2
(c) 506 N/m^2
(d) 594 N/m^2

$\tau = \mu\frac{V}{t} = \rho\nu\frac{V}{t}$

$\tau = 850\frac{kg}{m^3}\left(3.4\times10^{-4}\frac{m^2}{s}\right)\frac{0.35\,\cancel{m}/s}{0.0002\,\cancel{m}} = 506\, N/m^2$

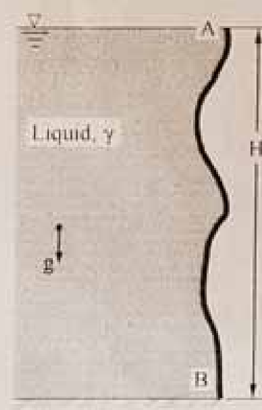


A4. Referring to the diagram, a liquid with density of ρ=1500 kg/m^3 is contained by an irregularly shaped vertical wall (A-B) with height H=5.3m. What is the horizontal hydrostatic force on the wall AB per unit depth (into the page)?

(a) 21.1 kN
(b) 78.0 kN
(c) 207 kN
(d) 413 kN
(e) Insufficient information (depends on the shape of the wall)

$F_H = \gamma h_{CG} A = 1500\frac{kg}{m^3}\left(9.8\frac{m}{s^2}\right)\frac{5.3}{2}m\,(5.3m)(1m)$

$= 207\,kN$

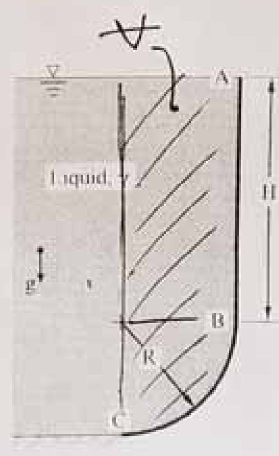


A5. Referring to the diagram, a liquid with specific weight (γ) is contained by a wall that consists of a straight vertical section (A-B) of height H and a curved section (B-C) with radius R. What is the total **vertical** force of the liquid on the wall A-B-C per unit depth (into the page)?

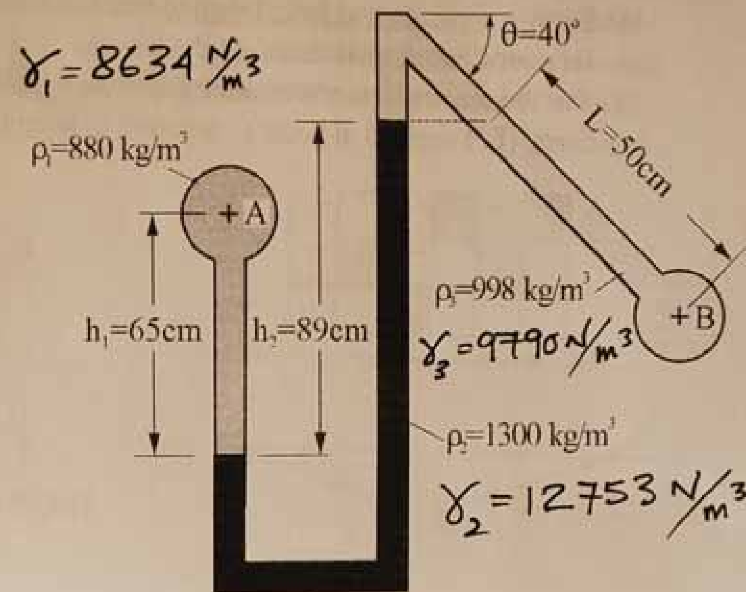
(a) $F_v = \gamma[HR + \pi R^2/4]$
(b) $F_v = \gamma[\pi R^2/4]$
(c) $F_v = \gamma[H + R]^2/2$
(d) $F_v = \gamma[HR + \pi R^2/2]$

$F_V = W = \gamma\forall$

$F_V = \gamma\left[H\cdot R + \frac{\pi R^2}{4}\right]$



B1. (a) Two pipes are connected by a manometer, as shown in the sketch. Calculate the pressure difference between the two pipes ($p_A - p_B$) based on the manometer heights (h_1, h_2), the angle of inclination and length (θ, L) and the densities of the three fluids (ρ_1, ρ_2, ρ_3) in the diagram. Work in symbolic for before substituting any values! ***(7 marks)***



$P_B - \gamma_3 L \sin\theta + \gamma_2 h - \gamma_1 h_1 = P_A$

$$\boxed{P_A - P_B = -\gamma_3 L \sin\theta + h_2 \gamma_2 - \gamma_1 h_1}$$

$P_A - P_B = -9790\,N/m^3\ 0.5m \sin 40^\circ + 12753\,N/m^3 (0.89m) - 8634\,N/m^3 (0.65m)$

$P_A - P_B = -3146\,N/m^2 + 11{,}350\,N/m^2 - 5611\,N/m^2 = 2590\,N/m^2$

ANS/

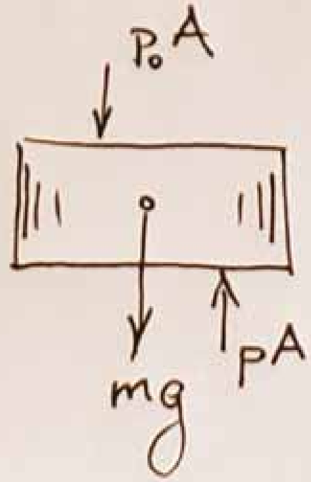
(b) Air is contained in a sealed frictionless piston-cylinder, shown in the sketch. The weight of the piston pressurizes the air in the cylinder. Local atmospheric pressure is 100 kPa. The cylinder has an inside diameter of D=5.5cm and a mass of m=3.2 kg. What pressure does the Bourdon gauge read?

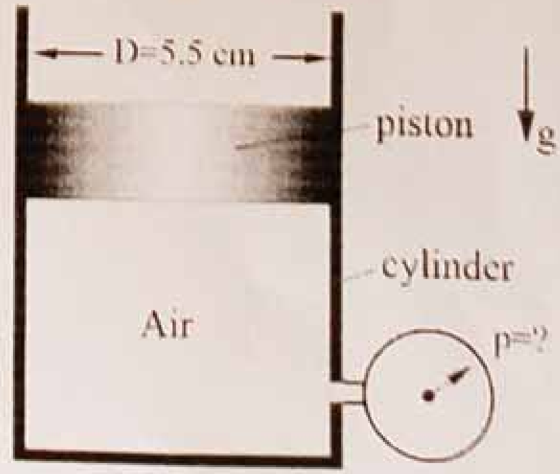
(3 marks)

$$\Sigma F_z = 0$$

$$(p - p_o)A = mg$$

$$\underbrace{p - p_o}_{\text{gauge press.}} = mg/A$$





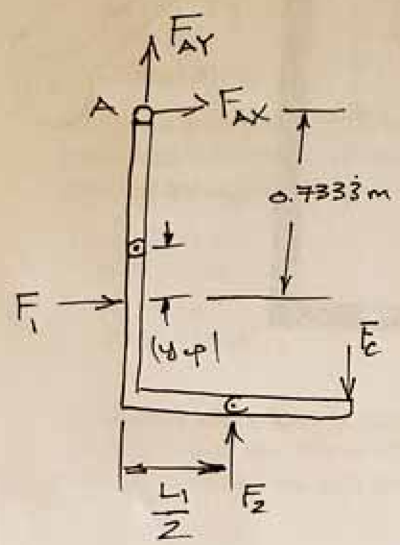
$$p - p_o = \frac{3.2\,\text{kg}\,(9.81\,\text{m/s}^2)}{\frac{\pi\,(.055)^2\,\text{m}^2}{4}} = 13.2\,\text{kPa (g)}$$

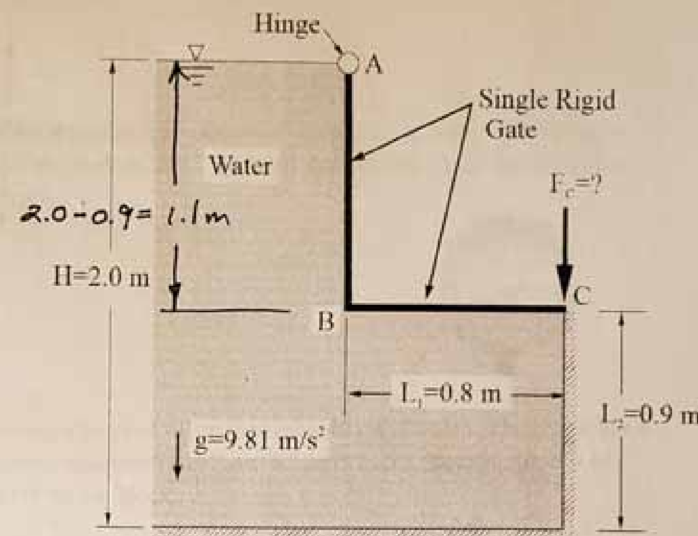
ANS

B2. Water (ρ=998 kg/m^3) is contained behind a rigid L-shaped gate (A-B-C). The gate has a depth of 0.5 m (into the page). The entire gate (A-B-C) is rigid and rotates about a hinge at point A. The weight of the gate can be neglected.

(a) Draw a <u>separate and fully labelled</u> free body diagram of the gate. Show the locations and directions of all the forces. ***(2 marks)***

(b) For the dimensions shown on the sketch, calculate the minimum vertical force (F_C) applied at point C required to keep the gate from opening. ***(8 marks)***





$$F_1 = \gamma h_{CG} A_{AB} = 9790\,\frac{N}{m^3}\left(\frac{1.1m}{2}\right)1.1m\,(0.5m) = 2961\,N \rightarrow$$

$$I_{xx} = \frac{bh^3}{12} = \frac{0.5m\,(1.1m)^3}{12} = 0.05546\,m^4$$

$$y_{cp} = \frac{-I_{xx}\sin\theta}{h_{CG}A} = \frac{-0.05546\,m^4 \sin 90^\circ}{(0.55m)\;0.55m^2} = -0.183\dot{3}$$

$$\text{Moment arm }(F_1):\quad \frac{1.1m}{2} + 0.183\dot{3} = 0.733\dot{3}\,m$$

$$F_2 = \gamma h_{BC} A_{BC} = 9790\,\frac{N}{m^3}(1.1\,m)(0.4\,m^2) = 4308\,N \uparrow$$

$\Sigma M_A = 0$ (AVOIDS UNKNOWN HINGE FORCES)

$$F_c L_1 = F_1\left(\frac{H-L_2}{2} - y_{cp}\right) + F_2\frac{L_1}{2}$$

$$F_c = \frac{2961N\,(0.7333m) + 4308N\,(0.4m)}{0.8m} = 4870\,N \downarrow$$

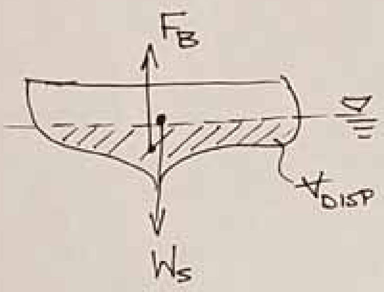
B3. The hull of a ship has a total internal volume of 24,300 m³. Without cargo, the ship weighs 26,500 kN. The ship floats in sea water with a density of ρ=1025 kg/m³.

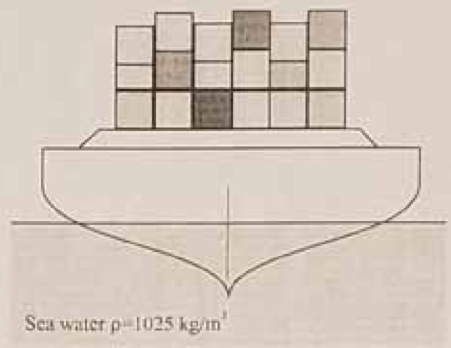
$\llcorner\ \gamma_W = \rho g = 10{,}055\ \mathrm{N/m^3}$

(a) Draw a separate free body diagram for the ship without cargo. Calculate the percentage of the hull's volume that will be **above** the waterline when the ship has no cargo. *(4 marks)*

(b) When fully loaded, one half of the ship's hull volume is below the waterline. Draw a separate free body diagram for the ship with cargo. Calculate the mass of the cargo in metric tonnes. Note: 1 tonne=1000 kg *(6 marks)*

(a)



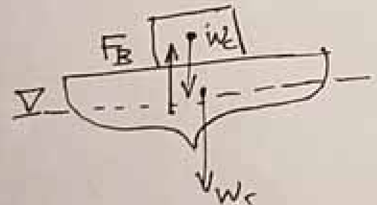


FLOATING $\therefore\ W_S = F_B = \gamma_W \forall_{DISP}$

$$\forall_{DISP} = \frac{W_S}{\gamma_W} = \frac{26500\times10^3\ \mathrm{N}}{1025\,\frac{kg}{m^3}\left(9.8\,\frac{m}{s^2}\right)} = 2635\ m^3$$

$$\%\ \text{HULL } \underline{\text{ABOVE}} \text{ WATERLINE} = \frac{24{,}300 - 2635}{24{,}300} \times 100\% = 89.1\% \quad \text{ANS/}$$

(b)



FLOATING

$W_C + W_S = F_B$

$$W_C = F_B - W_S = \gamma_W \forall_{DISP} - W_S$$

$$W_C = 10{,}055\,\frac{N}{M^3}\left(\frac{24300}{2}\,m^3\right) - 26{,}500\times10^3\ N$$

$$= 122{,}171\times10^3\ N - 26{,}500\times10^3\ N = 95671\ kN$$

$$(9.567\times10^8\ N)$$

$$m_C = \frac{W_C}{g} = \frac{9.567\times10^8\ N}{9.81\ m/s^2} = 9.75\times10^6\ kg$$

$$m_C = 9.75\times10^6\ kg\ \frac{1}{1000}\frac{tonne}{kg} = 9750\ \text{tonnes} \quad \text{ANS/}$$