

Print Name: SOLUTIONS Section # _____ Student ID: _____

RYERSON UNIVERSITY
Department of Mechanical and Industrial Engineering
MEC 516/BME 516
Fluids Mechanics
Common Midterm Test
Friday, Oct 17, 2014
4:10 to 6:00pm
Examiners: Drs. Alan Fung, David Naylor, and Tooraj Yousefi

- 1) All questions are to be answered. All answers must be written within this test booklet. You can use both sides of the page if you need to.
 - 2) Students are allowed to have 1) a **calculator**, and 2) **stationery** (pen, pencil, eraser, ruler). There should be no other materials including cell phone, computer, PDA, communication device, etc. with the student. All other personal belongings have to be stowed away under the black/whiteboard in front of the classroom.
 - 3) Please make sure you present your solution in a proper, clear and methodological way, including schematics and proper units, that is required by this course, otherwise, marks will be deducted even if you get the correct answer!
 - 4) Please present your solution step by step clearly and label all variables to indicate where they are coming from and the equation number whenever applicable. Highlight your final answer(s) with proper unit(s). Failure to do so will result in mark deduction.
 - 5) Make sure you write down your name, section #, and student ID.
 - 6) All questions are clear enough to conduct the analysis. There should be no reason for you to ask question. Your TAs DO NOT know anything about the test and they will not answer any question related to the test. (If you are really in doubt, make the necessary assumption.)
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READ THIS PAGE CAREFULLY

DO NOT TURN THE PAPER OVER

Print Name: SOLUTION Section # _____ Student ID: _____
 Question #1 (25%): The pressure difference, ΔP , across a partial blockage in an artery (called a *Stenosis*) is approximated by the equation

$$\Delta P = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity ($FL^{-2}T$), ρ is the blood density (ML^{-3}), D is the artery diameter, A_0 is the area of unobstructed artery, A_1 is the area of the stenosis. Determine the dimensions of the constants K_v and K_u . Would this equation be valid in any system of units? (Hint: Prove this equation is dimensionally homogeneous)

Solution: USE OF FLT θ SYSTEM

$$A_0 = \{L^2\}$$

$$\Delta P = \left\{ \frac{F}{L^2} \right\}$$

$$V = \left\{ \frac{L}{T} \right\}$$

$$D = \{L\}$$

$$A_1 = \{L^2\}$$

$$\mu = \left\{ \frac{FT}{L^2} \right\}$$

$$\rho = \left\{ \frac{M}{L^3} \right\}$$

$$\{F\} = \left\{ \frac{ML}{T^2} \right\}$$

$$\therefore M = \left\{ \frac{FT^2}{L} \right\}$$

(FROM $F = ma$)

$$\therefore \rho = \left\{ \frac{FT^2}{L^4} \right\}$$

EACH TERM MUST HAVE SAME UNITS AS LHS (ΔP)

$$\text{1ST TERM: } \left\{ \frac{F}{L^2} \right\} = \{K_v\} \left\{ \frac{FT}{L^2} \right\} \left\{ \frac{L}{T} \right\} \left\{ \frac{1}{L} \right\} = \{K_v\} \left\{ \frac{F}{L^2} \right\}$$

$$\therefore \{K_v\} = \{-\} \text{ UNITLESS / DIMENSIONLESS.}$$

2ND TERM:

$$\left\{ \frac{F}{L^2} \right\} = \{K_u\} \left\{ \frac{\{L^2\} - \{-\}}{\{L^2\}} \right\}^2 \left\{ \frac{FT^2}{L^4} \right\} \left\{ \frac{L^2}{T^2} \right\} = \{K_u\} \left\{ \frac{F}{L^2} \right\}$$

UNITLESS

$$\therefore \{K_u\} = \{-\} \text{ UNITLESS. (DIMENSIONLESS)}$$

YES. THIS EQUATION WOULD WORK IN ANY CONSISTENT SET OF UNITS.

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Question #1 (25%): The pressure difference, ΔP , across a partial blockage in an artery (called a *Stenosis*) is approximated by the equation

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where V is the blood velocity, μ the blood viscosity ($FL^{-2}T$), ρ is the blood density (ML^{-3}), D is the artery diameter, A_0 is the area of unobstructed artery, A_1 is the area of the stenosis. Determine the dimensions of the constants K_v and K_u . Would this equation be valid in any system of units? (Hint: Prove this equation is dimensionally homogeneous)

Solution: USE OF MLT θ SYSTEM FROM $\vec{F} = m\vec{a}$

$$\Delta P = \left\{ \frac{F}{L^2} \right\} \quad \text{USE } \{F\} = \left\{ \frac{ML}{T^2} \right\} \quad \therefore \Delta P = \left\{ \frac{M}{LT^2} \right\}$$

$$\mu = \left\{ \frac{FT}{L^2} \right\} = \left\{ \frac{M}{LT} \right\} \quad V = \left\{ \frac{L}{T} \right\} \quad D = \{L\}$$

$$A_1 = \{L^2\} \quad A_0 = \{L^2\} \quad \rho = \left\{ \frac{M}{L^3} \right\}$$

$$\text{1ST TERM: } \left\{ \frac{M}{LT^2} \right\} = \{K_v\} \left\{ \frac{M}{LT} \right\} \left\{ \frac{L}{T} \right\} \left\{ \frac{L}{L} \right\} = \{K_v\} \left\{ \frac{M}{LT^2} \right\}$$

$$\therefore \{K_v\} = \{-\} \quad \text{UNITLESS}$$

$$\text{2ND TERM: } \left\{ \frac{M}{LT^2} \right\} = \{K_u\} \left(\frac{\{L^2\}}{\{L^2\}} - \{-\} \right)^2 \left\{ \frac{M}{L^3} \right\} \left\{ \frac{L^2}{T^2} \right\} = \{K_u\} \left\{ \frac{M}{LT^2} \right\}$$

NO UNITS

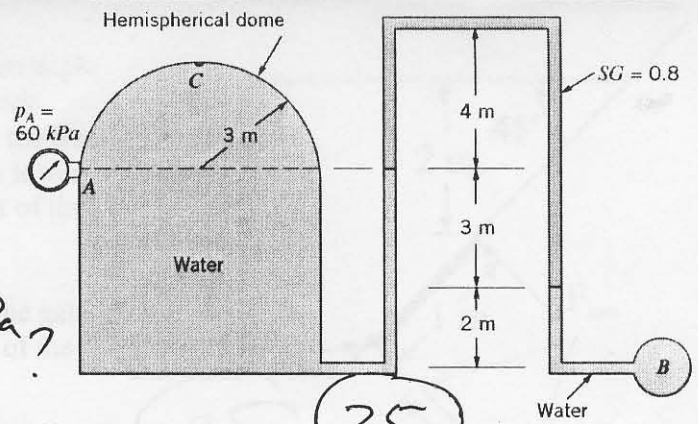
$$\therefore \{K_u\} = \{-\} \quad \text{UNITLESS. (DIMENSIONLESS)}$$

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Question #2 (25%): A closed cylindrical tank completely filled with water has a hemispherical dome and is connected to an inverted piping system as shown in the attached figure. The liquid in the top part of the piping system has a specific gravity of 0.8, and the remaining parts of the system are filled with water. γ of water is $9,800 \text{ N/m}^3$. If the pressure gage reading at A is 60 kPa, determine



- 10 a) the pressure in pipe B in kPa, and P_B in kPa?
 15 b) the pressure head, in millimeters of mercury, at the top of the dome (point C). (γ of mercury is $133,000 \text{ N/m}^3$)

P_C in mm Hg?

25

Solution:

(a) $P_A + SG \gamma_{H_2O} (3m) + \gamma_{H_2O} (2m) = P_B$ $P_A = 60 \text{ kPa}$

$$\Rightarrow P_B = 60 \text{ kPa} + [(0.8)(9800)(3) + (9800)(2)] / 1000$$

$$P_B = 60 \text{ kPa} + 43120 \text{ Pa} \quad (10)$$

$$P_B = 103.12 \text{ kPa} \quad \checkmark$$

(b) $P_C = P_A - \gamma_{H_2O} (3m)$

$$= 60 \text{ kPa} - 9800 (3) / 1000$$

$$= 30.6 \text{ kPa} \quad \text{or} \quad 30.6 \times 10^3 \text{ Pa}$$

Since $h_c = \frac{P_C}{\gamma_{Hg}}$

$$= \frac{30.6 \times 10^3}{133000}$$

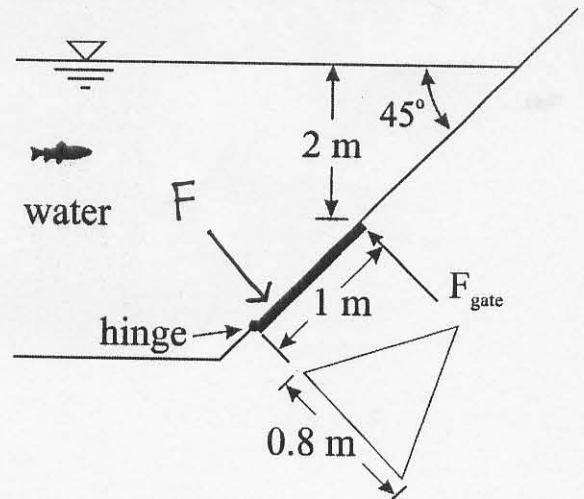
$$= 0.230 \text{ m of Hg}$$

$$\Rightarrow h = 230 \text{ mm Hg}$$

15

Question #3 (25%): As shown in the attached figure, a flat triangular gate is located on the side of a reservoir that has an angle of 45° . The triangular gate is 1 metre high and 0.8 metres wide. The apex of the triangular gate is located two metres below the free surface, as shown in the sketch. The gate is hinged along its lower edge. The density of water is 998 kg/m^3 . Neglect the weight of the gate. Calculate:

- 17 (a) The total hydrostatic force and its line of action on the gate.
 8 (b) The minimum force (F_{gate}) applied at the apex (top) of the gate needed to keep the gate closed.

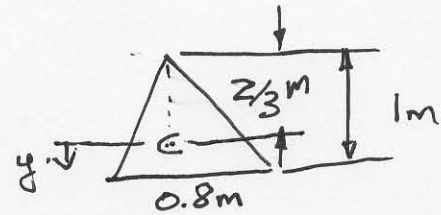


Solution:

$$F = \gamma h_{CG} A = \rho g h_{CG} A$$

$$h_{CG} = 2 \text{ m} + \frac{2}{3} \text{ m} \sin 45^\circ = 2.471 \text{ m}$$

$$F = 998 \frac{\text{kg}}{\text{m}^3} (9.81 \frac{\text{m}}{\text{s}^2}) 2.471 \text{ m} (0.4 \text{ m}^2) = 9678 \text{ N}$$



$$A = \frac{1}{2} b h = \frac{1}{2} (0.8) (1) \text{ m}^2 = 0.4 \text{ m}^2$$

$$y_{CP} = \frac{-I_{xx} \sin \theta}{h_{CG} A}$$

$$I_{xx} = \frac{b h^3}{36} = \frac{0.8 (1)^3}{36} = 0.0222 \text{ m}^4$$

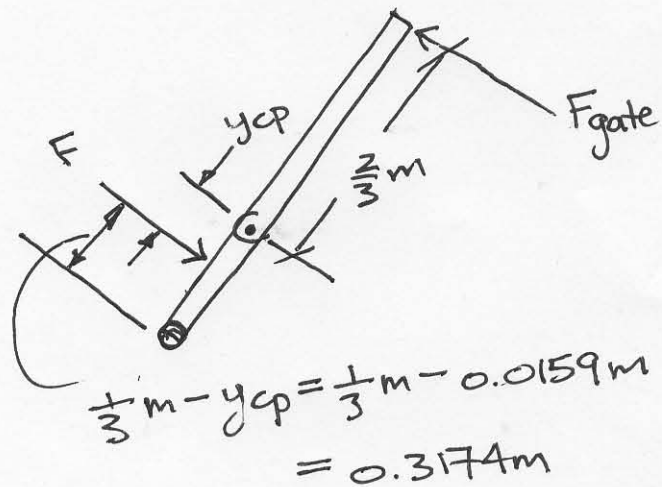
$$y_{CP} = \frac{-0.0222 \text{ m}^4 \sin 45^\circ}{2.471 \text{ m} (0.4 \text{ m}^2)} = -0.0159 \text{ m}$$

$$\sum M_{\text{HINGE}} = 0$$

$$F \left(\frac{1}{3} \text{ m} - 0.0159 \text{ m} \right) = F_{\text{gate}} (1)$$

$$F_{\text{gate}} = \frac{0.3174}{1} F$$

$$F_{\text{gate}} = 0.3174 (9678 \text{ N}) = 3070 \text{ N} \curvearrowright$$

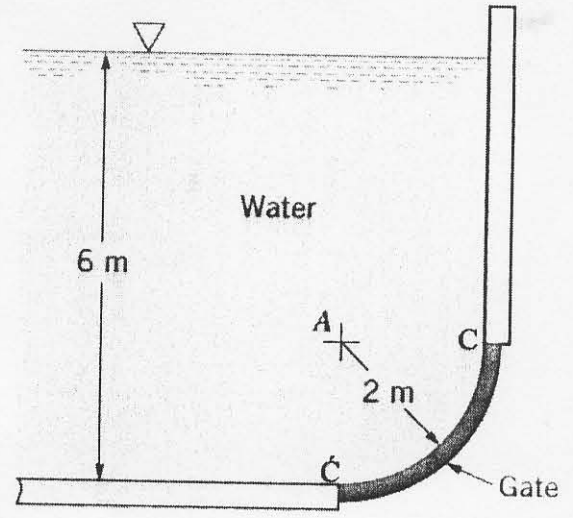


Question #4 (25%): A 3-m-long (into the paper) curved gate C-C' is located in the side of a reservoir containing water as shown in the attached figure. Determine

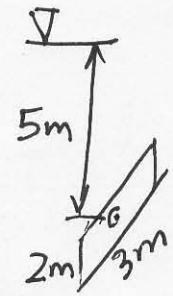
- I (a) The magnitude of the horizontal and vertical components of the force of the water on the gate.
- II (b) The net force to the gate and its direction when is passing through point A.

$\gamma_{\text{water}} = 9806 \text{ N/m}^3$

Solution:



(a) $F_H = \gamma_w h_{CG} A_{\text{proj}}$

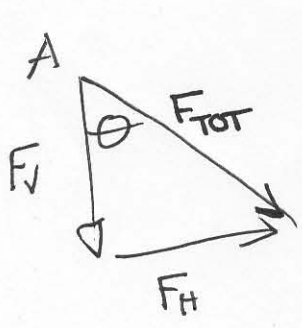


$$F_H = (9806 \text{ N/m}^3) (5 \text{ m}) (6 \text{ m}^2) = 294.1 \text{ kN} \rightarrow$$

$$F_V = \gamma V = \gamma (V_{\text{ABOVE CC'}}) = 9806 \frac{\text{N}}{\text{m}^3} \left(4 \times 2 + \frac{\pi (2)^2}{4} \right) 3 \text{ m}^3 = 327.8 \text{ kN} \downarrow$$

33.428 m^3

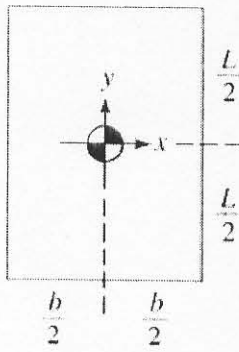
(b) $F_{\text{TOT}} = \sqrt{F_V^2 + F_H^2} = \sqrt{294.1^2 + 327.8^2} = 440 \text{ kN}$



$\tan \theta = \frac{F_H}{F_V}$

$\theta = \tan^{-1} \left(\frac{294.1}{327.8} \right) = \tan^{-1} (0.897)$

$\theta = 41.9^\circ \text{ ANS.}$

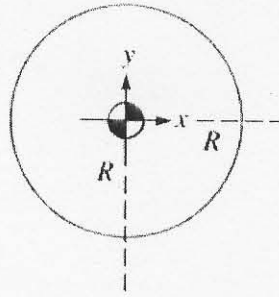


$$A = bL$$

$$I_{xx} = \frac{bL^3}{12}$$

$$I_{yy} = 0$$

(a)

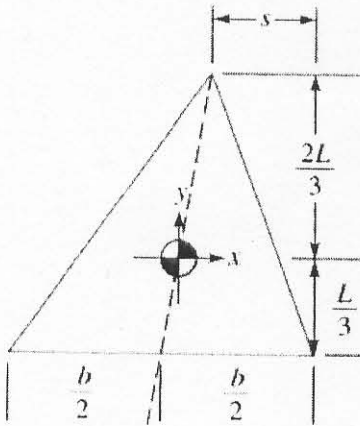


$$A = \pi R^2$$

$$I_{xx} = \frac{\pi R^4}{4}$$

$$I_{yy} = 0$$

(b)

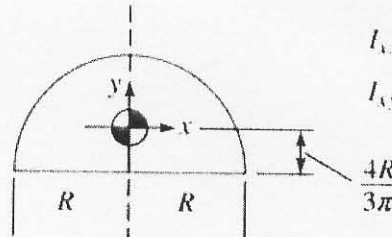


$$A = \frac{bL}{2}$$

$$I_{xx} = \frac{bL^3}{36}$$

$$I_{yy} = \frac{b(b-2s)L^2}{72}$$

(c)



$$A = \frac{\pi R^2}{2}$$

$$I_{xx} = 0.10976R^4$$

$$I_{yy} = 0$$

(d)

Some Useful Equations:

Ideal Gas Law: $P = \rho RT$

Pressure: $P_{absolute} = P_{gage} + P_{atmospheric}$

Shear Stress for Newtonian Fluids: $\tau = \frac{F}{A} = \mu \frac{du}{dy}$

Rise in Fluid Column due to Capillary Effect of Surface Tension: $h = \frac{2Y \cos \theta}{\gamma R}$

Hydrostatic Pressure: $\frac{dP}{dz} = -\gamma$

Hydrostatic Forces on Plane Surfaces: $F = \gamma h_{CG} A$ $y_{CP} = -\frac{I_{xx} \sin \theta}{h_{CG} A}$ $x_{CP} = -\frac{I_{xy} \sin \theta}{h_{CG} A}$

Buoyancy Force: $F_B = \gamma V_{displaced} = W_{floatingbody}$