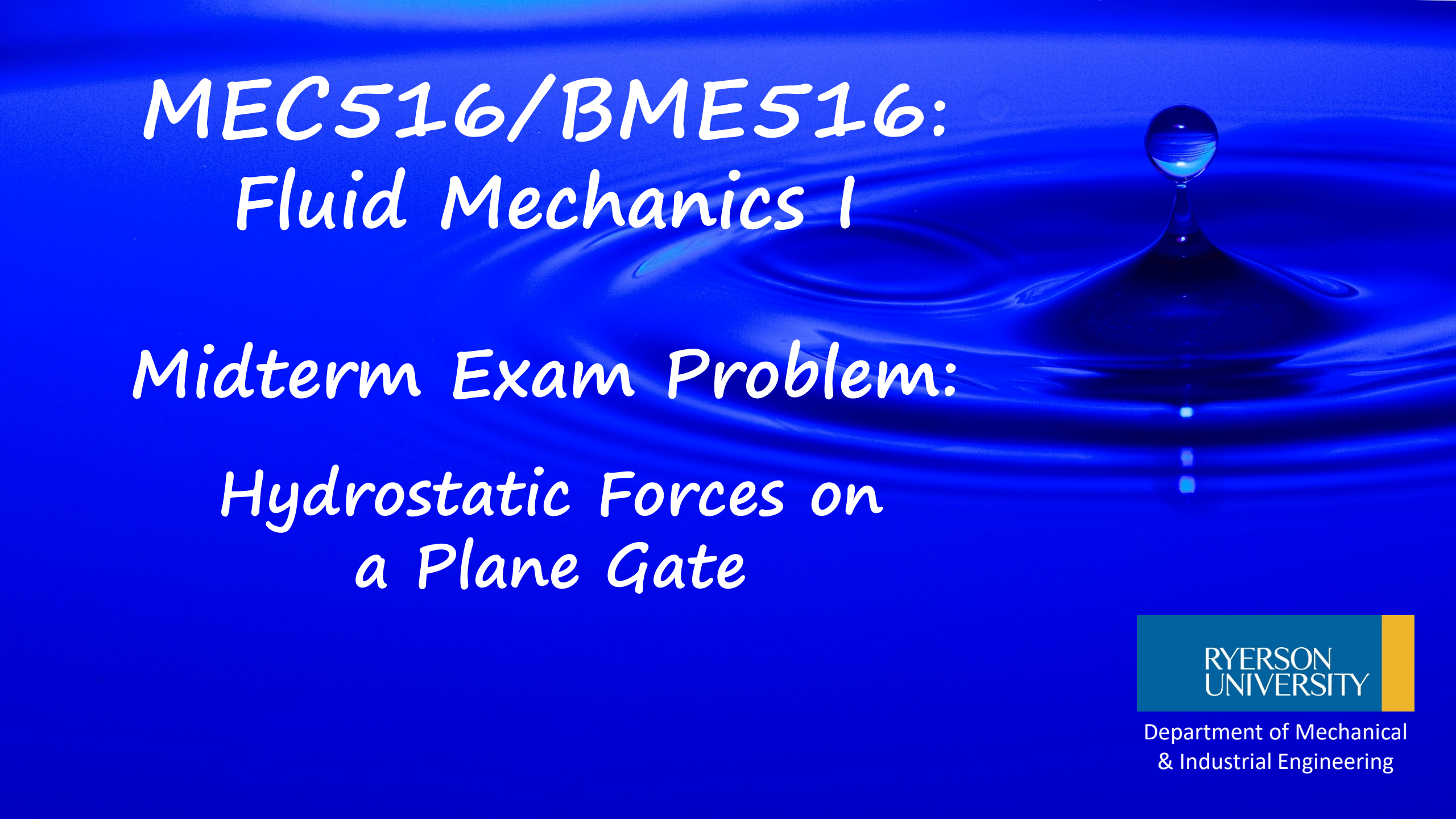
MEC516/BME516: Fluid Mechanics I

Midterm Exam Problem:

Hydrostatic Forces on a Plane Gate



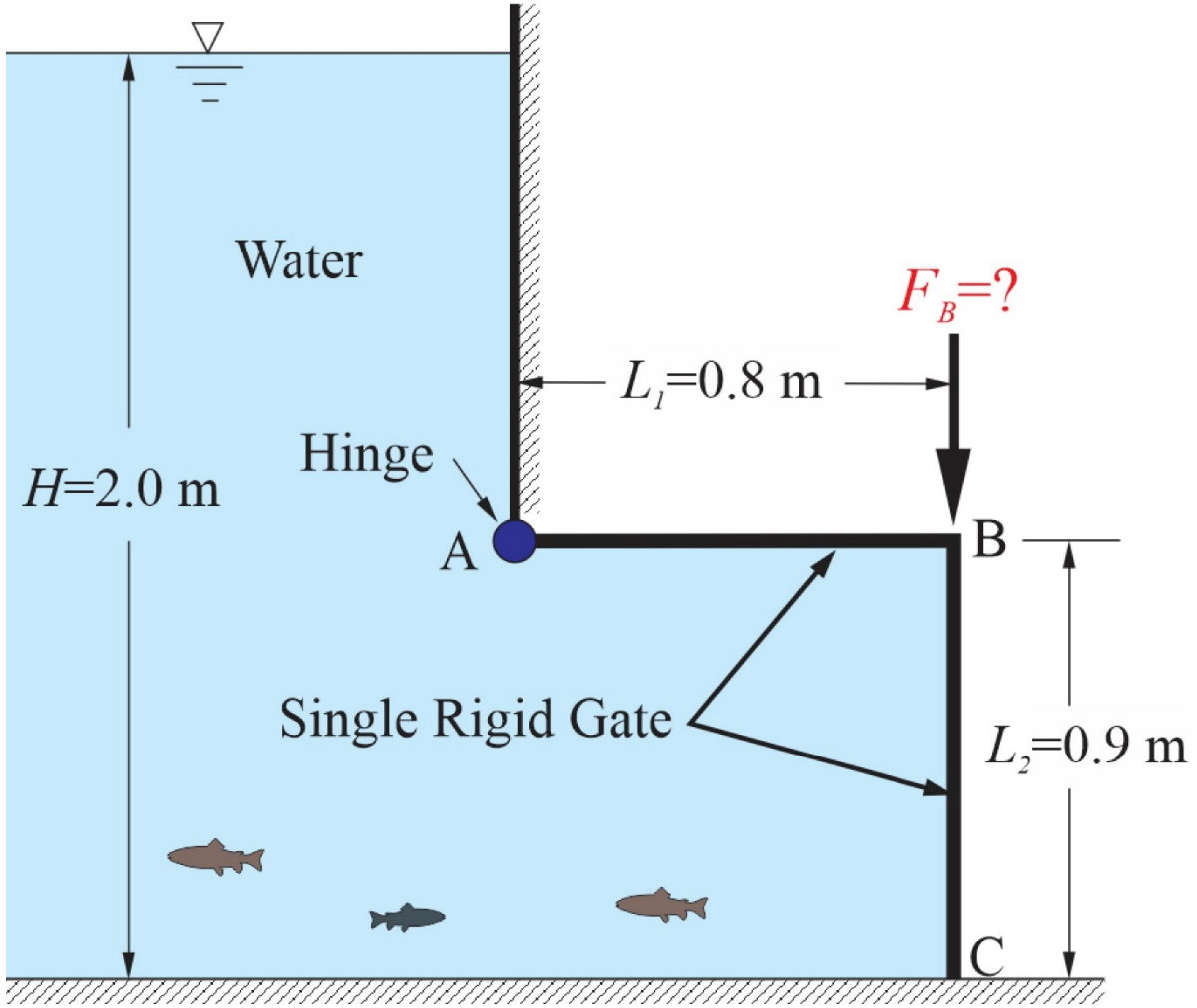
RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

Midterm Problem (Fall 2019)

Water (ρ=998 kg/m^3) is contained behind a rigid L-shaped gate (ABC). The gate has a depth of 0.5 m (into the page). The gate rotates about a hinge at point A. Neglect the weight of the gate.

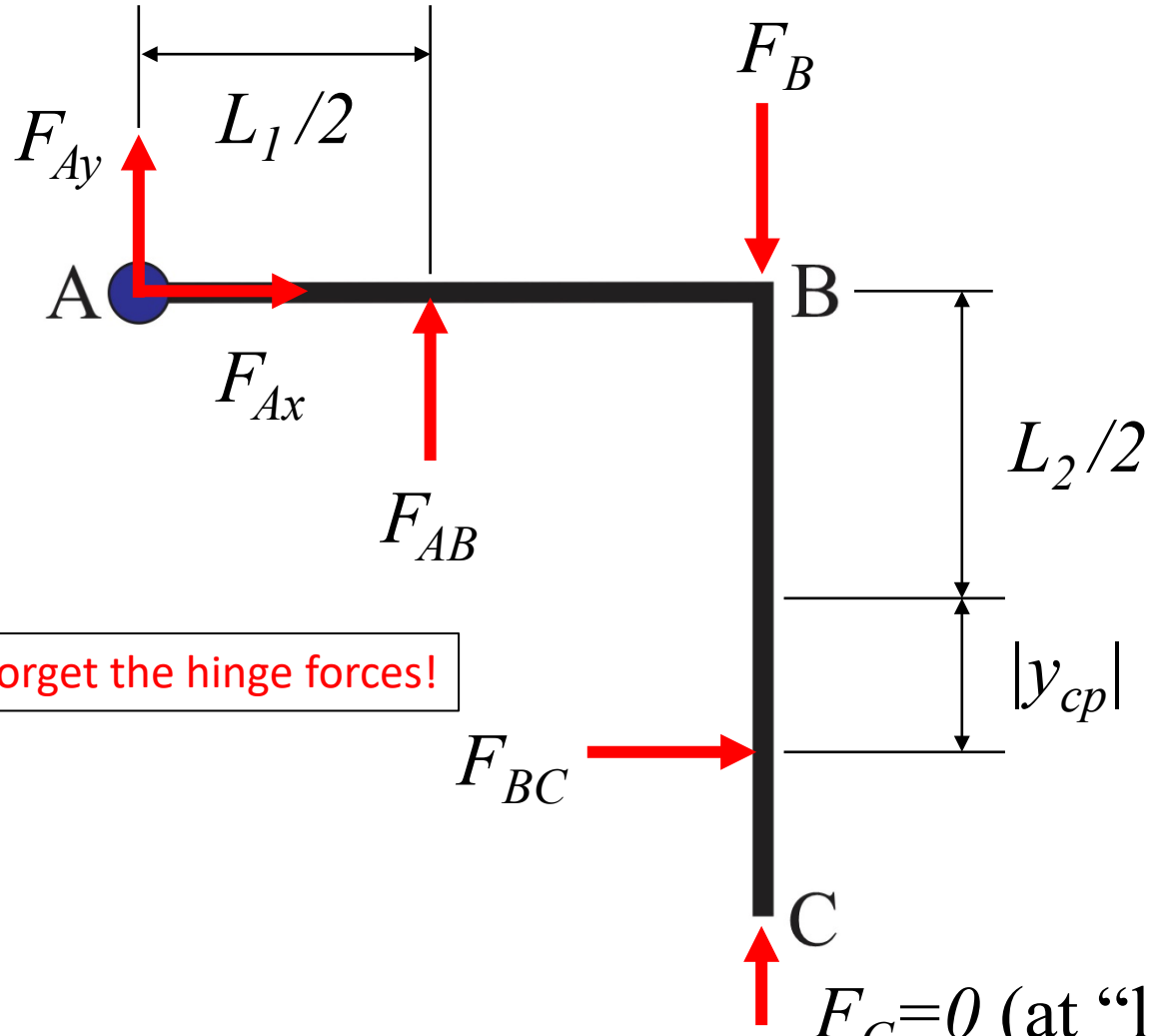
(a) Draw a fully-labelled free body diagram of the gate

(b) Calculate the minimum vertical force (F_B) applied at point B required to keep the gate from opening

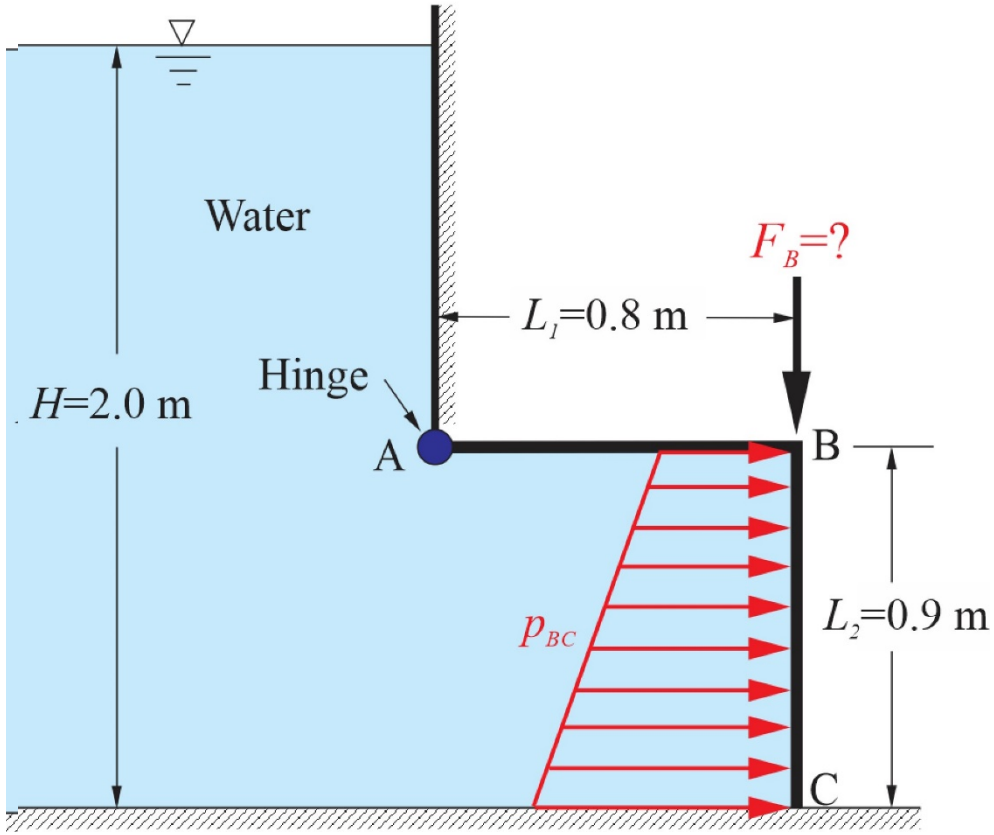


Solution

(a) Free body diagram of the gate ABC

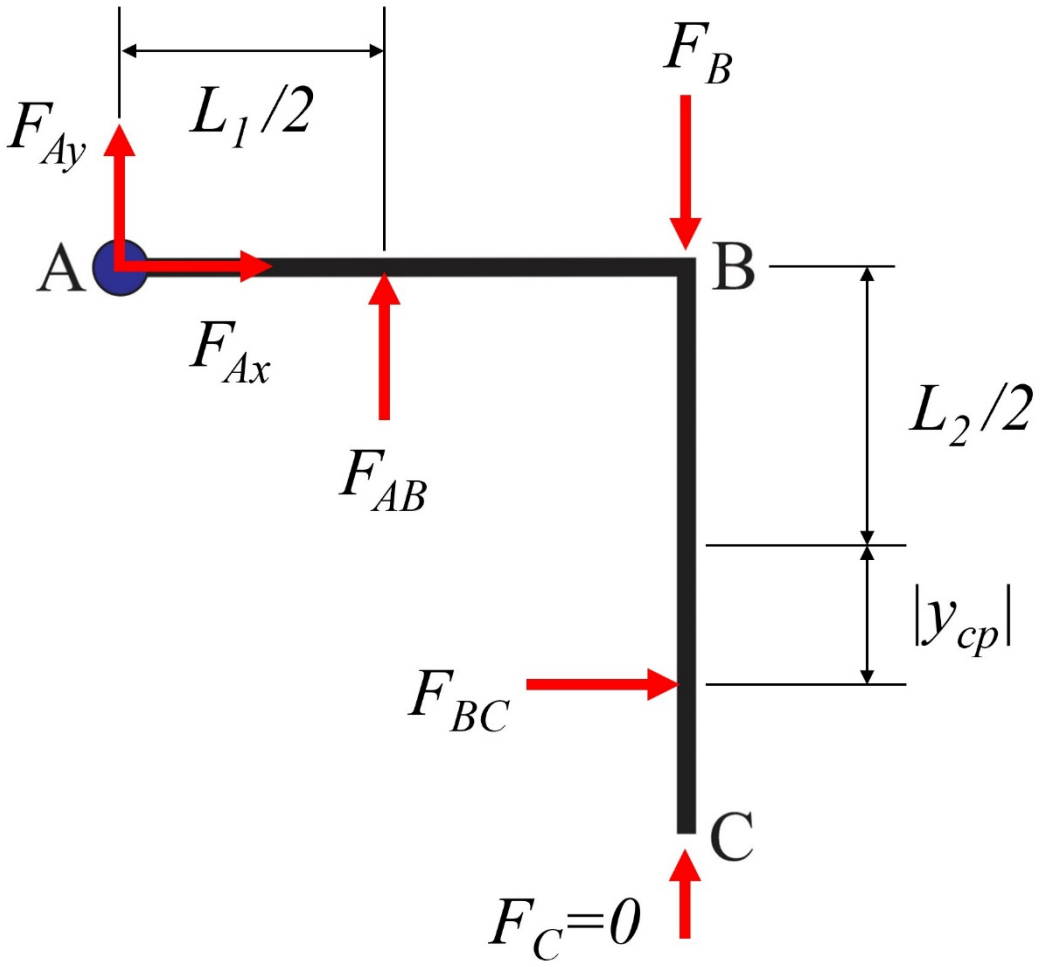


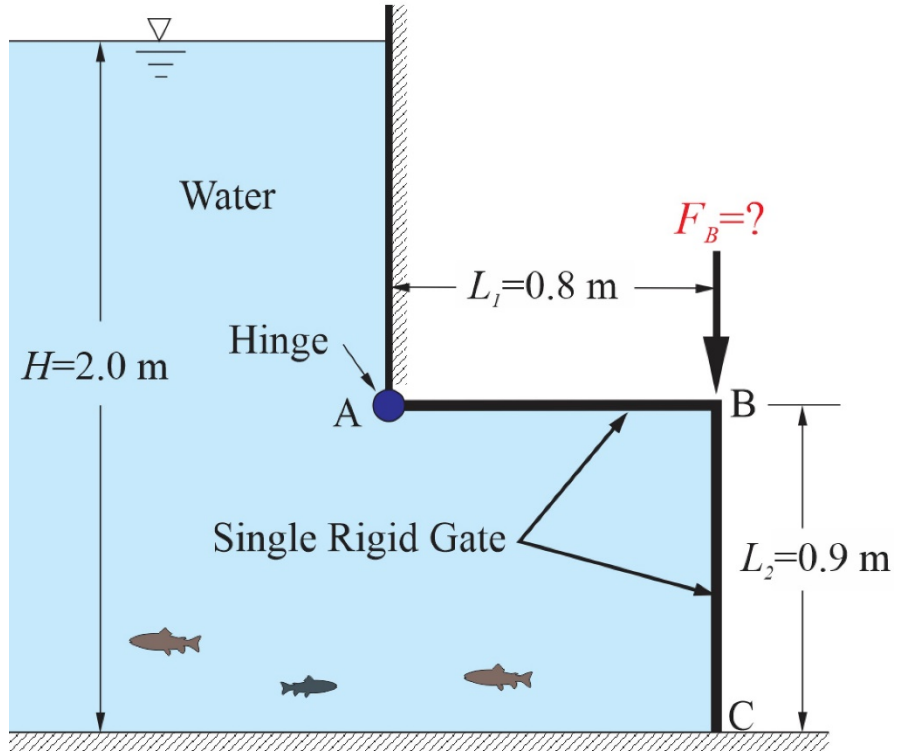
Don't forget the hinge forces!



Solution

(a) Free body diagram of the gate ABC



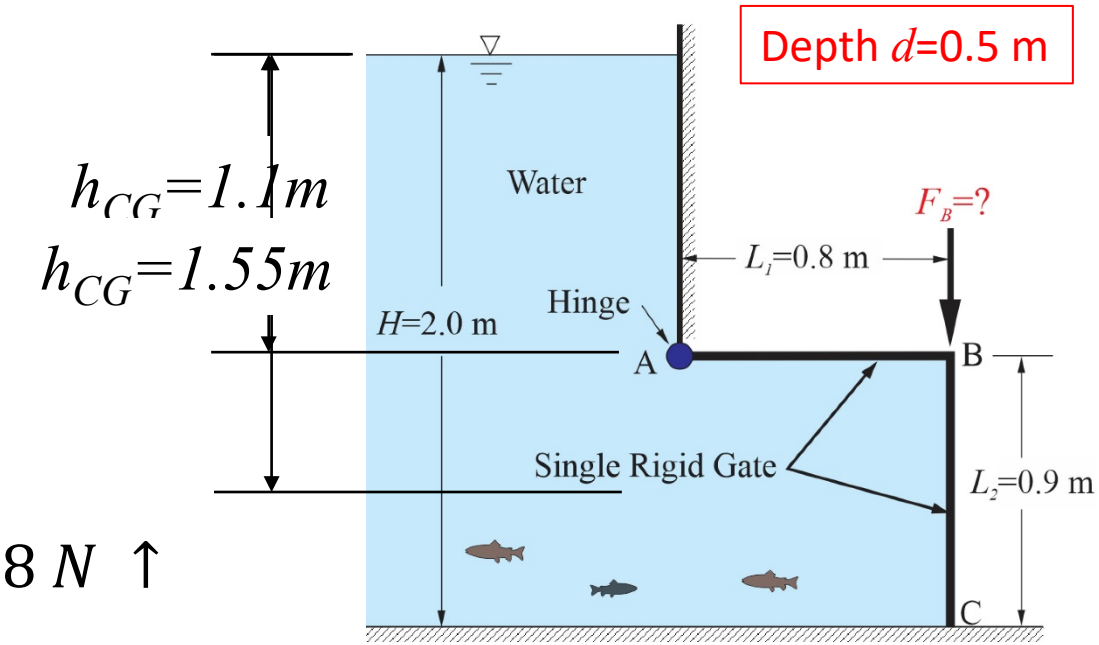


Do not put forces on problem diagram. Not a FBD!

Answer (a)

Solution

(b) Calculate the minimum vertical force (F_B)



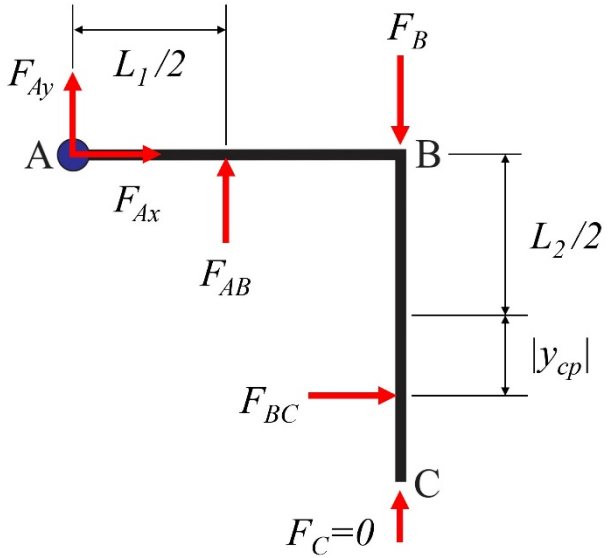
- Force on horizontal surface AB: $F_{AB} = \gamma_w h_{CG} A_{AB}$

$$F_{AB} = 9790 \frac{N}{m^3} (2.0 - 0.9) m \left(0.4\, m^2\right) = 4308\ N \uparrow$$

(at centre of AB)

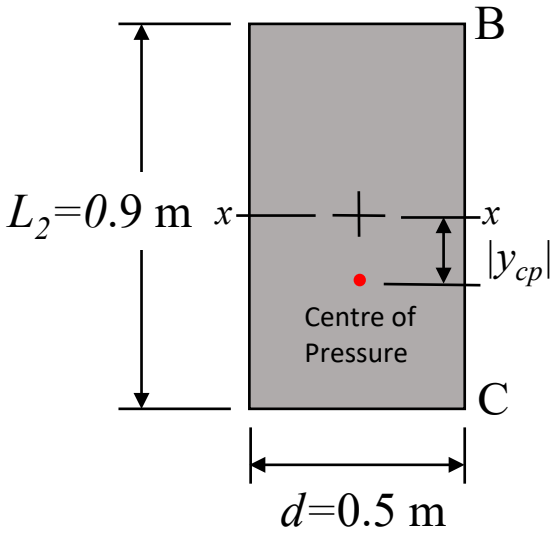
- Force on vertical surface BC: $F_{BC} = \gamma_w h_{CG} A_{BC}$

$$F_{BC} = 9790 \frac{N}{m^3} \left(2.0 - \frac{0.9}{2}\right) m \left(0.45\, m^2\right) = 6828\ N \rightarrow$$

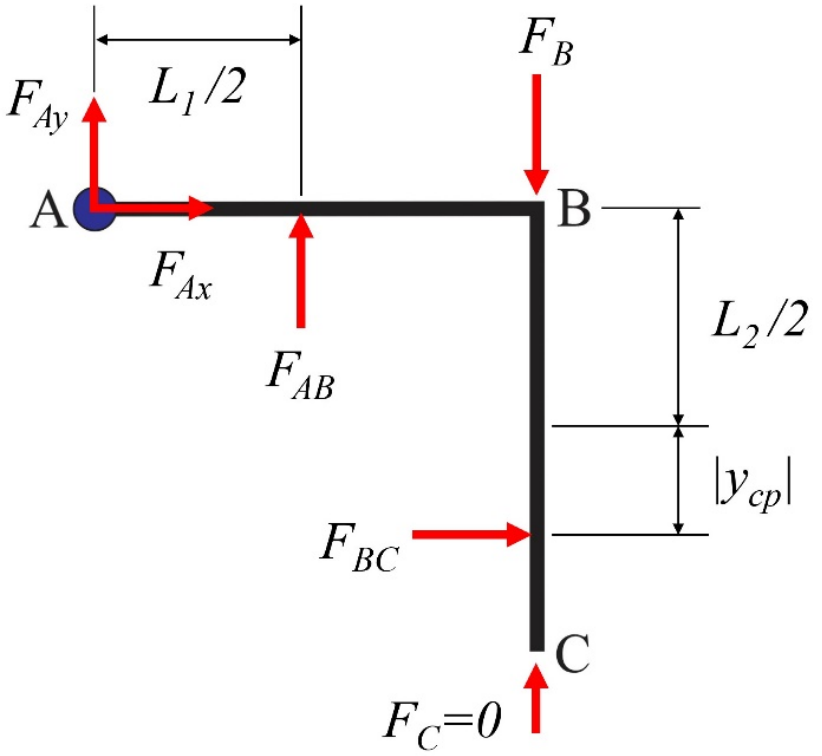


Solution

(b) Location of force F_{BC}



Depth d=0.5 m



$$y_{cp} = -\frac{I_{xx}\sin\theta}{h_{CG}A_{BC}}$$

$$I_{xx} = \frac{dL_2^3}{12} = \frac{0.5m(0.9m)^3}{12} = 0.03037\ m^4$$

Vertical gate

$$y_{cp} = -\frac{I_{xx}\sin\theta}{h_{CG}A_{BC}} = -\frac{0.03037\ m^4(\sin 90^o)}{(2.0-0.45)m(0.45\ m^2)} = -0.04355\ m$$

Indicates below centroid of BC

- We have all the forces and their locations

Solution

(a) Take moments about hinge at A:

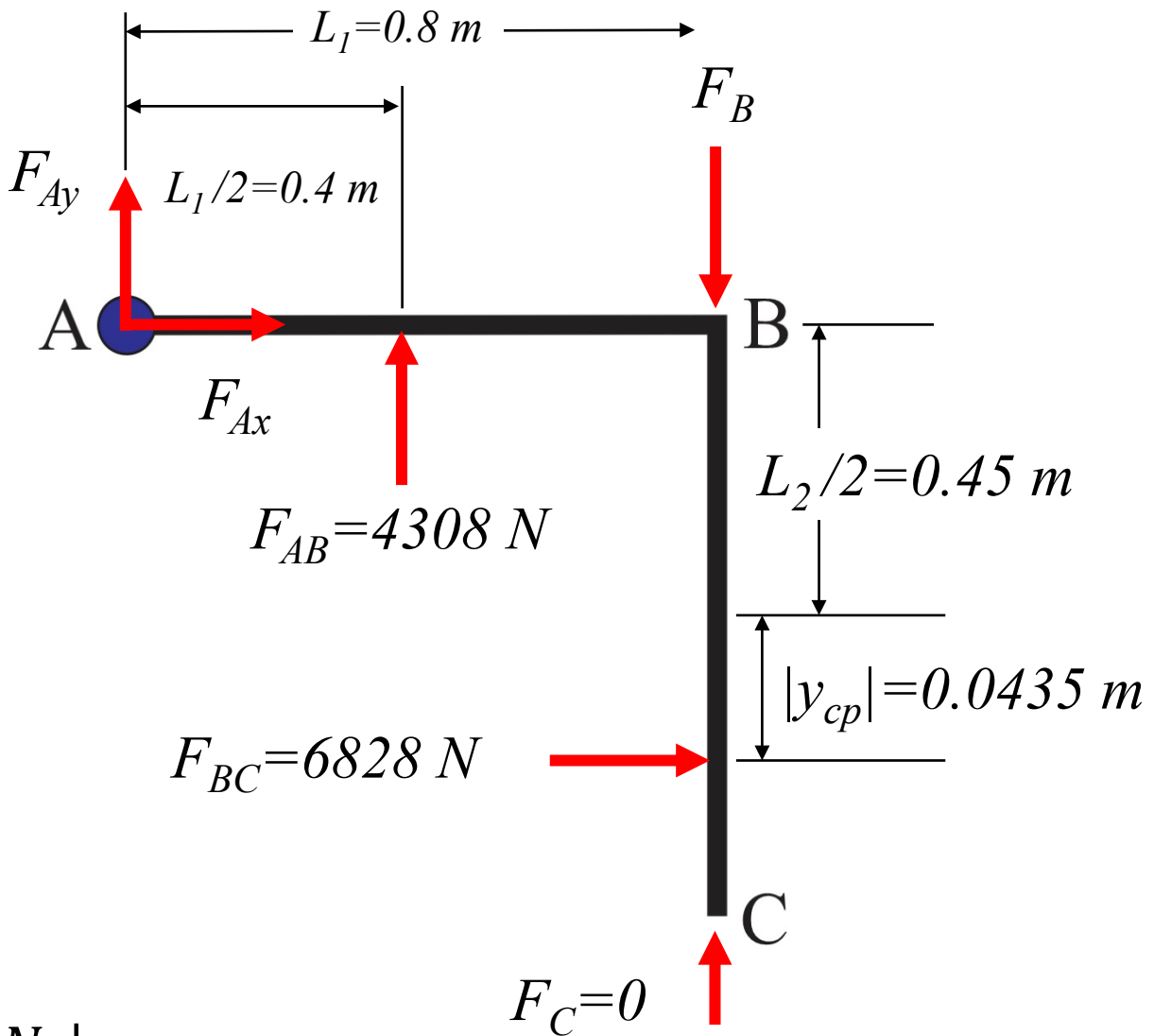
$$\sum M_A = 0 \qquad +\circlearrowleft$$

$$F_B L_1 - F_{AB}\frac{L_1}{2} - F_{BC}\left(\frac{L_2}{2} + |y_{cp}|\right) = 0$$

$$F_B = \left[F_{AB}\frac{L_1}{2} + F_{BC}\left(\frac{L_2}{2} + |y_{cp}|\right)\right] / L_1$$

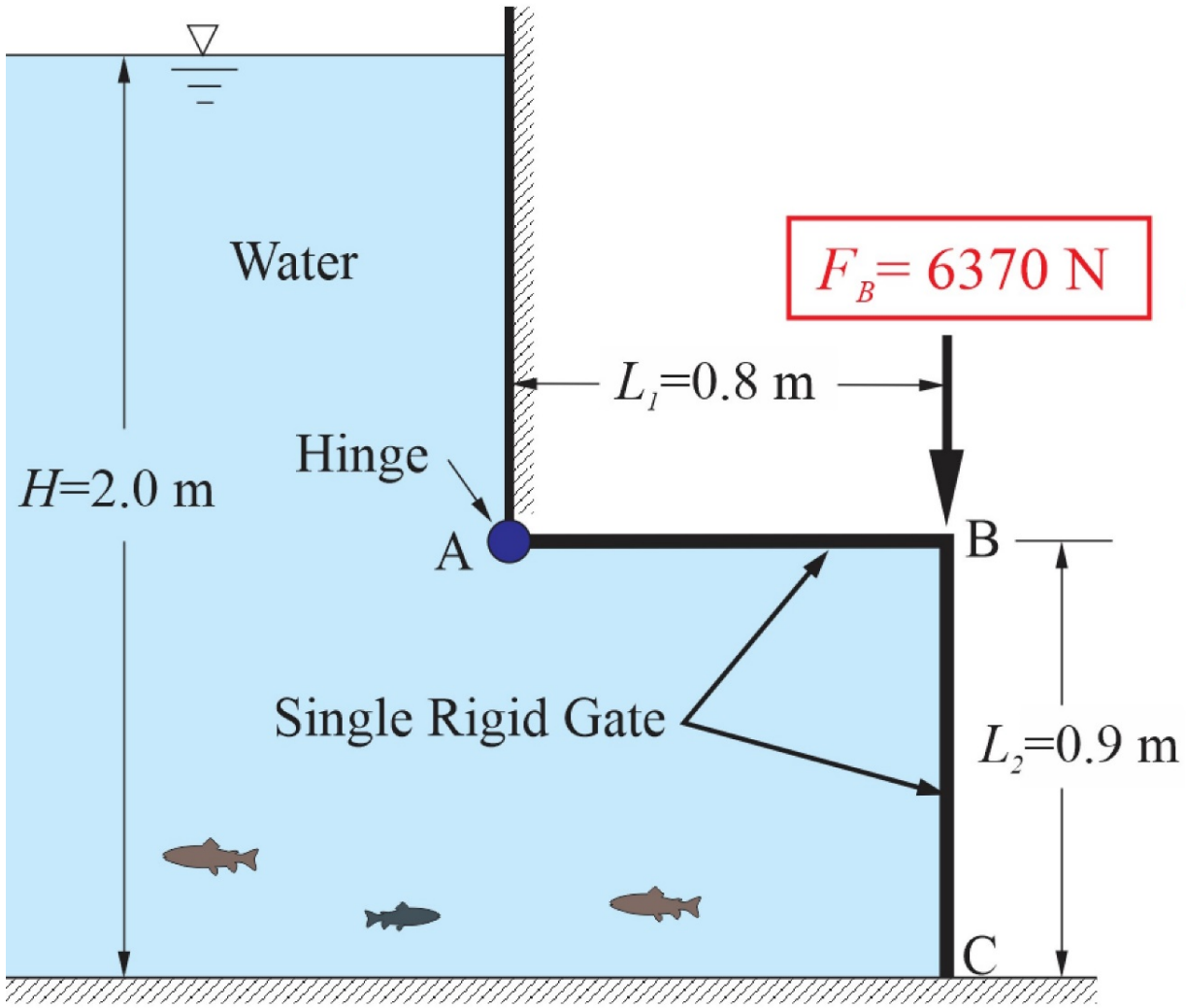
Minimum vertical force at B:

$$F_B = \frac{4308\ N\ (0.4\ m) + 6828\ N(0.4935\ m)}{0.8\ m} = 6370\ N \downarrow$$



Answer (b)

Solution



F_B= 6370 N Answer