Department of Mechanical and Industrial Engineering

FLUID MECHANICS I – MEC 516 (Online Lectures)

MIDTERM EXAM

DATE: Monday, October 16, 2023

TIME: 8:10am - 10:00am   Room: ENGLG11

EXAMINER: Dr. D. Naylor

DURATION: 1 hr 50 minutes

INSTRUCTIONS:

1. This is a CLOSED BOOK EXAM. Permitted aids are a non-communicating electronic calculator, and drawing/writing instruments (i.e., ruler, pens and pencils).
2. A table of centroids and second moments of area is included with this exam paper. A basic formula sheet is also included with this exam paper.
3. Prohibited items include textbooks, personal notes of any kind, cell phones and other wireless devices, laptop computers, etc. **Possession of a communication device will trigger charges of academic misconduct.**
4. A valid student identification card must be presented when attendance is taken.
5. Students may not borrow or lend any materials while the test is in progress.
6. Answer all questions on the exam paper in the space provided. Marks are indicated beside each question and in the table below.
7. To get full marks you must clearly show the formulas, methods and numbers used to solve the problem.
8. **For part marks you must use the nomenclature (variable symbols) given in the problem statement.** Also, for part marks be sure to give the correct units on intermediate results.
9. Marks will be deducted for incorrect or missing units.
10. Do not detach any sheets from this test paper.

Student Name (Please Print): SOLUTIONS

Student Number: ______________________ Section Number: ___________

Question	Mark
A1-A5	/10
Q1	/10
Q2	/10
Q3	/10
Total	/40

PART A - MULTIPLE CHOICE QUESTIONS

Each of the questions below is followed by several suggested answers. On the exam paper, **circle the ONE that is best**.

Questions *A1* to *A5* are worth ***2 marks*** each for a total of 10 marks.

A1. In this course we will be assuming that fluids are "Newtonian". A fluid is considered Newtonian if:

(a) The fluid obeys Newton's second law (F=ma).
(b) The local shear stress is linearly proportional to the local velocity gradient.
(c) The dynamic viscosity is not a function of temperature.
(d) The fluid cannot resist shear stress no matter how small.
(e) The viscosity is a function of the local velocity gradient, but constant over time.

A2. Using the nomenclature of Chapter 1, what are the ***dimensions*** of the square root of viscous shear stress (τ) divided by fluid density (ρ)?

$$\sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{N}{m^2}\frac{m^3}{kg}} = \sqrt{\frac{kg\,m}{s^2}\frac{m}{kg}} = \frac{m}{s}$$

$$= \left\{\frac{L}{T}\right\}$$

(a) $\{LT^{-1}\}$
(b) $\{L^2T^{-2}\}$
(c) $\{M^{-1}LT^{-1}\}$
(d) $\{L^2T^{-1}\}$
(e) $\{M^{-1/2}LT^{-1}\}$

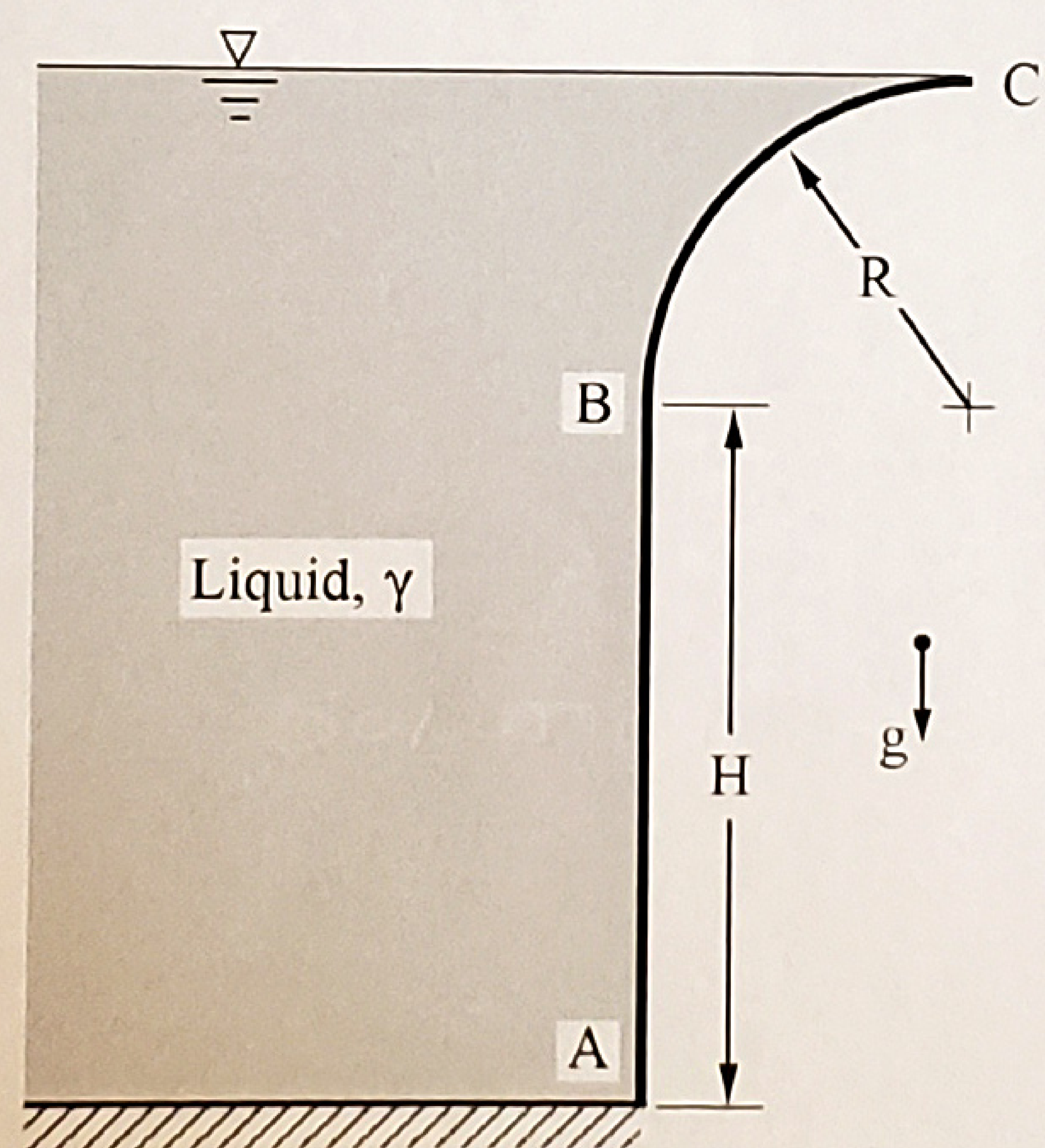
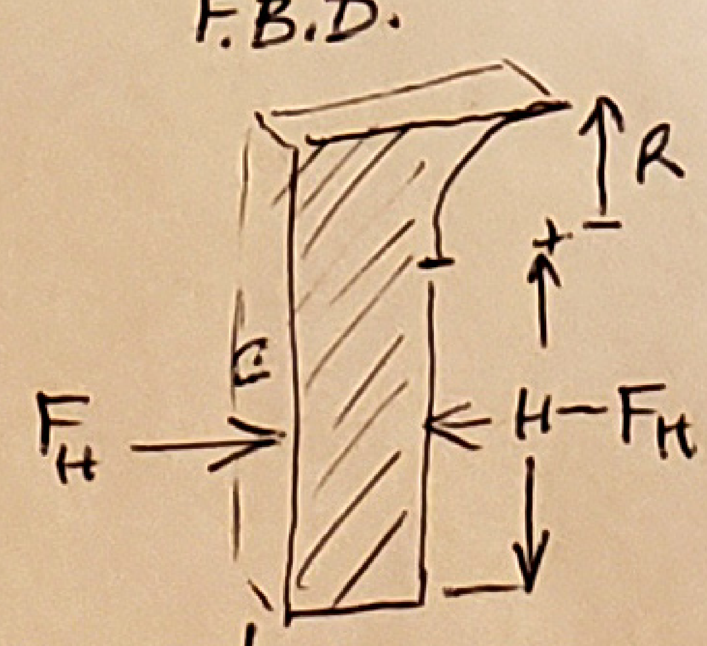


Figure for Questions *A3 and A4.*

A3. Referring to the diagram above, a liquid with specific weight (γ) is contained by a wall that consists of a straight vertical section (A-B) of height H and a curved section (B-C) with radius R. What is the total **horizontal** force of the liquid on wall A-B-C per unit depth (into the page)?

(a) $F_H = \gamma(R+H)^2$
(b) $F_H = \gamma(R+H)^2/2$
(c) $F_H = \gamma\left(R+\frac{H}{2}\right)H$
(d) $F_H = \gamma R^2(1-\frac{\pi}{4})$
(e) $F_H = \gamma(R+\frac{H}{2})(R+H)$

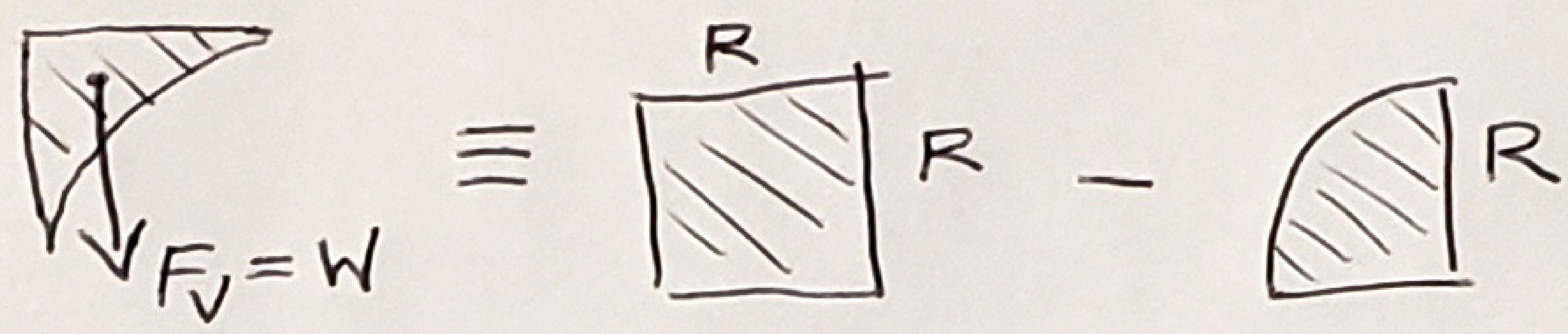
F.B.D.



$$F_H = \gamma h_{CG} A$$
$$= \gamma\left(\frac{R+H}{2}\right)(R+H)(1)$$

A4. Referring to the diagram above, a liquid with specific weight (γ) is contained by a wall that consists of a straight vertical section (A-B) of height H and a curved section (B-C) with radius R. What is the total **vertical** force of the liquid on wall A-B-C per unit depth (into the page)?

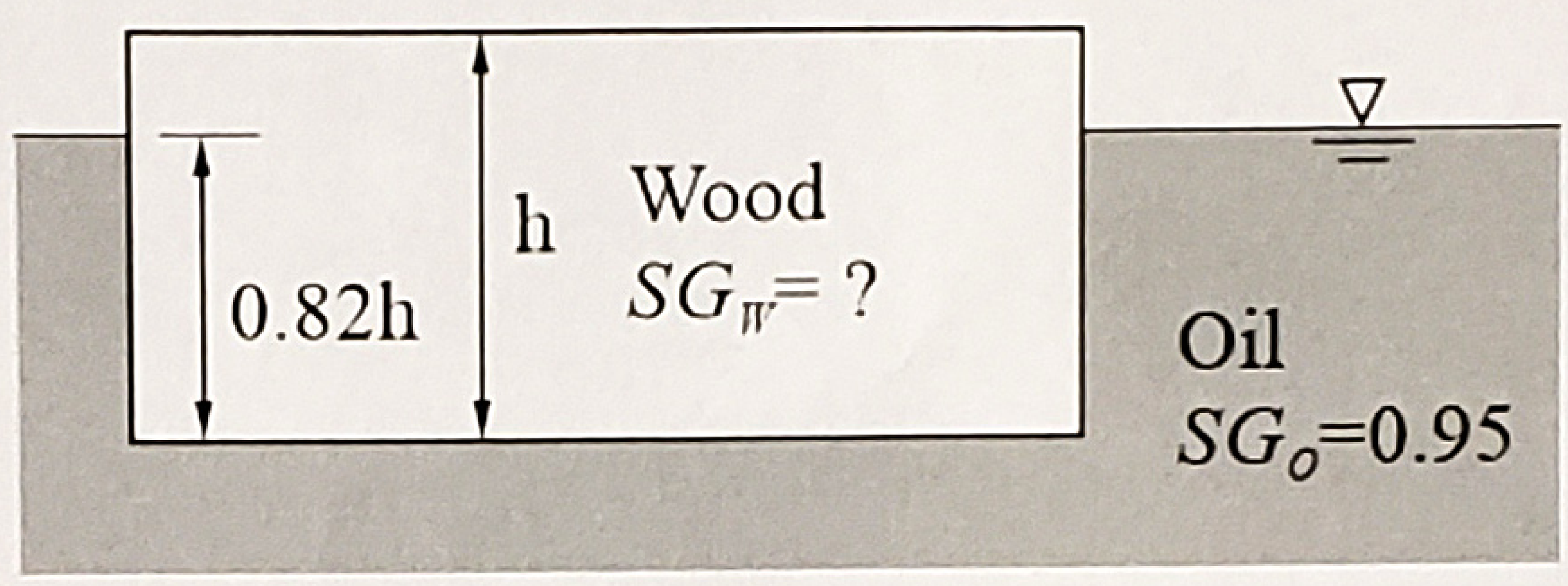
(a) $F_V = \gamma(R+H)^2/2$
(b) $F_V = \gamma\left(HR + \frac{\pi R^2}{4}\right)$
(c) $F_V = \gamma R^2(1 - \frac{\pi}{4})$
(d) $F_V = \gamma\, R^2\left(1 - \frac{\pi}{2}\right)$
(e) $F_V = \gamma\, \pi R^2/4$



$$F_V = \gamma(R^2 - \pi R^2/4)$$

A5. A solid block of wood floats on the surface of oil with a specific gravity of $SG_o = 0.95$. The wood floats with 82% of its volume below the oil's surface. What is the specific gravity of the wood?

(a) 0.62
(b) 0.78
(c) 0.86
(d) 0.91
(e) 1.15



$W_w = F_B$

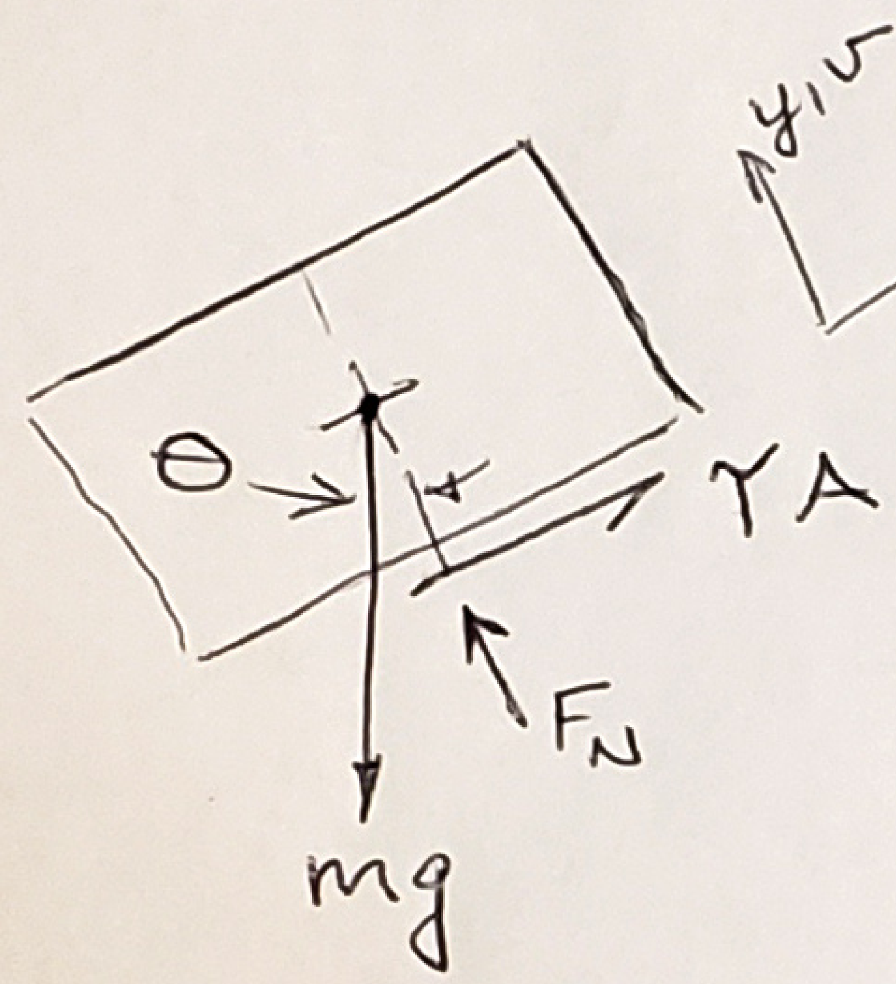
$SG_w \cancel{\rho_{H_2O}}\cancel{g}\cancel{V_w} = SG_o \cancel{\rho_{H_2O}}\cancel{g}\, 0.82 \cancel{V_w}$

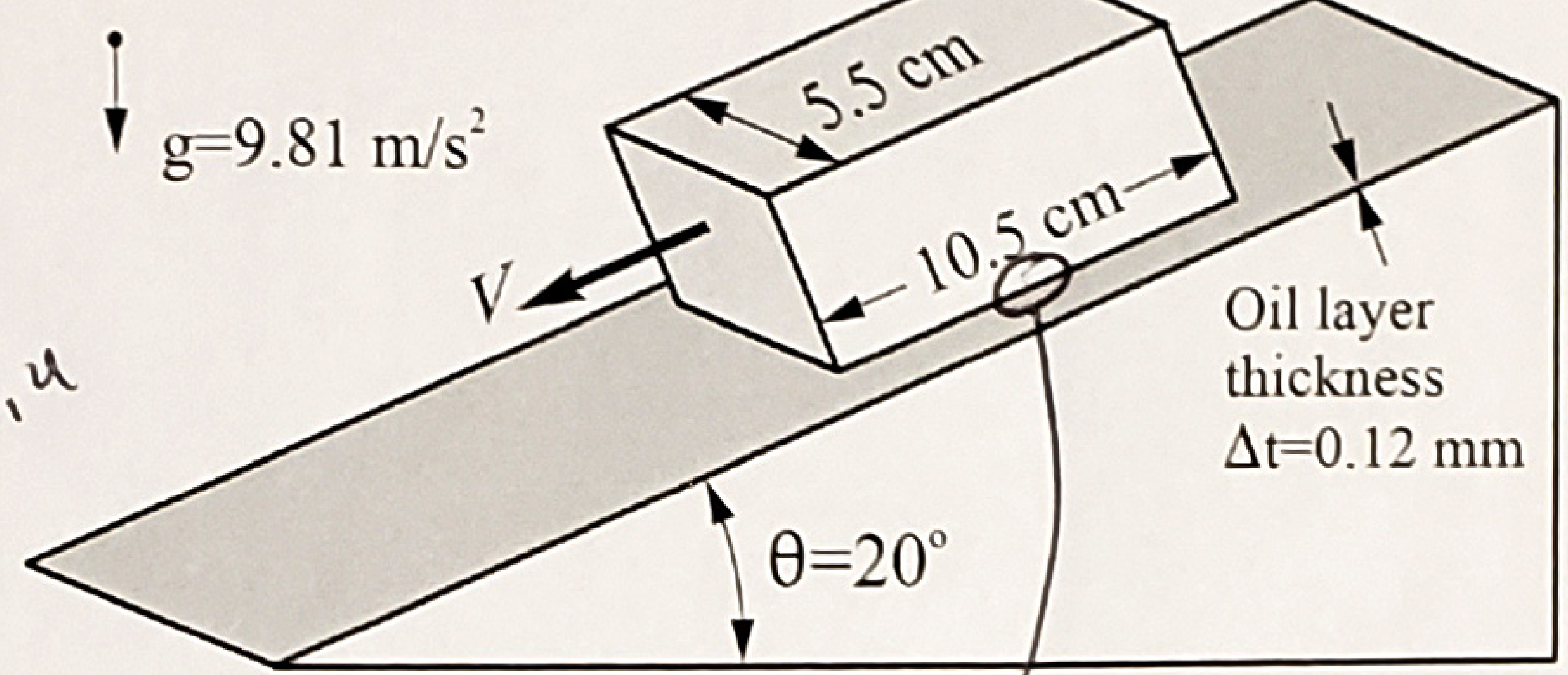
$SG_w = SG_o(0.82) = 0.95(0.82) = 0.78$

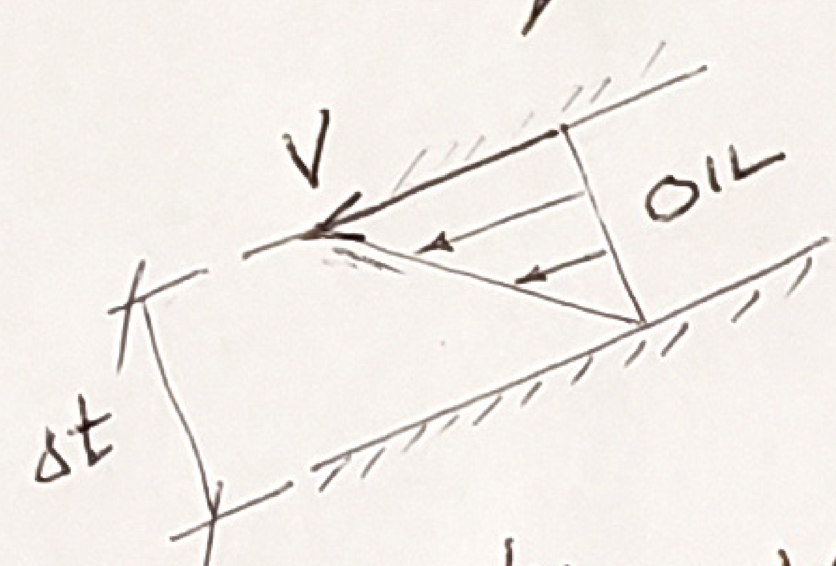
Q1. As shown in the sketch, a solid rectangular block with mass m=50g slides down a long plane under the influence of gravity. The plane is inclined at θ=20° from horizontal. The lower surface of the block has dimensions 5.5cm by 10.5cm and is lubricated with a thin layer of oil with constant thickness Δt=0.12 mm. The oil has a dynamic viscosity of μ=0.874 Ns/m². The aerodynamic drag of the air surrounding the block is negligible compared to the viscous shear force of the oil layer. Assuming a linear velocity profile in the lubricating oil layer, calculate the steady-state speed (V) of the sliding block.

(10 marks)

F.B.D.







$$\Sigma F_x = m a_x = 0 \qquad \frac{du}{dy} = \frac{V}{t}$$

$$mg\sin\theta = \tau A = \mu \frac{du}{dy} A$$

$$mg\sin\theta = \mu \frac{V}{\Delta t} A$$

$$V = \frac{mg\sin\theta\,\Delta t}{\mu A}$$

$$V = \frac{0.05\,kg\,(9.81\,\frac{m}{s^2})\overbrace{\sin 20^\circ}^{0.342}(0.00012\,m)}{0.874\,\frac{kg}{m\cdot s}\underbrace{(0.105\cdot 0.055)\,m^2}_{5.775\times10^{-3}\,m^2}}$$

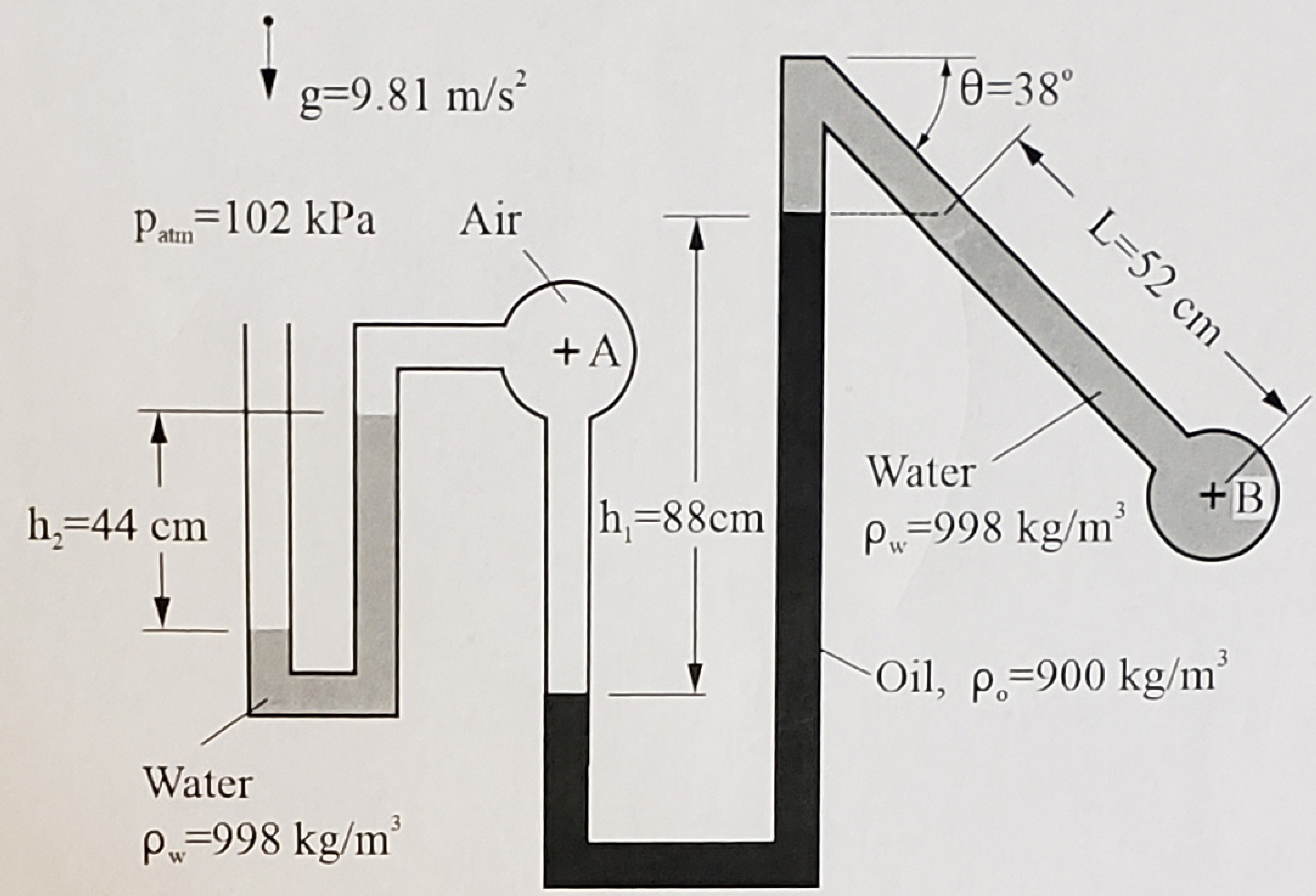
$$V = 3.99\times10^{-3}\ m/s = 3.99\ \frac{mm}{s} \quad \text{ANS}$$

Q2. Consider the complex manometer shown below that contains water (ρ_w=998 kg/m^3) and oil (ρ_o=900 kg/m^3) and air. The open end is at an atmospheric pressure of p_{atm}=102 kPa. For the dimensions shown in the sketch:

(a) Calculate the absolute pressure of the air at point A in kilopascals (kPa). *(3 marks)*

(b) Derive **an expression** for the pressure difference $p_A - p_B$ in terms of the gravitational acceleration (g), the column dimensions (h_1, L), the angle (θ) and the density of the fluids (ρ_w, ρ_o). **Do not substitute any numerical values!** *(4 marks)*

(c) Calculate pressure difference, $p_A - p_B$. *(3 marks)*



(a) $P_A = P_{atm} - \rho_w g h_2 = 102\times10^3 \frac{N}{m^2} - 9790 \, N/m^3 \, (0.44m)$

$= 97.7$ kPa ANS.

(b) $P_B - \rho_w g L \sin\theta + \rho_o g h_1 = P_A$

$P_A - P_B = \rho_o g h_1 - \rho_w g L \sin\theta$ ANS/

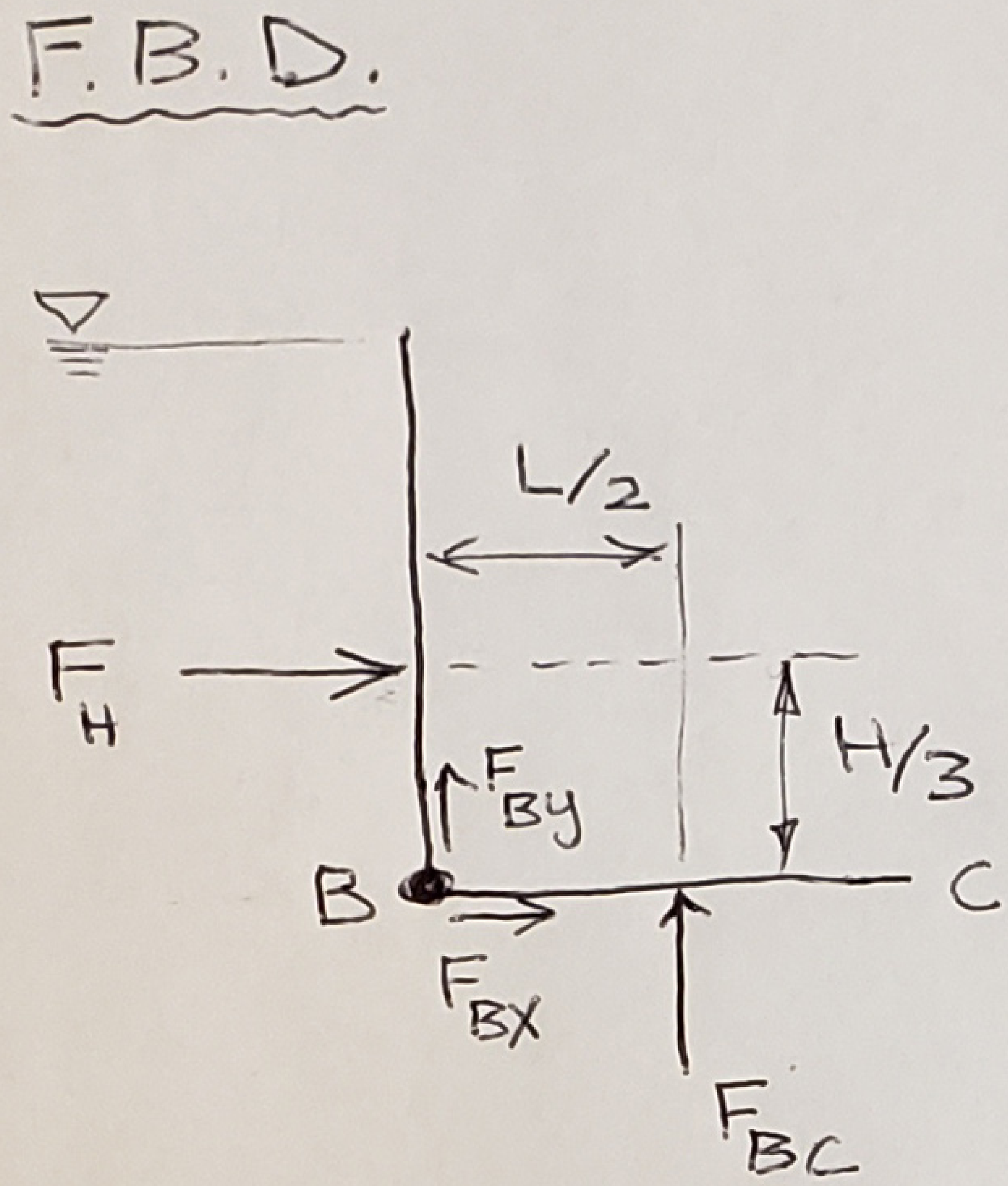
(c) $P_A - P_B = 900 \frac{kg}{m^3} (9.81 m/s^2)\, 0.88m - 9790 \, N/m^3 \, (0.52m) \sin 38°$

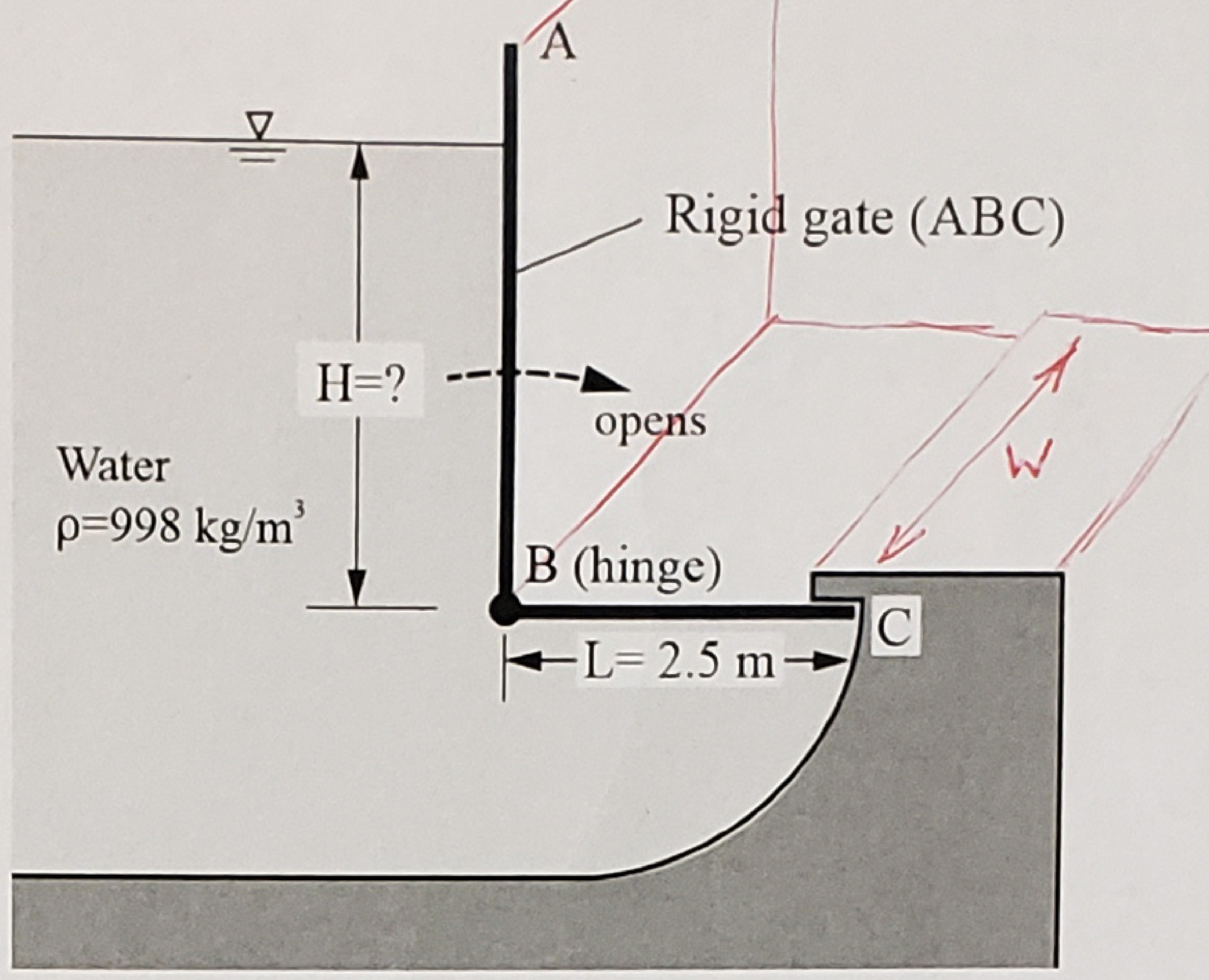
$= 7769 \, N/m^2 - 3134 \, N/m^2 = 4635$ Pa ANS/.

Q3. Water (ρ=998 kg/m^3) is contained behind a long rigid L-shaped gate (ABC) shown in the sketch below. The entire L-shaped gate rotates about a frictionless hinge at point B. The horizontal section of the gate (BC) has length L=2.5 m. The weight of the gate is negligible. Calculate the water height (H) that will cause the gate to open (i.e., rotate clockwise) as the water rises. ***(10 marks)***

Hints: Consider a unit depth (w) of the gate into the page. I recommend working in symbolic form for as long as possible.

F.B.D.

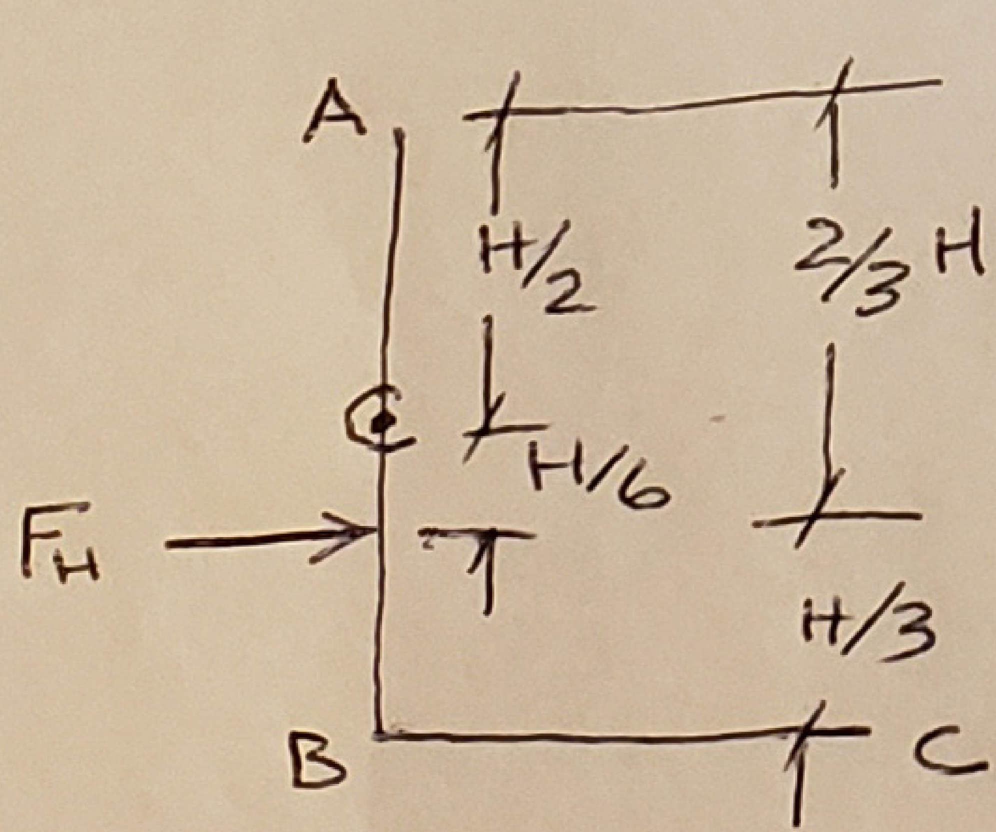




$\Sigma M_B = 0$ $\quad F_H = \gamma h_{CG} A_{AB} = \gamma \frac{H}{2}(HW) = \frac{\gamma H^2 W}{2}$ ① →

LOCATION OF F_H $\quad y_{cp} = -\frac{I_{xx}\sin\theta}{h_{CG} A_{AB}}$ 90° $\quad I_{xx} = \frac{H^3 W}{12}$

CENTRE OF PRESSURE $\quad y_{cp} = -\frac{H^3 (W)}{12 \frac{H}{2} H (W)} = -H/6$



$F_{BC} = \gamma H L W$ ↑ ②

$\Sigma M_B = 0$ $\quad F_H \frac{H}{3} = F_{BC} \frac{H}{2}$ ③

SUB. ① & ② INTO ③ $\left(\frac{\cancel{\gamma} H^2 \cancel{W}}{2}\right)\frac{\cancel{H}}{3} = (\cancel{\gamma} \cancel{H} L \cancel{W})\frac{L}{\cancel{2}}$

$H^2 = 3L \quad H = \sqrt{3}L$

$H = \sqrt{3}(2.5m) = 4.33m$ ANS/

FORMULA SHEET

Gravitational Acceleration: $g=9.81\ m/s^2$
Standard Atmospheric Pressure at Sea Level: $p_{atm}=101.3\ kPa$
Celsius to Kelvin Temperature Conversion: $K = {}^{o}C+273.15$

Ideal gas equation of state: $p = \rho R T$ For air: $R=0.287\ kJ/(kg\ K)$

Specific Gravity: $SG = \dfrac{\rho}{\rho_{water\ @\ 4^oC}} = \dfrac{\rho}{1000\ kg/m^3}$

Hydrostatic Pressure: $\dfrac{dp}{dz} = -\gamma = -\rho g$

Fluid (viscous) Shear Stress: $\tau = \dfrac{F}{A} = \mu \dfrac{du}{dy}$ Kinematic viscosity: $\nu = \dfrac{\mu}{\rho}$

Hydrostatic Forces on Plane Surfaces: $F = \gamma\, h_{CG} A$ $\quad y_{CP} = -\dfrac{I_{xx}\sin\theta}{h_{CG}\, A}$

Buoyancy Force: $F_B = \gamma \forall_{displaced}$

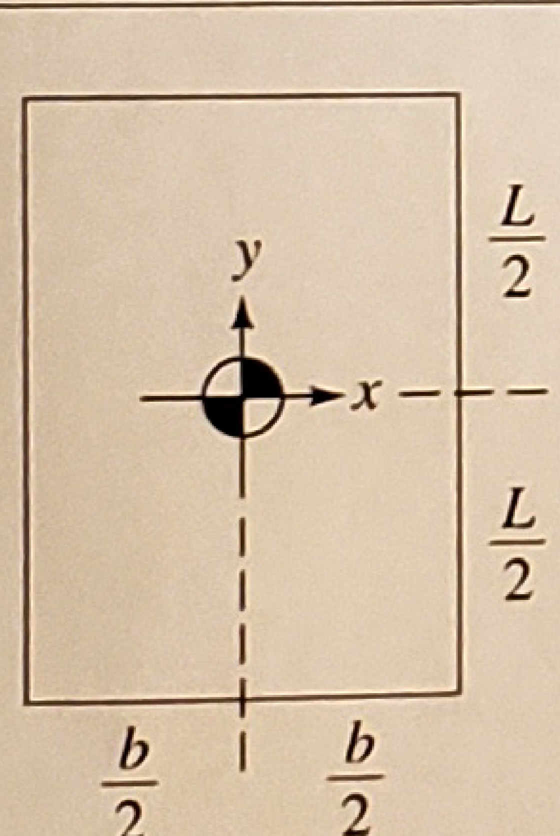
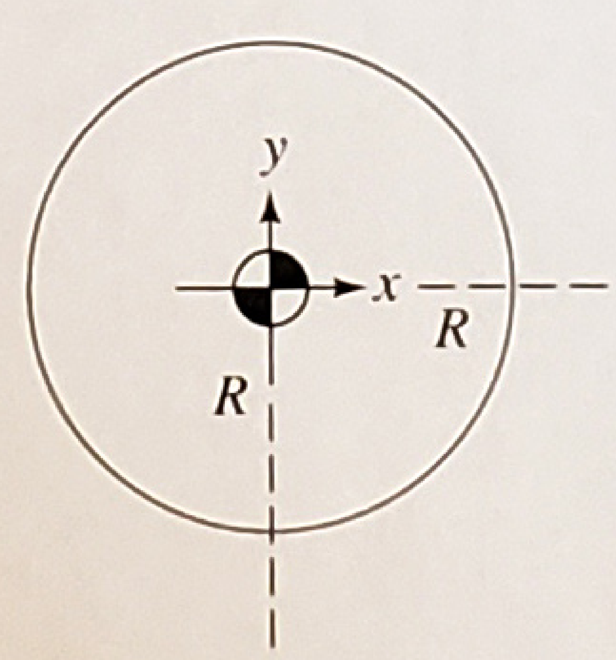
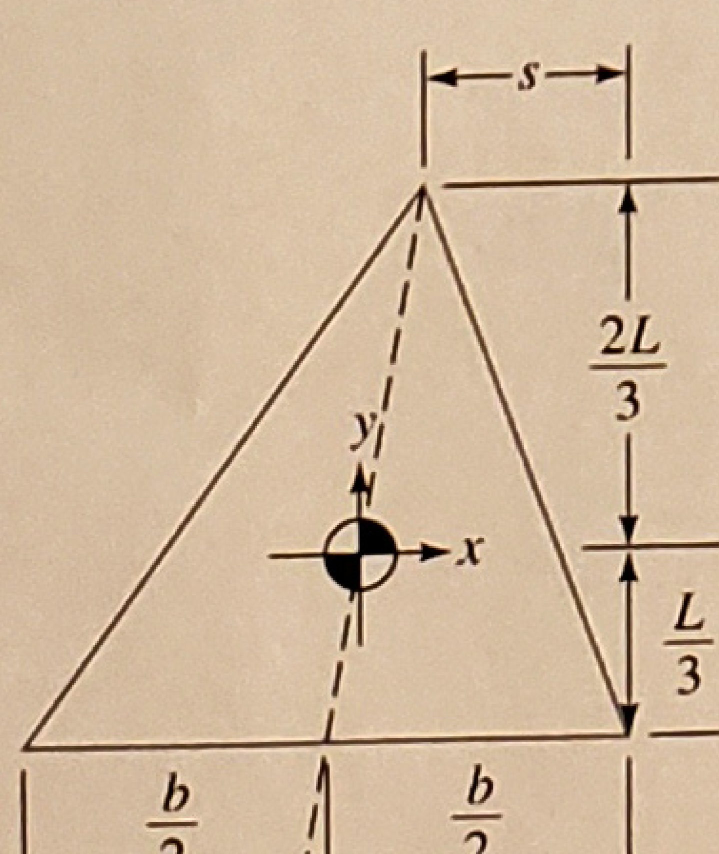
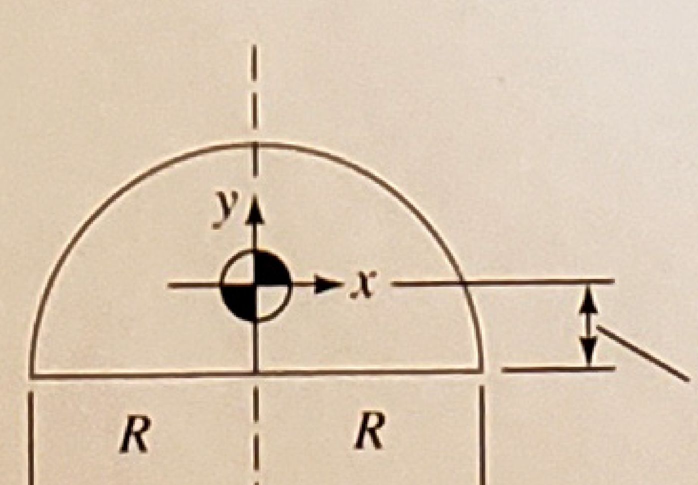
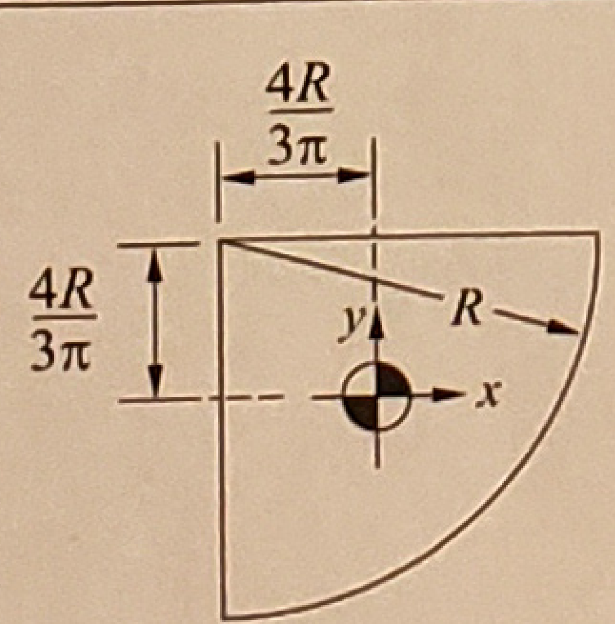
Reynolds Number: $Re = \dfrac{\rho V D}{\mu}$

Volume of a Sphere: $\forall_{sphere} = \dfrac{4}{3}\pi r^3$ Area of a Sphere: $A_{sphere} = 4\pi r^2$

Surface Tension: Liquid rise in a round capillary tube: $\Delta h = \dfrac{2\Upsilon\cos(\theta)}{\gamma R}$

Pressure in a spherical liquid droplet: $\Delta p = \dfrac{2\Upsilon}{R}$

Areas, centroids and second moments of area of common shapes

 $A = bL$ $I_{xx} = \dfrac{bL^3}{12}$ $I_{xy} = 0$	 $A = \pi R^2$ $I_{xx} = \dfrac{\pi R^4}{4}$ $I_{xy} = 0$
 $A = \dfrac{bL}{2}$ $I_{xx} = \dfrac{bL^3}{36}$ $I_{xy} = \dfrac{b(b-2s)L^2}{72}$	 $A = \dfrac{\pi R^2}{2}$ $I_{xx} = 0.10976R^4$ $I_{xy} = 0$
 $A = \dfrac{\pi R^2}{4}$ $I_{xx} = 0.05488\, R^4$	