RYERSON UNIVERSITY
DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING
MEC 516/BME 516 FLUID MECHANICS I
FINAL EXAMINATION

Time: Thursday December 4, 2014 **Examiners:** A. Fung / D. Naylor/ T. Yousefi

INSTRUCTIONS:

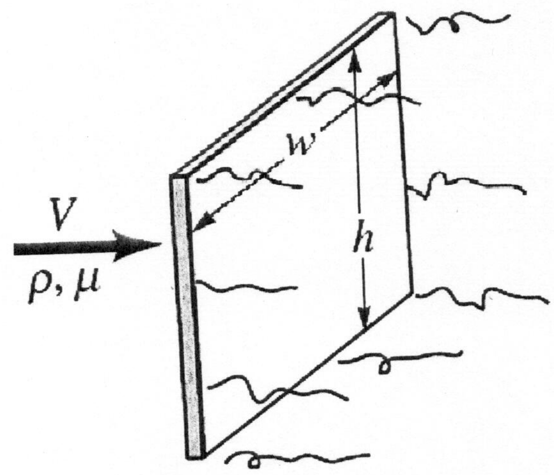
1. There are total of **eleven** sheets in this exam booklet (including this cover sheet).
2. This exam consists of **five** problems. Marks are as shown in brackets for each of the five problems in this exam booklet.
3. Solve **all** five problems in this exam.
4. This is a **closed book** exam. A non-communicable calculator is allowed. Students are also allowed with **one** US legal size (double-sided) equation sheet **without any solved problem and solution**.
5. Read the questions carefully. The back of each sheet can be used if your solution needs more space. It is your responsibility to write your solution clearly, coherently and legibly. Illegible and/or hard to follow answers will be penalized.
6. Always make a sketch (or FBD) and show your system boundaries with dashed lines. State **all** assumptions and analyses in detail and clearly. Marks are given based on procedures and steps.
7. Make sure to include your formula sheet (**with your printed name and student ID**) along with this exam booklet. Failure to do so may result in mark deduction.

Print Name:__________________________**ID:**______________________**Section:**________

Problem #1	
Problem #2	
Problem #3	
Problem #4	
Problem #5	
Total	

Problem #1 (20 points)

A thin rectangular plate having a width, w, and height, h, is located so that it is normal to a moving stream of fluid. Assume the drag, F_D, that the fluid exerts on the plate is a function of width, w, and height, h, of the plate, the fluid viscosity and density, μ and ρ, respectively, and the velocity V of the fluid approaching the plate. Use both the **Buckingham Pi Theorem** method and the **Step by Step Method by Ipsen** to derive a suitable set of **Pi** (dimensionless) terms to study this problem experimentally.



Hint:
If you need 2 repeating variables, use w and V
If you need 3 repeating variables, use w, V and ρ
If you need 4 repeating variables, use w, V, ρ, and μ

(a) **Solution:**

Buckingham Pi Theorem:

$F_D = f(w, h, \mu, \rho, V)$

$F_D \doteq MLT^{-2}$, $w \doteq L$, $h \doteq L$

$\mu \doteq ML^{-1}T^{-1}$, $\rho \doteq ML^{-3}$

$V \doteq LT^{-1}$

We have 6 variables ($k=6$)

3 primary dimensions ($r=3$)

$\Rightarrow$ There may be min of $(6-3)=3$ repeating variables. $\Rightarrow w, V, \rho$

(Note: w, V, ρ are dimensionally independent since each one contains a basic dimension not included in the others)

Let's start w/ F_D:

$\Pi_1 = F_D w^a V^b \rho^c$

$\Rightarrow (MLT^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$

$$\left.\begin{aligned} M&: 1+c=0 \\ L&: 1+a+b-3c=0 \\ T&: -2-b=0 \end{aligned}\right] \quad \begin{aligned} a&=-2 \\ b&=-2 \\ c&=-1 \end{aligned}$$

$\Pi_1 = \dfrac{F_D}{w^2V^2\rho}$ #

Let's look at h:

$\Pi_2 = h w^a V^b \rho^c$

$\Rightarrow (L)(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$

$$\left.\begin{aligned} M&: c=0 \\ L&: 1+a+b-3c=0 \\ T&: b=0 \end{aligned}\right] \quad a=-1,\ b=0,\ c=0$$

$\therefore \Pi_2 = \dfrac{h}{w}$ #

Now, let's look at μ :

$\Pi_3 = \mu w^a V^b \rho^c$

$\Rightarrow (ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$

$$\left.\begin{array}{l} M: 1+c=0 \\ L: -1+a+b-3c=0 \\ T: -1-b=0 \end{array}\right] a=-1,\ b=-1,\ c=-1$$

$\Rightarrow \Pi_3 = \dfrac{\mu}{wV\rho}$ #

(b) Ipsen :

Since $F_D = f(w,\ h,\ \mu,\ \rho,\ V)$

$\{MLT^{-1}\} = \{L\}\quad \{L\}\quad \{ML^{-1}T^{-1}\}\quad \{ML^{-3}\}\quad \{LT^{-1}\}$

There are 3 primary dimensions $\{M, L, T\}$

We need to eliminate the primary dimensions successively.

Let's start w/ M :

$\Rightarrow \dfrac{F_D}{\rho} = f\left(w,\ h,\ \dfrac{\mu}{\rho},\ \dfrac{\rho}{\rho},\ V\right)$

$\dfrac{MLT^{-2}}{ML^{-3}} = f\left(w,\ h,\ \dfrac{ML^{-1}T^{-1}}{ML^{-3}},\ 1,\ V\right)$

$L^4T^{-2} = f(w,\ h,\ L^2T^{-1},\ 1,\ V)$

Let's do T :

$\dfrac{F_D}{\rho V^2} = f\left(w,\ h,\ \dfrac{\mu}{\rho V},\ 1,\ \dfrac{V}{V}\right)$

$\dfrac{L^4T^{-2}}{L^2T^{-2}} = f\left(w,\ h,\ \dfrac{L^2T^{-1}}{LT^{-1}},\ 1,\ 1\right)$

$L^2 = f(w,\ h,\ L,\ 1,\ 1)$

Now, we do L:

$$\frac{F_D}{\rho V^2 W^2} = f\left(\frac{W}{W}, \frac{h}{W}, \frac{\mu}{\rho V W}, 1, 1\right)$$

$$\frac{L^2}{L^2} = f\left(1, \frac{L}{L}, \frac{L}{L}, 1, 1\right)$$

$$\Rightarrow \frac{F_D}{\rho V^2 W^2} = f\left(\frac{h}{W}, \frac{\mu}{\rho V W}\right)$$

Therefore, $\Pi_1 = \dfrac{F_D}{\rho V^2 W^2}$ #

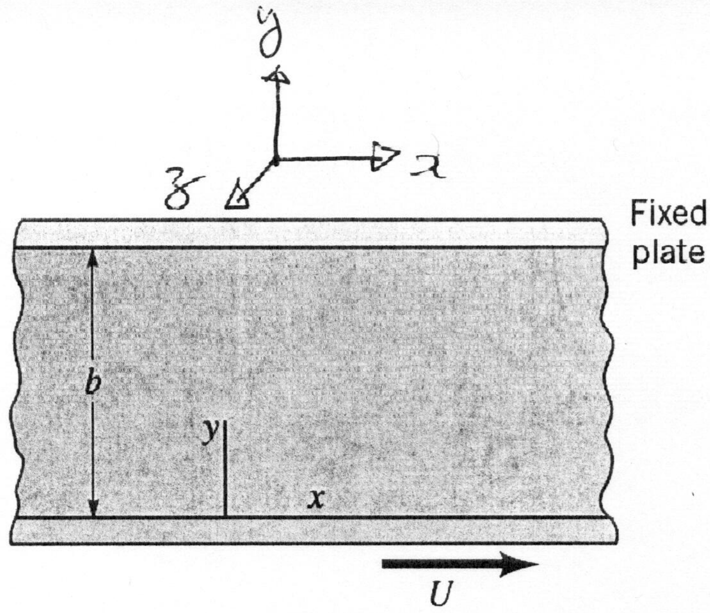
$\Pi_2 = \dfrac{h}{W}$ #

$\Pi_3 = \dfrac{\mu}{\rho V W}$ #

This is the same as the solution from the Buckingham Pi Theorem.

Problem #2 (20 points)

The viscous, incompressible flow between the two large parallel plates, shown in the attached figure, is caused by both the motion of the bottom plate and a pressure gradient, $\frac{\partial P}{\partial x}$. Derive the fluid velocity function. In addition, present your final fluid velocity equation in the dimensionless from. State clearly all equations, assumptions, variables and terms used to derive the final equation(s).



Solution:

Continuity: $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}} + \cancel{\frac{\partial w}{\partial z}} = 0$

$\Rightarrow u = u(y)$

N-S Eq:

$$\rho \cancel{g_x} - \frac{\partial P}{\partial x} + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} + \cancel{\frac{\partial^2 u}{\partial z^2}}\right) = \rho \frac{du}{dt} = \rho\left(\cancel{\frac{\partial u}{\partial t}} + \cancel{u\frac{\partial u}{\partial x}} + \cancel{v\frac{\partial u}{\partial y}} + \cancel{w\frac{\partial u}{\partial z}}\right)$$

$$\Rightarrow \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}\frac{1}{\mu}$$

$$\Rightarrow u(y) = \frac{1}{2\mu}\frac{\partial P}{\partial x}y^2 + C_1 y + C_2$$

Apply BCs: $y = 0 \Rightarrow u = U$; $y = b \Rightarrow u = 0$

$C_2 = U$; $C_1 = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)b - \frac{U}{b}$

$$\therefore u(y) = \frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(y^2 - by) + U\left(1 - \frac{y}{b}\right) \quad \#$$

or $$\frac{u(y)}{U} = \frac{b^2}{2\mu U}\left(\frac{\partial P}{\partial x}\right)\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \left(\frac{y}{b}\right) + 1$$

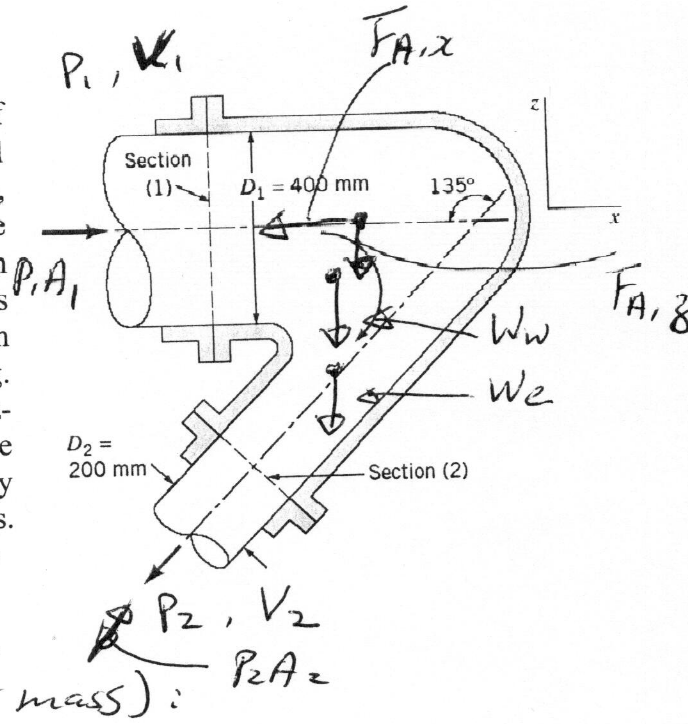
$$\frac{u(y)}{U} = -P\left(\frac{y}{b}\right)\left(\frac{y}{b} - 1\right) - \left(\frac{y}{b}\right) + 1$$

\# dimensionless Equation

where $P = \frac{-b^2}{2\mu U}\left(\frac{\partial P}{\partial x}\right)$

Problem #3 (20 points)

A converging elbow turns water through an angle of 135° in a vertical plane. The flow cross-sectional diameter is 400mm at the elbow inlet, section (1), and 200mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m^3 between sections (1) and (2). The water volume flowrate is 0.4 m^3/s., and the elbow inlet and outlet pressures (in gage) are 150 and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x-direction) and vertical (z-direction) anchoring forces required to hold the elbow in place. Make you draw a detailed free-body diagram with all the forces ~~and their~~ directions. Assume density of water of 999 m^3/kg. kg/m^3



Solution:

From Continuity (Conservation of mass):

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_2 = \rho Q$$

From Linear momentum: x-direction (→)

$$-V_1 \rho u_1 A_1 - V_2 \cos 45^\circ \rho V_2 A_2 = P_1 A_1 - F_{A,x} + P_2 A_2 \cos 45^\circ$$

Thus,

$$F_{A,x} = \frac{\rho Q^2}{A_1} + \frac{\rho Q^2}{A_2}\cos 45^\circ + P_1 A_1 + P_2 A_2 \cos 45^\circ$$

$$= \frac{\rho Q^2}{\frac{\pi}{4}D_1^2} + \frac{\rho Q^2}{\frac{\pi}{4}D_2^2}\cos 45^\circ + P_1 \frac{\pi}{4} D_1^2 + P_2 \frac{\pi}{4} D_2^2 \cos 45^\circ$$

$$\Rightarrow F_{A,x} = \left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \frac{1}{\pi/4}\left[\frac{1}{(0.4m)^2} + \frac{\cos 45^\circ}{(0.2m)^2}\right] + \frac{\pi}{4}\left[(150\times 10^3\, Pa)(0.4m)^2 + (90\times 10^3\, Pa)(0.2)^2 \cos 45^\circ\right]$$

$$\Rightarrow F_{A,x} = 25{,}700\, N \ (\leftarrow)\ \#$$

From Linear momentum: z-direction (↑)

$$-V_2 \sin 45^\circ \rho V_2 A_2 = P_2 A_2 \sin 45^\circ - F_{A,z} - W_w - W_e$$

$$\Rightarrow F_{A,z} = \frac{\rho Q^2}{A_2}\sin 45^\circ + P_2 A_2 \sin 45^\circ - W_w - W_e$$

$$= \frac{\rho Q^2}{\frac{\pi}{4}D_2^2}\sin 45^\circ + \frac{\pi}{4} P_2 D_2^2 \sin 45^\circ - \rho g \forall_w - m_e g$$

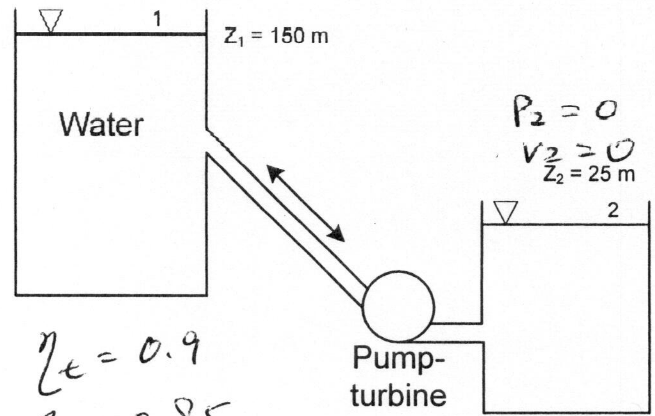
$$= \frac{\left(999 \frac{kg}{m^3}\right)\left(0.4 \frac{m^3}{s}\right)^2 \sin 45^\circ}{\frac{\pi}{4}(0.2)^2} + \frac{\pi}{4}(90\times 10^3\, Pa)(0.2)^2 \sin 45^\circ - \left(999 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(0.2\, m^3) - (12 kg)\left(9.81 \frac{m}{s^2}\right)$$

$$\Rightarrow F_{A,z} = 3520\, N \downarrow\ \#$$

Problem #4 (20 points)

$V_1 = 0,\ P_1 = 0$

A pump-turbine system shown in the attached figure draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower reservoir to restore the situation. For a design flow rate of 120 m^3/min in either direction, the total friction head loss is 15 m. Estimate



$P_2 = 0$

$V_2 = 0$

a) ~~Theoretical~~ Mechanical power extracted by the turbine from the water in kW,

b) The electrical power generated by the generator in kW,

c) ~~Theoretical~~ mechanical power ~~delivered by~~ Input to the pump to the water in kW,

d) The electrical power required by the pump in kW, and

e) Overall efficiency of the system,

if the efficiency of the turbine, pump, generator, and motor are 90%, 85%, 85%, 90%, respectively?

$\eta_t = 0.9$

$\eta_p = 0.85$

$\eta_g = 0.85 \quad \eta_m = 0.9$

Hint: Overall efficiency of the system can be defined as net electricity generated by the turbine-generator over total electricity input to the pump-motor. Assume γ_{water} **= 9790 N/m^3**

Solution:

$Q = 120\ m^3/min = 2\ m^3/s$

$h_f = 15 m$

Turbine :

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f + h_t$$

$$150 = 25 + 15 + h_t$$

$$h_t = 110 m$$

$$P_t|_{max} = \gamma Q h_t = 9790(2)(110) = 2153.8\ kW$$

(a) $P_t = P_t|_{max}\, \eta_t = 2153.8 \times 0.9 = 1938.42\ kW$ #

(b) $P_g = P_t\, \eta_g = 1938.42 \times 0.85 = 1647.66\ kW$ #

Pump :

$$\cancel{\frac{P_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + z_2 = \cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_f - h_p$$

$$25 = 150 + 15 - h_p$$

$$h_p = 140\ m$$

$$P_p|_{min} = \gamma Q h_p = 9790(2)(140) = 2741.2\ kW$$

(c) $P_p = P_p|_{min} / \eta_p = 2741.2/0.85 = 3224.94\ kW$ #

(d) $P_m = P_p / \eta_m = 3224.94/0.9 = 3583.27\ kW$ #

(e) $\eta_{overall} = \dfrac{P_g}{P_m} = \dfrac{1647.66}{3583.27} = 0.459 = 45.9\%$ #

Problem #5 (20 points total, each of the ten questions below takes 2 points. Circle the most appropriate answer.)

1) When dealing with **inviscid** flow the following differential form of the linear momentum equation $\rho\vec{g} - \nabla P = \rho\frac{d\vec{V}}{dt}$ is called?

a) Euler Equation
b) Continuity Equation
c) Reynolds Transport Equation
d) Bernoulli Equation
e) Navier-Stokes Equation

2) In the study of fluid mechanics, the specified item for a control volume is:

a) the mass of the fluid contained in the control volume
b) the region of the control volume
c) the mean velocity within the control volume
d) the total energy stored in the control volume

3) A tank has two inlets and one outlet. Oil flows in at 1.5 *kg/s* through the 1st inlet, and 3.5 *kg/s* at the 2nd inlet. It flows out at 6 *kg/s*. What is $\frac{dm_{Tank}}{dt}$?

a) 11 *kg/s*
b) 1 *kg/s*
c) -1 *kg/s*
d) -11 *kg/s*

4) A steady flow means:

a) the flow behavior remains unchanged over the entire flow region
b) the flow must be of one dimensional nature
c) all properties of the flow at any arbitrarily chosen point remain constant with respect to time
d) the fluid of the flow is considered as inviscid

5) When two systems have dynamic similarity, the incorrect statement among the four statements given below is:

a) the geometric similarity holds for the two systems
b) the kinematic similarity holds for the two systems
c) corresponding forces are in the same ratio for the two systems
d) none of the above three statements is correct

6) When in fluid mechanics, the ***no-slip condition*** means:

a) Surface tension prevents a fluid from slipping at a liquid/solid interface.
b) Local shear stress in a fluid is linearly related to the shear rate.
c) Fluid behaviour is approximated as a continuum.
d) At a solid surface the fluid has the same velocity as the surface.
e) The shear stress in a fluid increases with time because of the lack of molecular slip.

7) Which of the following statements about pressure is **false**?

a) Pressure in a static fluid is a point property, independent of orientation.

b) For an incompressible fluid in a gravitational field, pressure increases linearly with depth, z.
c) In a gas the pressure gradient dp/dz is a constant (where z is the fluid depth).
d) Gauge pressure is the pressure relative to local atmospheric pressure.
e) Absolute pressure is the pressure relative to a perfect vacuum.

8) When The Bernoulli equation applies for:

a) Unsteady incompressible viscous flow.
b) Unsteady compressible viscous flow.
c) Steady compressible viscous flow
d) Steady incompressible inviscid (frictionless) flow.

9) In its most general application, the partial differential equation shown below describes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

(a) Conservation of mass for unsteady compressible flow.
(b) Conservation of mass for unsteady incompressible flow.
(c) Conservation of mass for a steady compressible flow.
(d) Conservation of momentum for unsteady compressible flow.
(e) Conservation of momentum for a steady incompressible flow.

10) The Reynolds number (Re) is one of the most important dimensionless parameters in fluid mechanics. In terms of fluid forces, the physical interpretation of the Reynolds number is:

(a) Inertia force divided by the viscous force.
(b) Inertial force divided by the gravity force.
(c) Viscous forces divided by the pressure force.
(d) Surface tension force divided by the viscous force.
(e) Gravity force divided by the viscous force.