MEC516/BME516: Fluid Mechanics I

Hydrostatic Forces on a Plane Gate

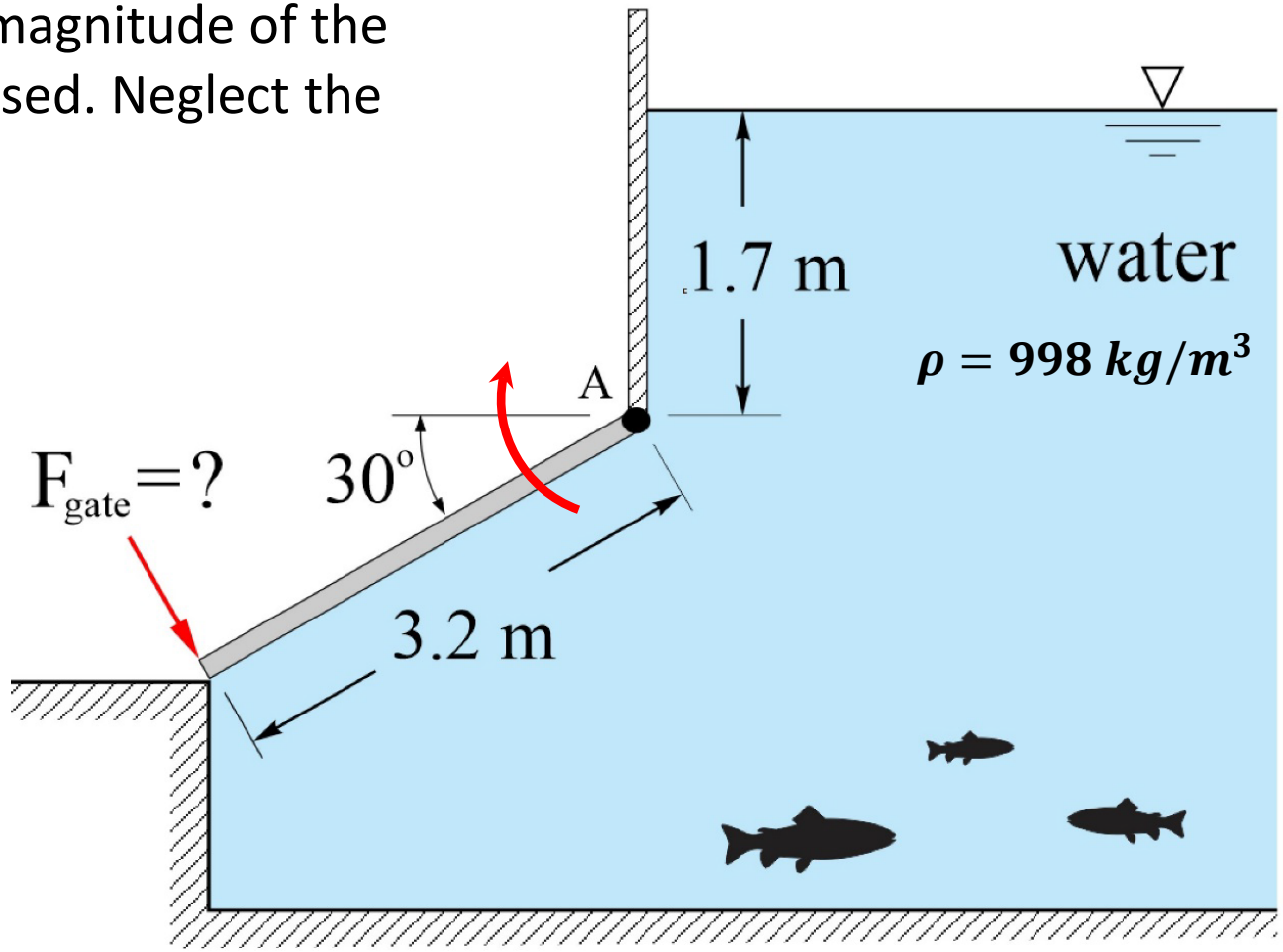
Solved Midterm Example

© Dr. David Naylor, 2024

Toronto Metropolitan University

Midterm Review

A hinged gate in a dam is inclined at 30^o from the horizontal. The gate is $3.2\ m$ long, $2.0\ m$ wide (into the page) and hinged at A, at a depth of $1.7\ m$. Calculate the magnitude of the force F_{gate} required to hold the gate closed. Neglect the weight of the gate.

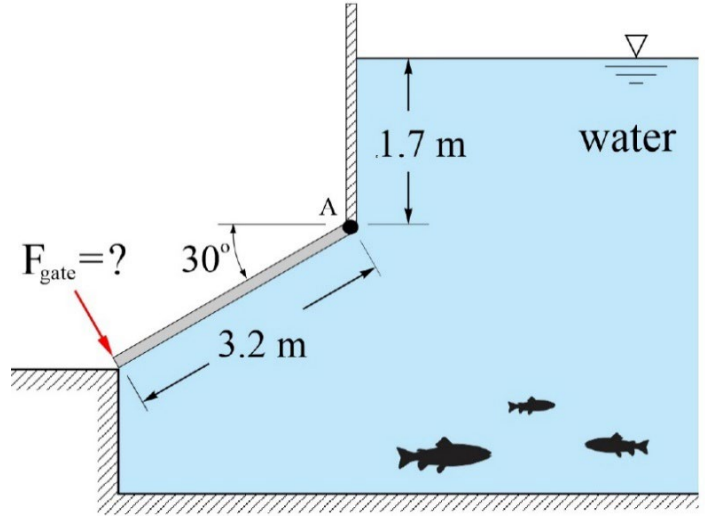


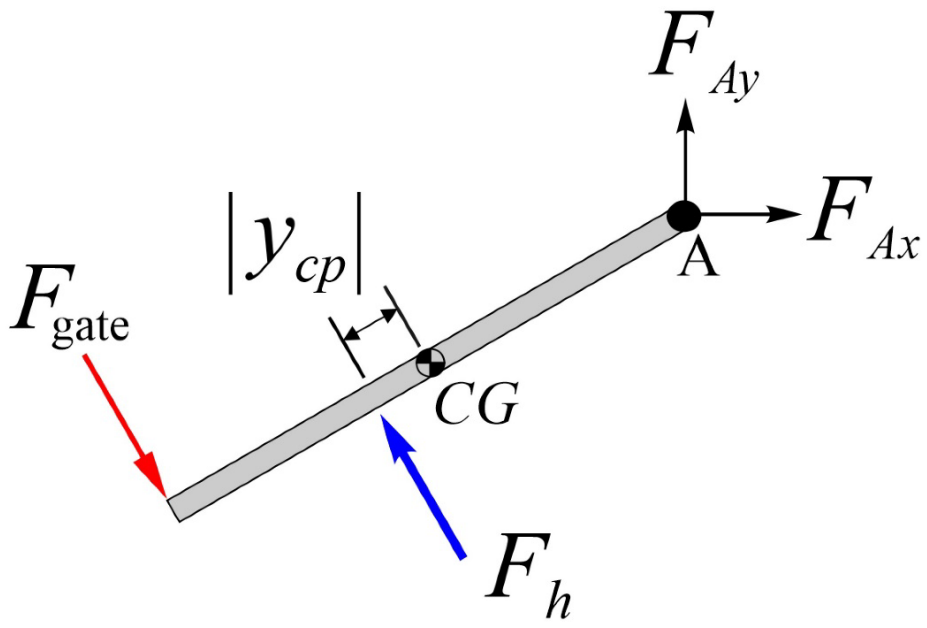
Hard copy (pdf) can be downloaded: www.drdavidnaylor.net

Link to "theory" for this presentation in the comments

Solution: Free body diagram

- Static equilibrium, $\sum M_A = 0$





Also, p_{atm} acts uniformly on both sides. Cancel out.

Solution: Hydrostatic force, F_h

$$F_h = \gamma h_{CG} A$$

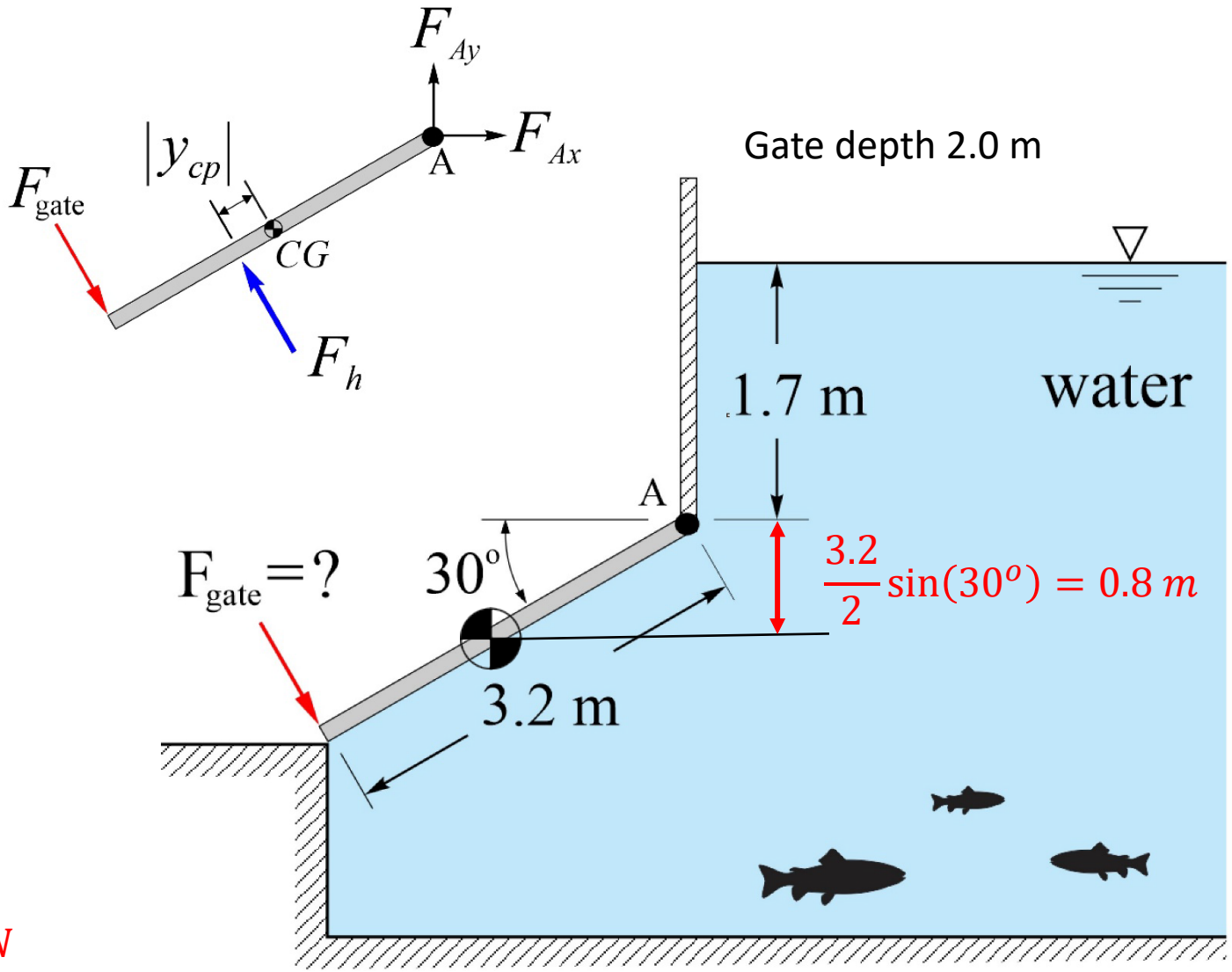
- h_{CG} is the depth of the centroid of the gate

$$h_{CG} = 1.7\ m + \frac{3.2\ m}{2}\sin(30^o) = 2.5\ m$$

$$A = 3.2\ m(2.0\ m) = 6.4\ m^2$$

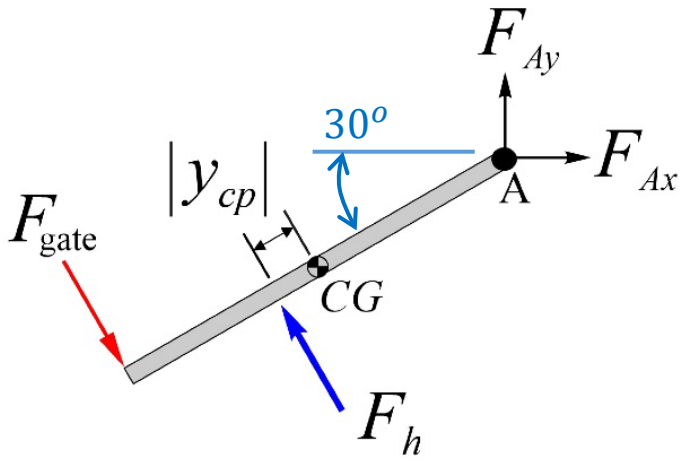
$$kg\frac{m}{s^2} \equiv N$$

$$F_h = \gamma h_{CG} A = \rho g h_{CG} A = 998\frac{kg}{m^3}\left(9.81\frac{m}{s^2}\right)2.5\ m\ (6.4\ m^2) = 156.6\ kN$$



Solution

$h_{CG} = 2.5\ m, A = 6.4\ m^2$



- Location of the centre of pressure, y_{cp}

Angle from horizontal, $\theta = 30^o$

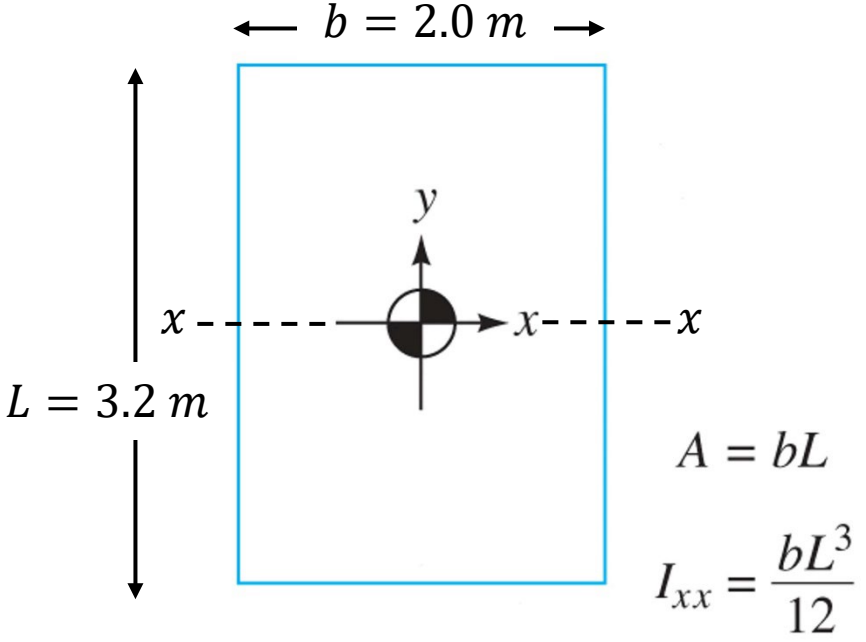
$$y_{cp} = -\frac{I_{xx}\sin(\theta)}{h_{CG}\,A}$$

Below centroid

- I_{xx} is the second moment of area about the centroid

$$I_{xx} = \frac{bL^3}{12} = \frac{2.0\ m(3.2\ m)^3}{12} = 5.461\ m^4$$

$$y_{cp} = -\frac{I_{xx}\sin(\theta)}{h_{CG}\,A} = -\frac{5.461\ m^4\ (\sin 30^o)}{2.5\ m\ (6.4\ m^2)} = -0.171\ m$$



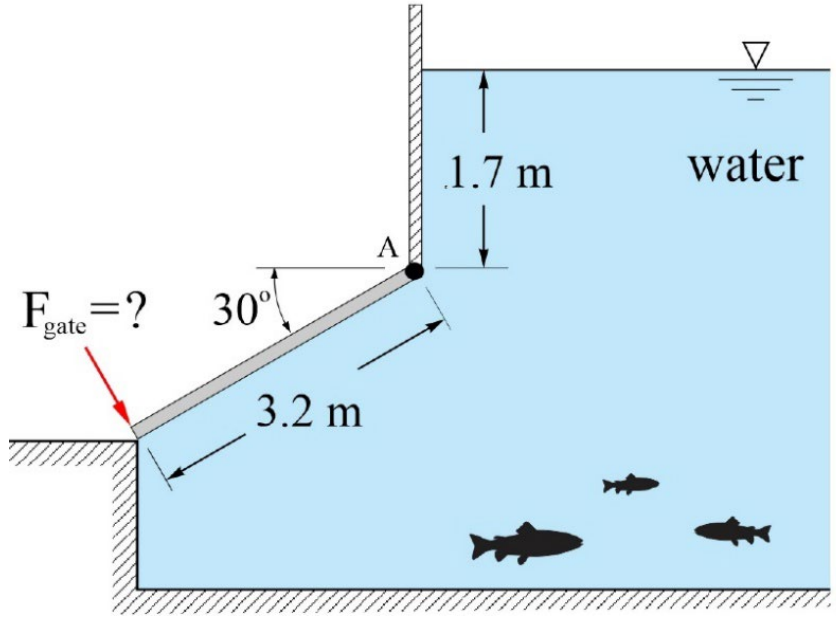
Solution: $y_{cp} = -0.171\ m,\ \ F_h = 156.6\ kN$

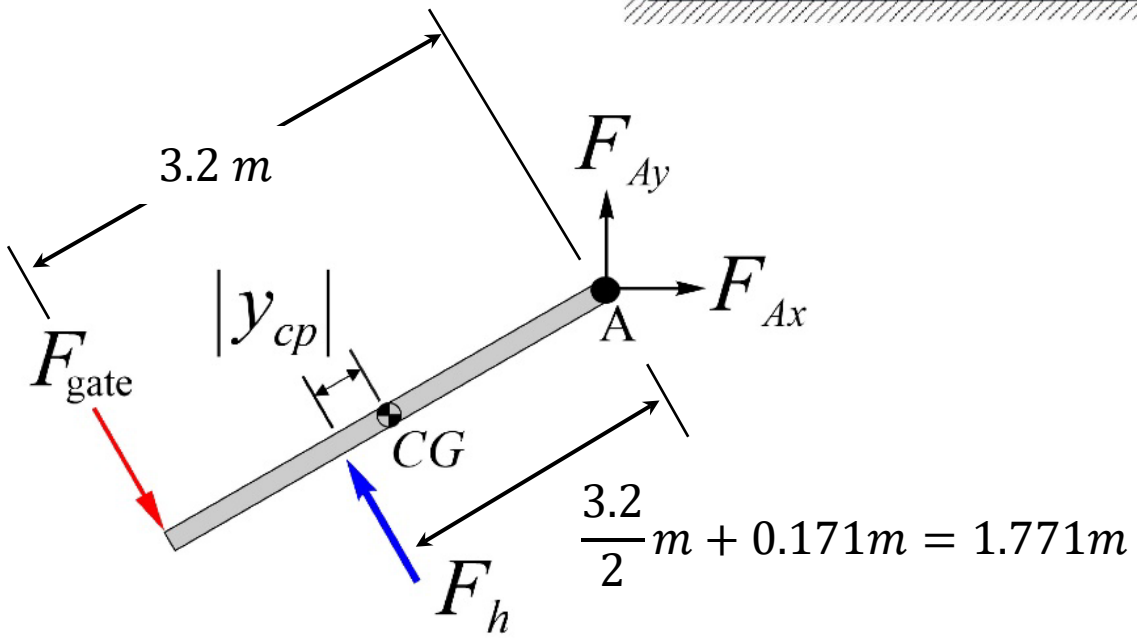
- Static Equilibrium, $\sum M_A = 0$ +

$$F_{gate}(3.2\ m) - F_h(1.771\ m) = 0$$

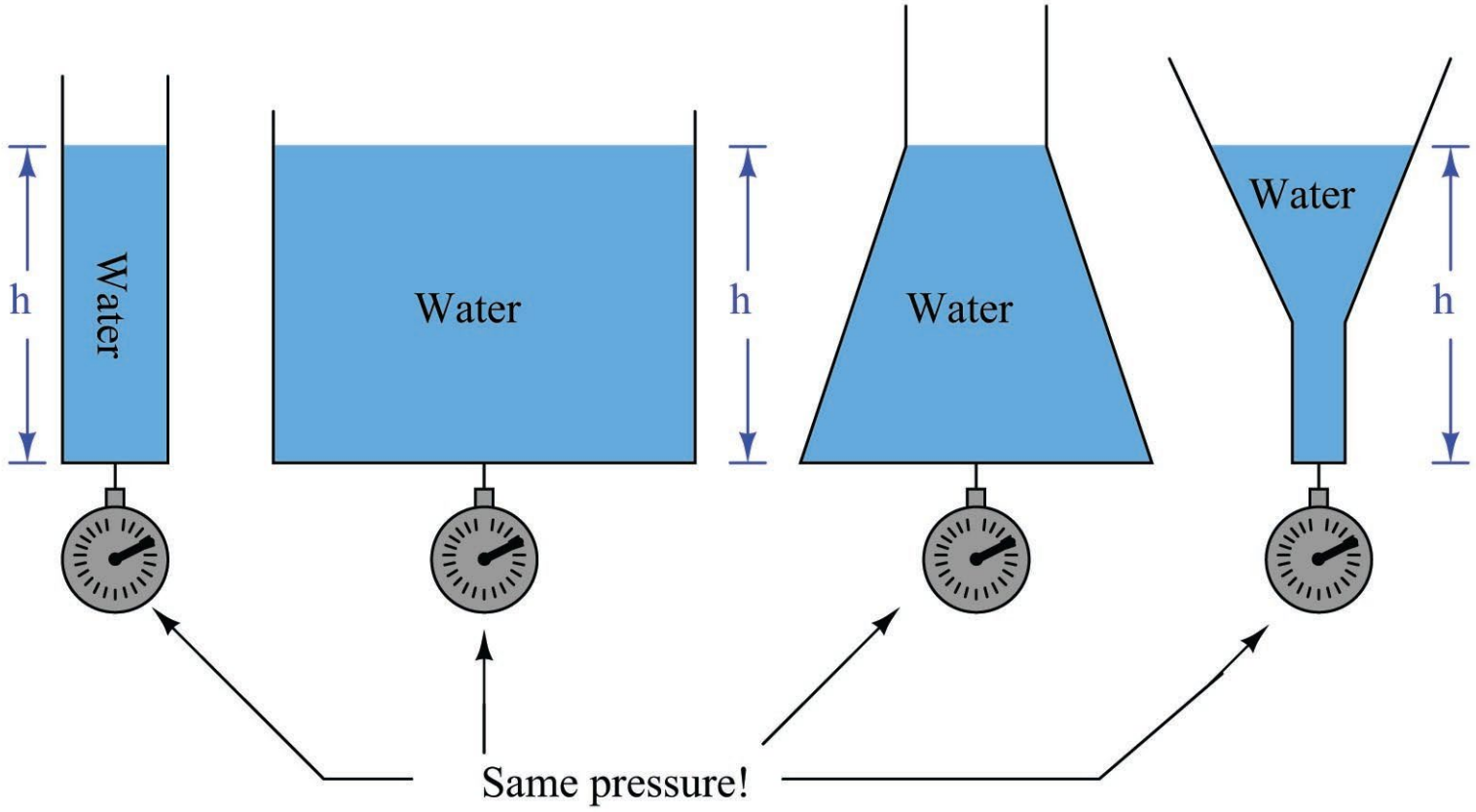
$$F_{gate} = F_h\left(\frac{1.771}{3.2}\right) = 156.6\ kN\left(\frac{1.771}{3.2}\right)$$

$$F_{gate} = 86.7\ kN \searrow$$ **Answer**





$$\frac{3.2}{2}m + 0.171m = 1.771m$$



$$p = \rho g h = \gamma h$$

Image credit: https://www.control.com/textbook/hydrostatic-pressure/

END NOTES

Prepared and delivered by Professor David Naylor

© Dr. David Naylor 2024

Toronto Metropolitan University