# MEC516/BME516: Fluid Mechanics I

## Hydrostatic Forces on Curved Surfaces

## Solved Midterm Example



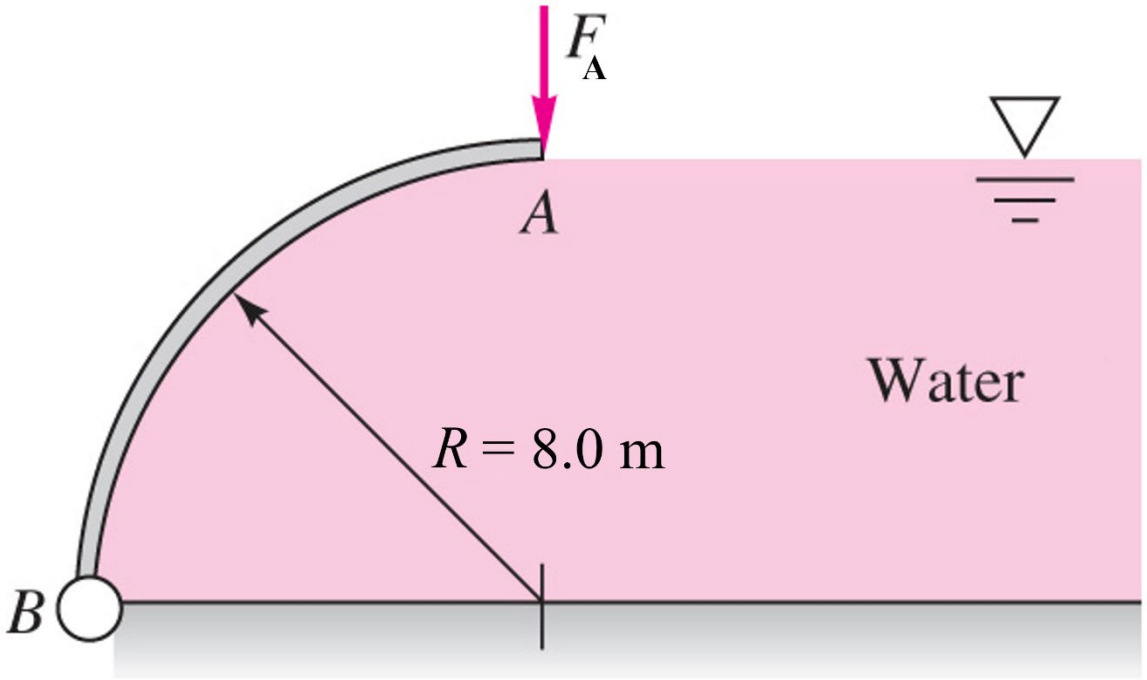
Toronto Metropolitan University
Department of Mechanical & Industrial Engineering

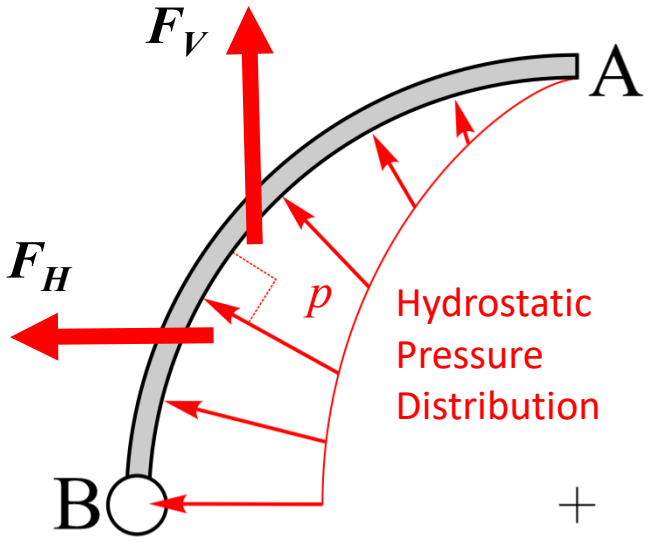
# Midterm (Makeup) Fall 2022

Gate AB is a quarter–circle with a radius $R = 8.0\ m$ and w $= 10\ m$ wide (into the page). The gate is hinged at B. The fluid is water, $\gamma = 9790\ N/m^3$. Calculate:

(a) The horizontal and vertical hydrostatic forces **on** gate AB. Magnitude and direction.

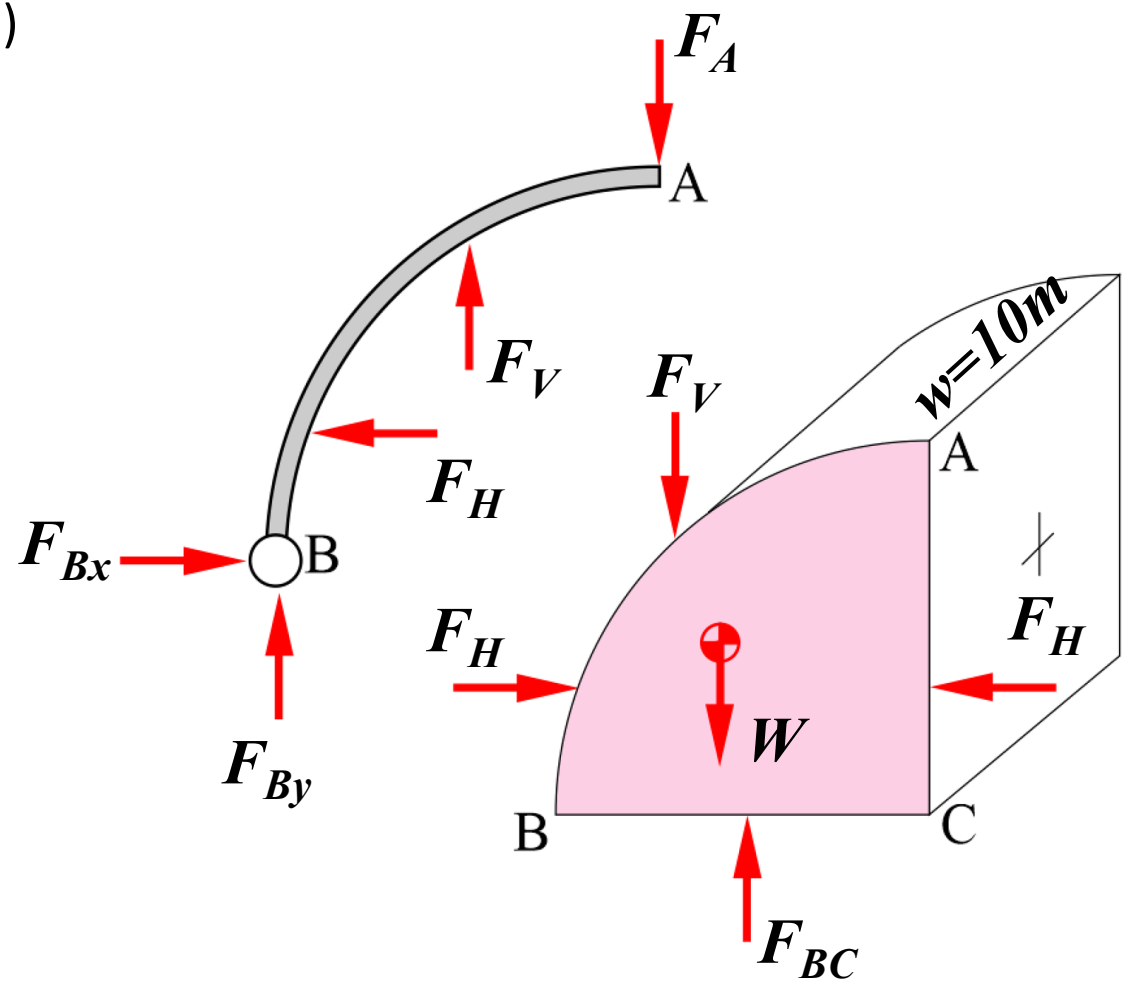
(b) The vertical force $F_A$ needed to keep the gate from opening. Neglect the weight of the gate.

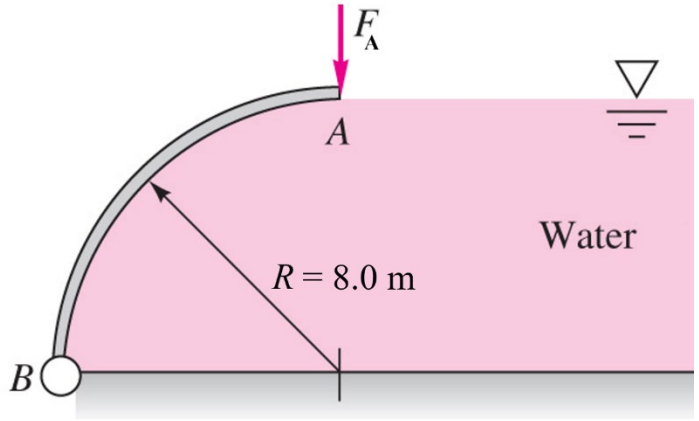


# (a) Horizontal and Vertical Hydrostatic Forces

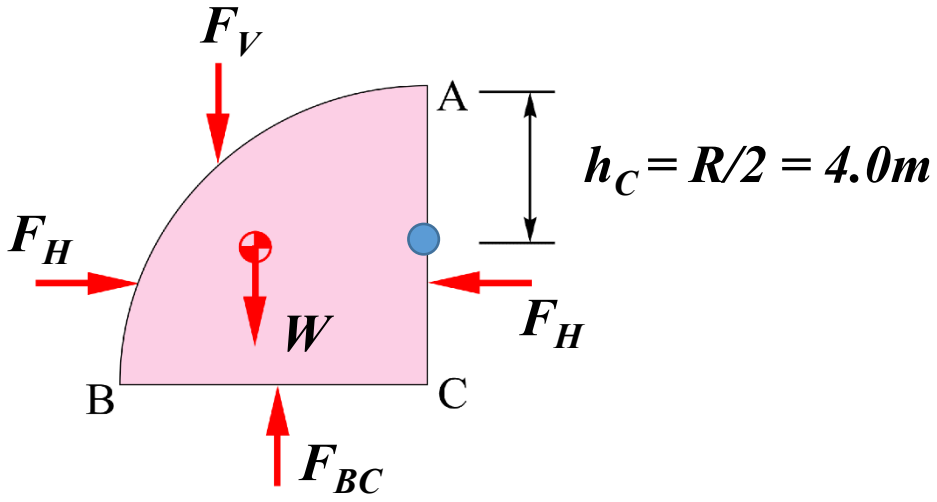
- Free Body Diagram (Water & Gate)

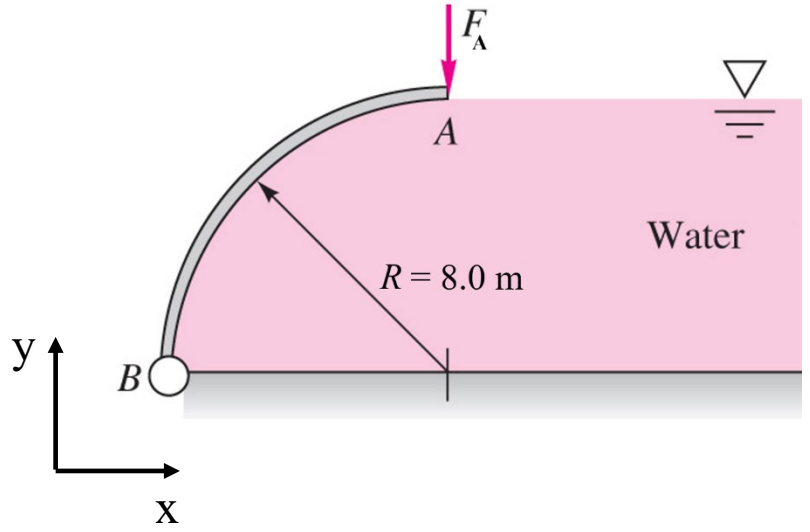


# (a) Horizontal and Vertical Hydrostatic Forces

- Free Body Diagram (Water)




- Horizontal Force

$$F_H = \gamma\, h_C A_{AC} = 9790\,\frac{N}{m^3}(4.0m)(8x10)m^2 = 3133\ kN \quad \leftarrow on\ gate\ AB$$

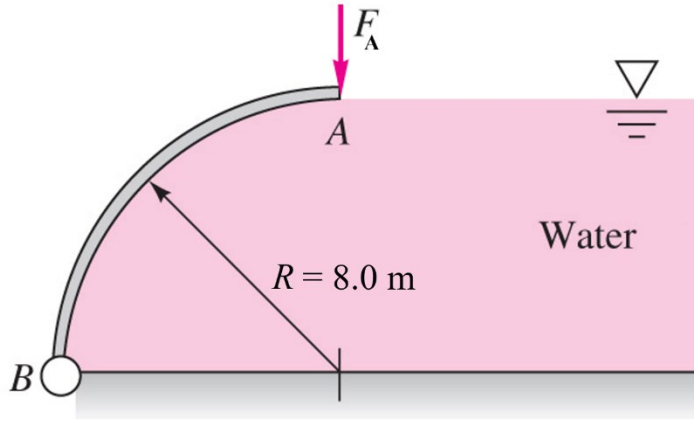
**Answer**

- Vertical Force

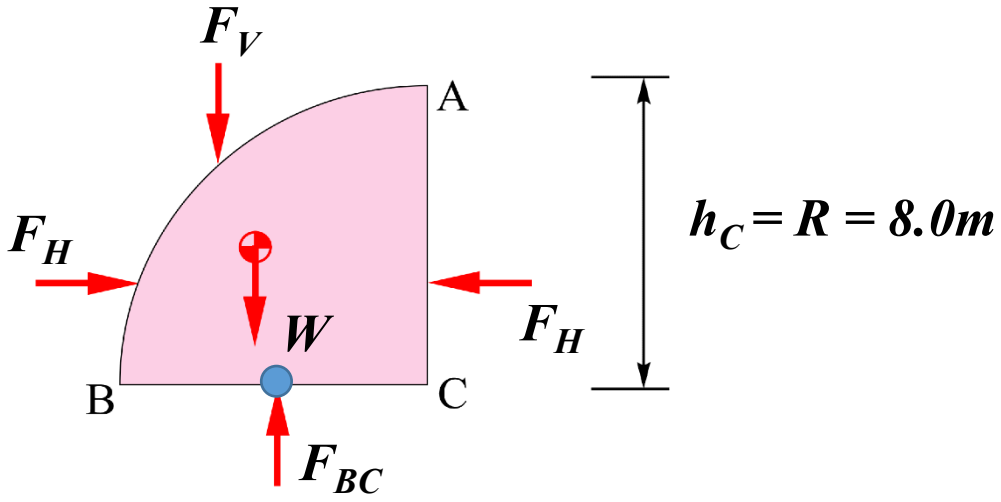
$$\sum F_y = 0 \qquad F_{BC} - W - F_V = 0$$

$$F_V = F_{BC} - W$$

# (a) Horizontal and Vertical Hydrostatic Forces




- Free Body Diagram




$$F_V = F_{BC} - W$$

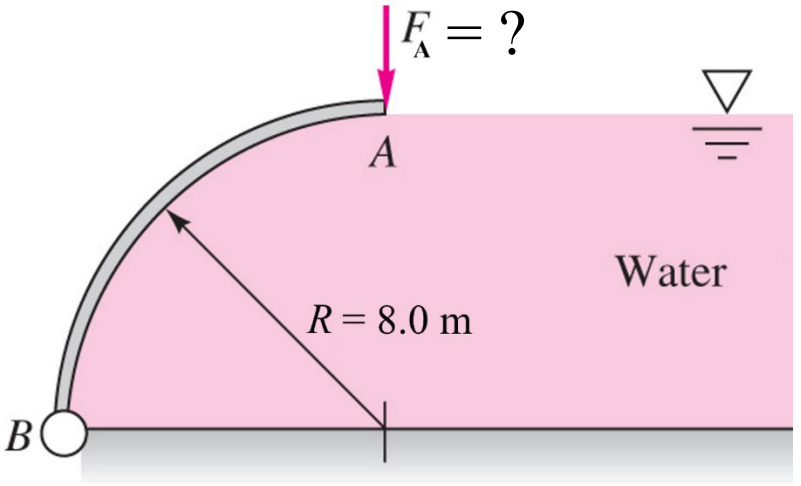
$$F_{BC} = \gamma\, h_C A_{BC} = 9790 \frac{N}{m^3}(8.0m)(8x10)m^2 = 6266\ kN$$

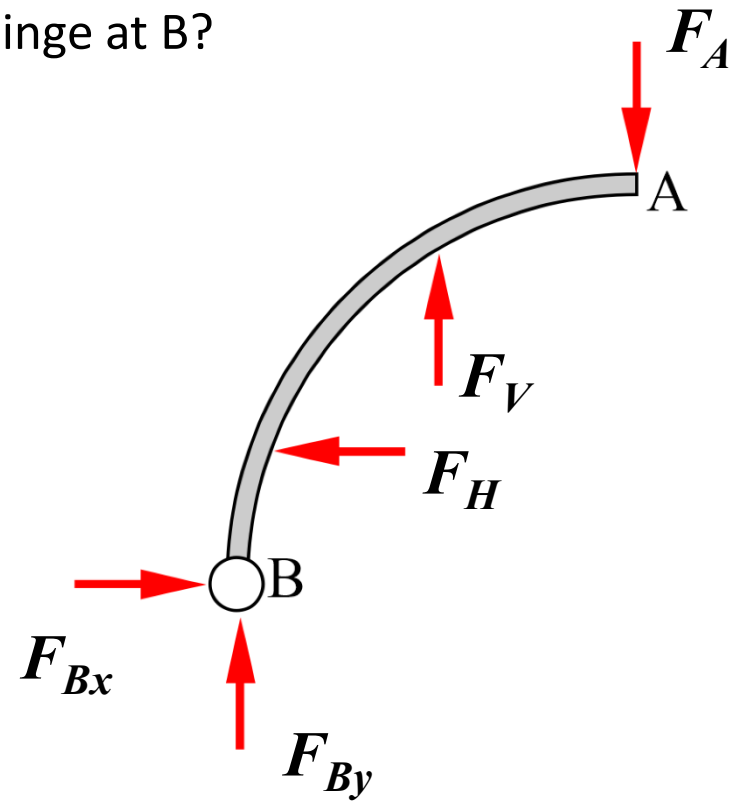
$$W = \gamma\ \forall_{ABC} = \gamma \frac{\pi R^2}{4} w = 9790 \frac{N}{m^3}\left(\frac{\pi\,(8.0)^2 m^2}{4}\right)(10m) = 4921\ kN$$

$$F_V = F_{BC} - W = 6266\ kN - 4921\ kN = 1345\ kN\ \uparrow\ \ on\ gate\ AB$$ **Answer**

# (b) Vertical Force $F_A$

- Requires the lines of action of $F_H$ and $F_V$
- Free Body Diagram for Gate AB → $\sum M_B = 0$
- Why take moments about the hinge at B?

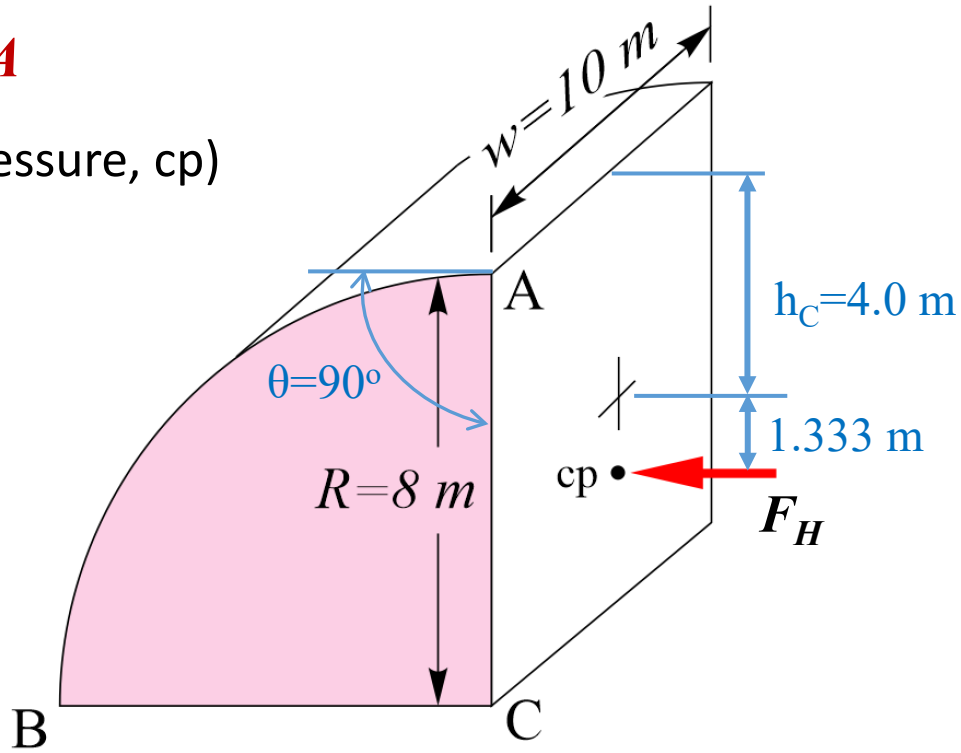


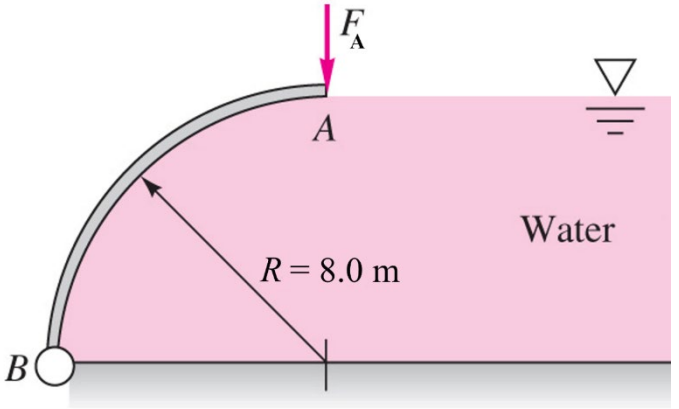



# (b) Vertical Force $F_A$

- Location of $F_H$ (center of pressure, cp)

Free Body Diagram for Water



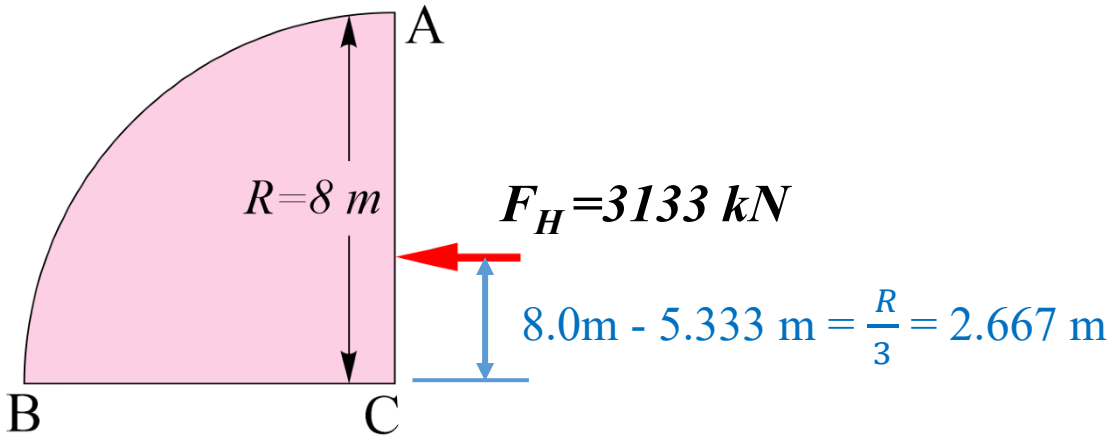

$$y_{cp} = -\frac{I_{xx}\sin(\theta)}{h_c A_{AC}}$$

For AC:

$$I_{xx} = \frac{wR^3}{12} = \frac{10m(8m)^3}{12} = 426.7\ m^4$$

$$y_{cp} = -\frac{426.7\ m^4\ (\sin 90^0)}{4.0\ m\ (80m^2)} = -1.333\ m$$

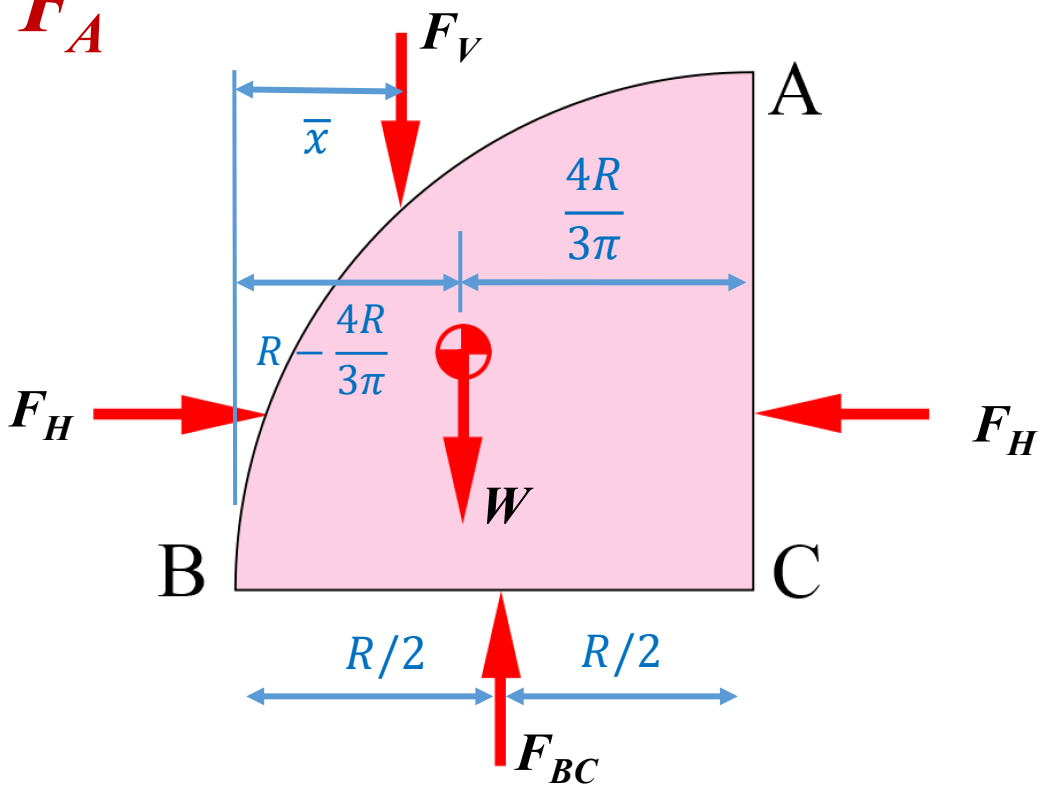
Below centroid

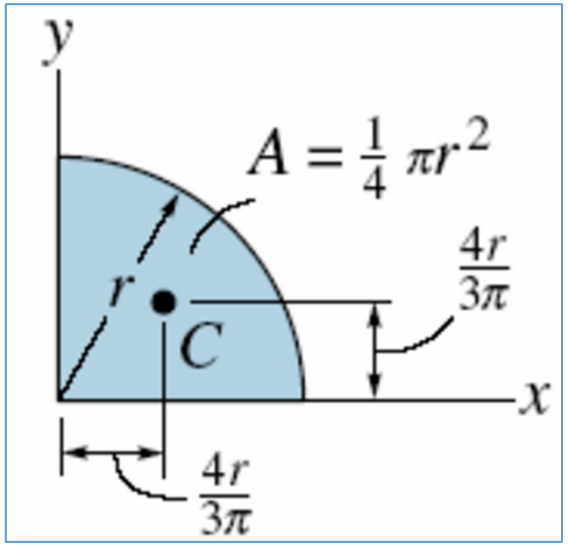


# (b) Vertical Force $F_A$

- Line of Action of $F_V$

Free Body Diagram (Water)




Centroid Location (given in exam)

- The moments (about *any* point) must sum to zero. I will pick point B: $\sum M_B = 0$

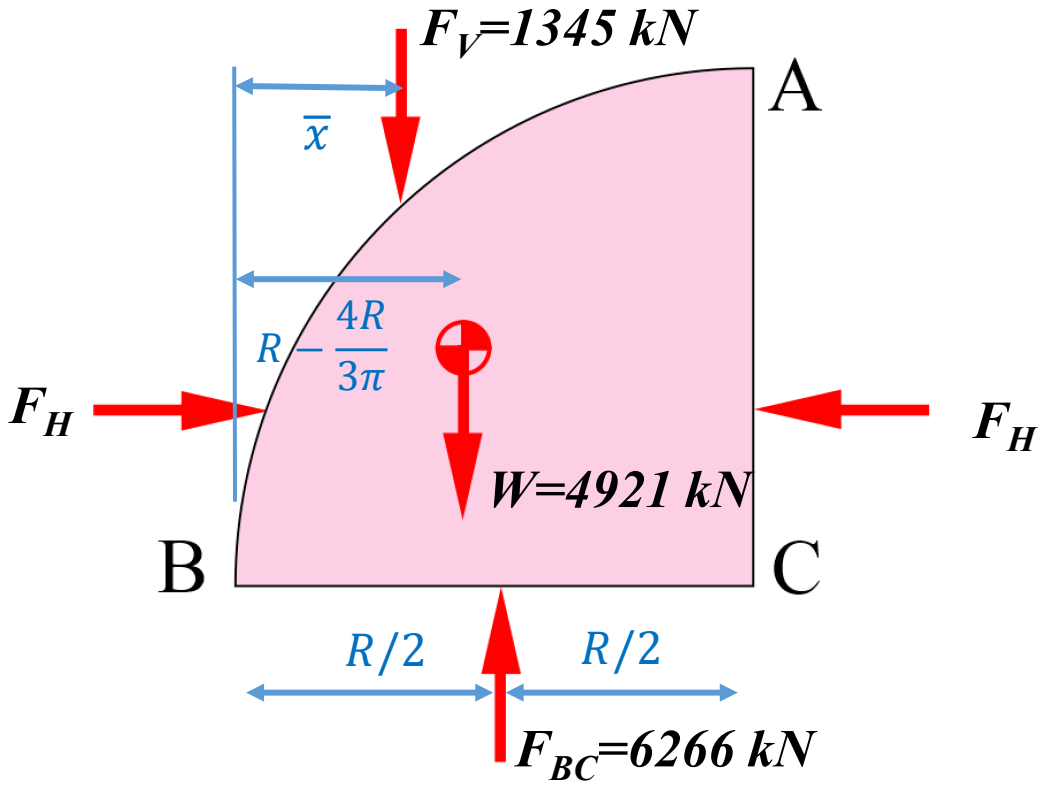
$$F_{BC}\frac{R}{2} - W\left(R - \frac{4R}{3\pi}\right) - F_V\,\overline{x} = 0 \quad \rightarrow \quad \overline{x} = \frac{F_{BC}\frac{R}{2} - W\left(R - \frac{4R}{3\pi}\right)}{F_V}$$

Note: Moments of $F_H$ cancel out

- Line of Action of $F_V$

$$\overline{x} = \frac{F_{BC}\frac{R}{2} - W\left(R - \frac{4R}{3\pi}\right)}{F_V}$$

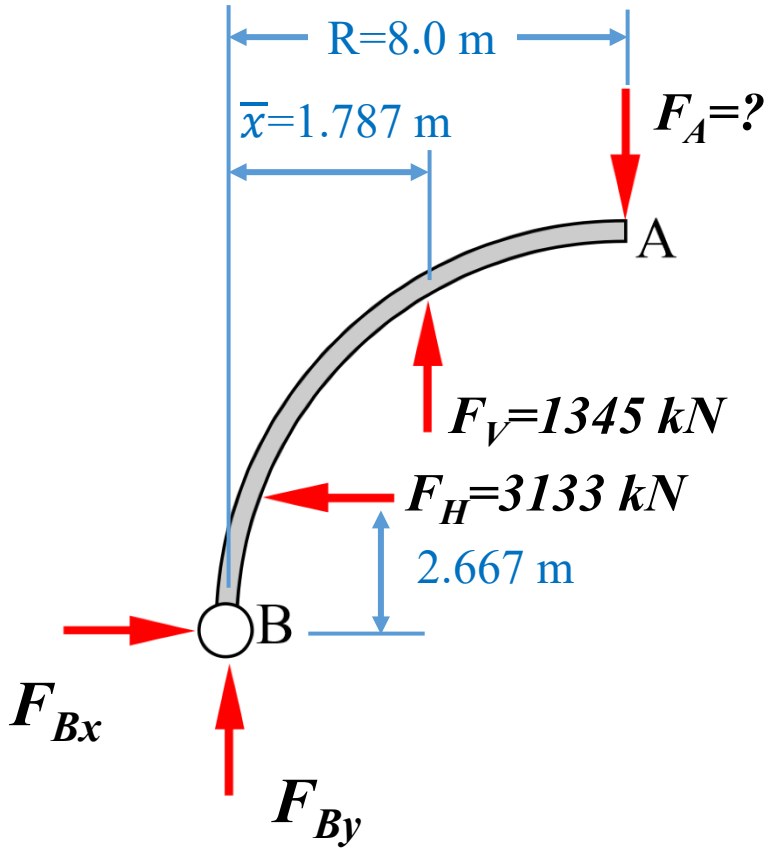
$$\overline{x} = \frac{6266\ kN\ (4m) - 4921kN\left(8 - \frac{4(8)}{3\pi}\right)}{1345\ kN} = 1.787\ m$$

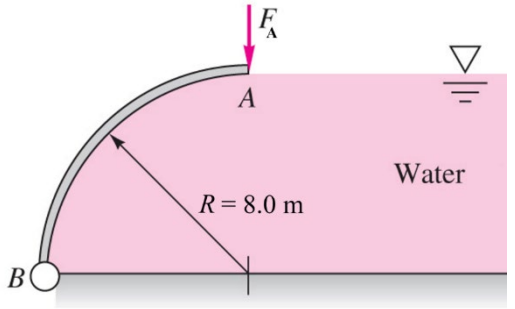



- We can now sum moments on the gate about the hinge, point B.

# (b) Vertical Force $F_A$

- Free Body Diagram (Gate)



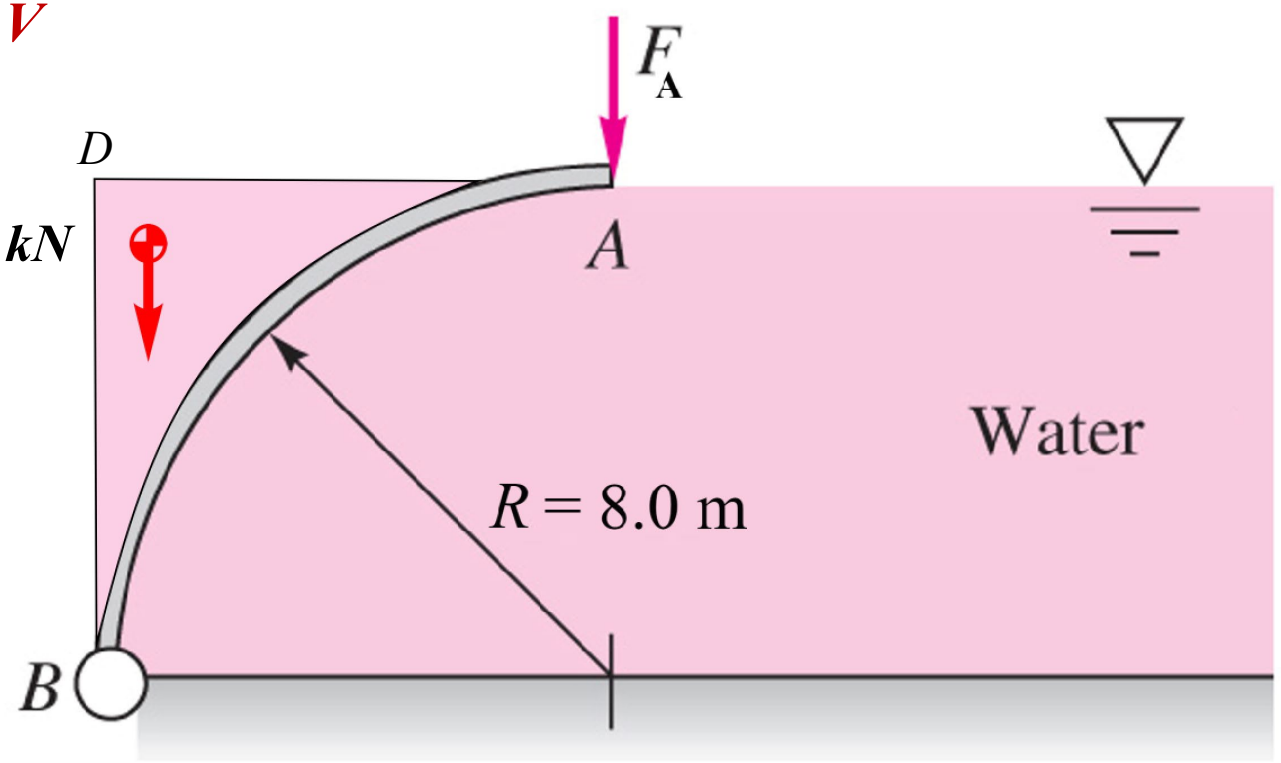

- $\sum M_B = 0$

$$F_A\,(8m) - F_H(2.667m) - F_V(1.787m) = 0$$

$$F_A = \frac{F_H(2.667m)+F_V(1.787m)}{8\ m} = \frac{3133\ kN(2.667m)+1345\ kN(1.787m)}{8\ m} = 1345\ kN\ \downarrow$$ ***Answer***

# Alternate Method for $F_V$

$$F_V = W_{BDA} = (R^2 - \frac{\pi R^2}{4}) w\ \gamma = 1345\ kN$$




## END NOTES

Prepared and delivered by Professor David Naylor



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