MEC516/BME516: Fluid Mechanics I

Review: Final Exam Question

General Energy Equation Example

RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

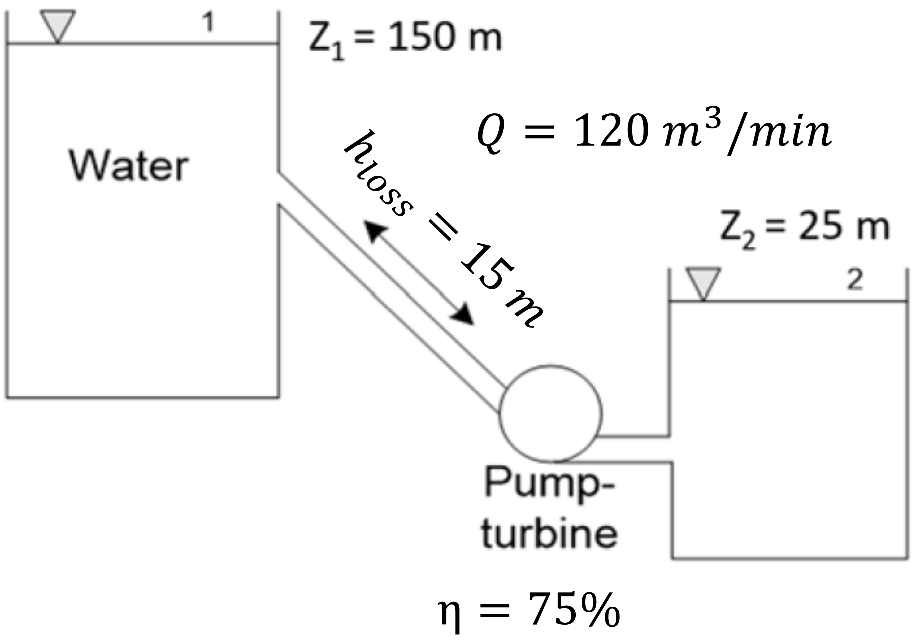
Example: General Energy Equation Problem

Final Exam 2018

A pump-turbine system shown in the figure draws water from the upper reservoir in the daytime to produce power for a city. At night, it pumps water from lower reservoir to restore the situation. For a design flow rate of 120 m^3/min in either direction, the total frictional head loss is 15 m. Assume that the turbine and pump both have an efficiency of 75%. Estimate:

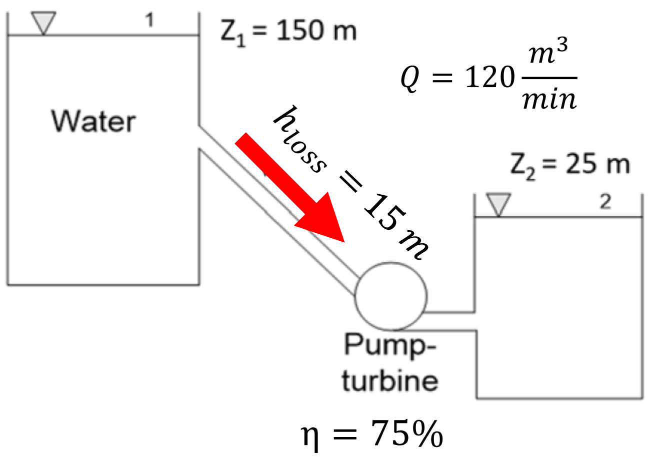
a) The electrical power output by the turbine in kW.

b) The electrical power required by the pump in kW.



Solution

(a) Daytime: Flow is from 1 to 2; Energy is extracted by the turbine



- General Energy Equation (All terms in energy per unit weight, $\frac{J}{N} = \frac{Nm}{N} = m$)

$V_1 \approx 0$ $\quad$ $V_2 \approx 0$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_{Turb} - h_{loss} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$p_1 = p_2 = p_{atm}$

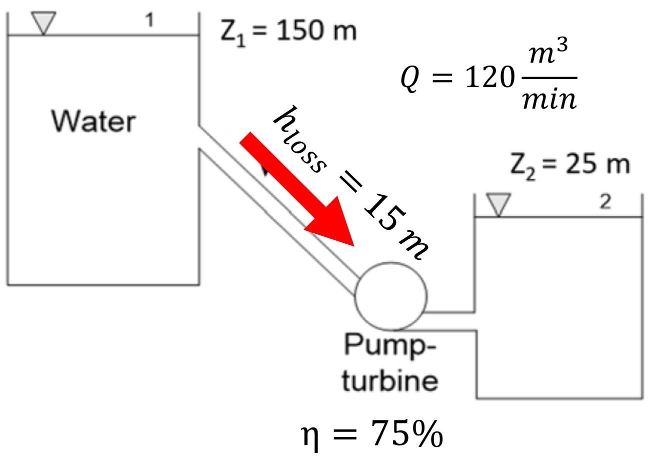
Total fluid energy per unit weight at point (1)

Total fluid energy per unit weight at point (2)

- Solve for the head of the turbine, h_{Turb}

- $h_{Turb} = z_1 - z_2 - h_{loss} = 150\ m - 25\ m - 15\ m = 110\ m$ (Energy extracted per unit weight from the fluid)

Solution



- Energy per unit weight extracted from the fluid: $h_{Turb} = 110\ m$

- The power output from the turbine is:

$$P_{Turb} = \eta\,\gamma\,h_{Turb}\,Q$$

(Only 75% of the fluid power gets converted into useful power by the turbine)

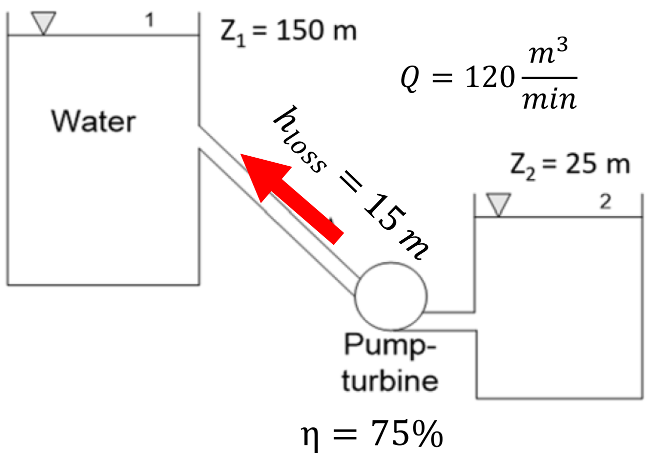
- Volume flow rate $Q = 120\frac{m^3}{min}\frac{min}{60s} = 2.0\ m^3/s$

- Specific weight of water: $\gamma = \rho g = 998\frac{kg}{m^3}\left(9.81\frac{m}{s^2}\right) = 9790\ \frac{N}{m^3}$

$$P_{Turb} = \eta\,\gamma\,h_{Turb}\,Q = 0.75\ \left(9790\ \frac{N}{m^3}\right)(110m)2.0\frac{m^3}{s} = 1615{,}300\ W = 1620\ kW$$

Ans.

Solution



(b) Night: Flow is from 2 to 1; Energy added by the pump

- General Energy Equation (All terms in energy per unit weight, $\frac{J}{N} = \frac{Nm}{N} = m$)

$V_2 \approx 0$ $\qquad$ $V_1 \approx 0$

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{Pump} - h_{loss} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$

$p_1 = p_2 = p_{atm}$

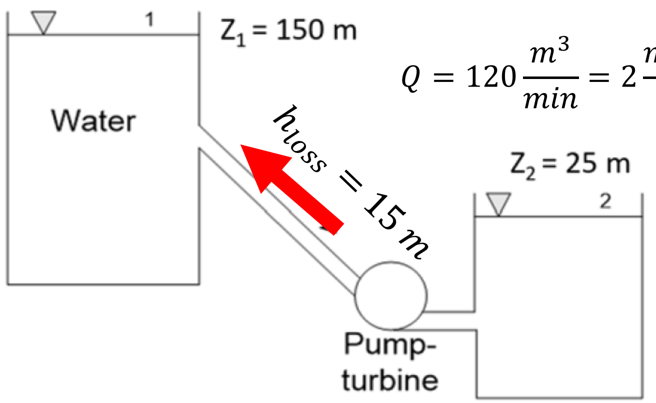
Total fluid energy per unit weight at point (2)

Total fluid energy per unit weight at point (1)

- Solve for the head of the pump, h_{Pump}

- $h_{Pump} = z_1 - z_2 + h_{loss} = 150\ m - 25\ m + 15\ m = 140\ m$ (Energy added per unit weight to the fluid)

Solution



$Q = 120 \frac{m^3}{min} = 2 \frac{m^3}{s}$

$\eta = 75\%$

- Energy per unit weight added to the fluid: $h_{Pump} = 140\ m$

- The power input to the pump shaft is:

$$P_{Pump} = \frac{\gamma\, h_{Pump}\, Q}{\eta}$$

(Pump is 75% efficient at adding power to the fluid. Pump requires MORE power input the energy added to the fluid.)

$$P_{Turb} = \frac{\gamma\, h_{Pump}\, Q}{\eta} = \frac{\left(9790\ \frac{N}{m^3}\right)(140m)2.0\frac{m^3}{s}}{0.75} = 3655000\ W = 3655\ kW$$

Ans.