FINAL EXAM W2017

Q4. The Eulerian velocity field for an incompressible flow is:

$$\boldsymbol{V} = \underbrace{(-2Kxy)}_{u}\ \mathbf{i} + \underbrace{(Ky^2 - Kx^2)}_{v}\ \mathbf{j}$$

where K is a constant.

(a) Determine if this velocity field satisfies conservation of mass.

(b) Derive an expression for the total (substantive) acceleration vector, $\boldsymbol{a} = \frac{dV}{dt}$. Collect terms to simplify the expression.

(a) $u = -2kxy$

$v = ky^2 - kx^2$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$-2ky + 2ky = 0$ ✓ $\therefore \vec{V}$ SATISFIES CONTINUITY

(b) $\vec{a} = a_x \hat{i} + a_y \hat{j}$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$a_x = 0 + (-2Kxy)(-2Ky) + (Ky^2 - Kx^2)(-2Kx)$$

$$a_x = 4K^2xy^2 - 2K^2y^2x + 2K^2x^3$$

$$= 2K^2y^2x + 2K^2x^3$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$a_y = 0 + (-2Kxy)(-2Kx) + (Ky^2 - Kx^2)\,2Ky$$

$$a_y = 4K^2x^2y + 2K^2y^3 - 2K^2x^2y = 2K^2x^2y + 2K^2y^3$$

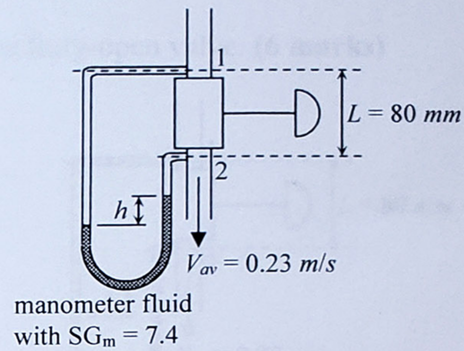
$$\vec{a} = (2K^2y^2x + 2K^2x^3)\hat{i} + (2K^2x^2y + 2K^2y^3)\hat{j}$$

B1. In a laboratory test, as shown below, a valve is connected to a 25-*mm*-diameter vertical pipe. The valve is fully open and water at 20°*C* is used for the test. The manometer height is $h = 4$ *mm*.

(a) What are the **volumetric flow rate**, in *L/s*, and **Reynolds number** of the flow in the pipe? Is the flow laminar, critical or turbulent?

(b) Calculate the **head loss**, in *m* of water, and **pressure loss**, in *Pa*, of the fully-open valve.

(c) Calculate the **pressure difference** $p_1 - p_2$, in *Pa*.



(a) $Q = VA = V\dfrac{\pi D^2}{4} = \left(0.23\,\dfrac{m}{s}\right)\dfrac{\pi(25\times10^{-3}\,m)^2}{4}$

$= 1.129\times10^{-4}\,\dfrac{m^3}{s}\left|\dfrac{1000\,L}{1\,m^3}\right|$

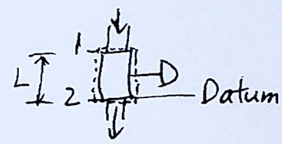
$= 0.1129\,\dfrac{L}{s}$

$Re_D = \dfrac{\rho V D}{\mu} = \dfrac{\left(998\,\frac{kg}{m^3}\right)\left(0.23\,\frac{m}{s}\right)\left(25\times10^{-3}\,m\right)}{1.003\times10^{-3}\,\frac{kg}{m\cdot s}}$

$= 5721$ ∴ turbulent flow

(b) Applying CV energy equation :

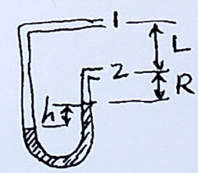
$\dfrac{P_1}{\gamma_w} + \dfrac{\alpha V_1^2}{2g} + z_1 = \dfrac{P_2}{\gamma_w} + \dfrac{\alpha V_2^2}{2g} + z_2 + h_{loss}$



$\therefore h_{loss} = \dfrac{P_1 - P_2}{\gamma_w} + \dfrac{\alpha}{2g}\underbrace{\cancel{(V_1^2 - V_2^2)}}_{0\ (\because V_1 = V_2)} + z_1 - z_2 = \dfrac{P_1 - P_2}{\gamma_w} + L$ ——— ①

Establishing the manometer equation for the manometer:

$P_1 + \gamma_w(L + R + h) - \gamma_m h - \gamma_w R = P_2$



$\Rightarrow P_1 - P_2 = -\gamma_w L - \cancel{\gamma_w R} - \gamma_w h + \gamma_m h + \cancel{\gamma_w R}$

$= -\gamma_w L + \gamma_w\left(\dfrac{\gamma_m}{\gamma_w}h - h\right)$

$\div\,\gamma_w :\ \dfrac{P_1 - P_2}{\gamma_w} = h(SG_m - 1) - L$ ——— ②

Substituting ② into ① yields:

$h_{loss} = h(SG_m - 1) - L + L = h(SG_m - 1) = (4\times10^{-3}\,m)(7.4 - 1) = 0.0256\ m$

$\Delta P_{loss} = \gamma_w h_{loss} = \rho_w g h_{loss} = \left(998\,\frac{kg}{m^3}\right)\left(9.807\,\frac{m}{s^2}\right)(0.0256\,m) = 250.6\ Pa$

(c) Eq. ② : $P_1 - P_2 = \gamma_w[h(SG_m - 1)] - \gamma_w L = \Delta P_{loss} - \gamma_w L$

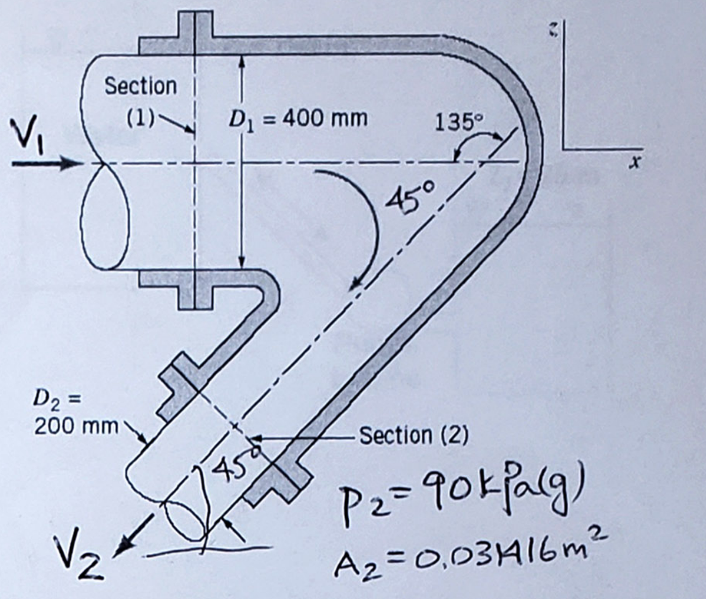
$= 250.6\ Pa - \left(998\,\frac{kg}{m^3}\right)\left(9.807\,\frac{m}{s^2}\right)(80\times10^{-3}\,m) = -532.4\ Pa$

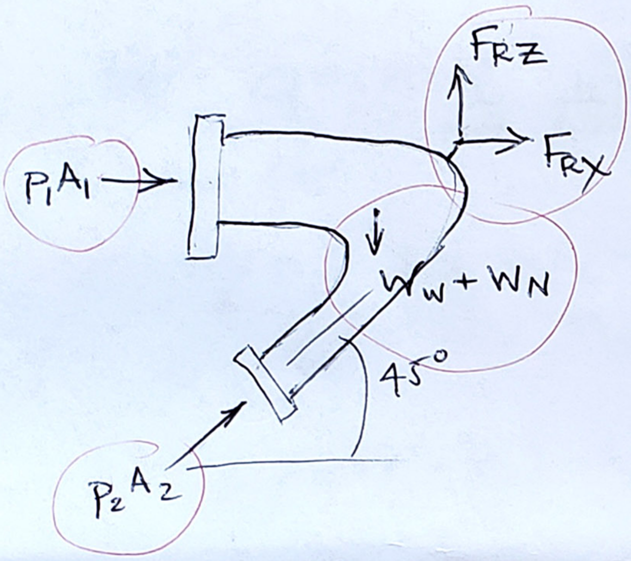
$P_1 = 150\ kPa(g) \quad A_2 = 0.1257 m^2$

Problem #1 (20 points)

A converging elbow turns water through an angle of 135° in a vertical plane. The flow cross-sectional diameter is 400 mm at the elbow inlet, section (1), and 200mm at the elbow outlet, section (2). The elbow flow passage volume is 0.2 m^3 between sections (1) and (2). The water volume flowrate is 0.4 m^3/s, and the elbow inlet and outlet pressures (in **gauge**) are 150 and 90 kPa. The elbow mass is 12 kg. Calculate the horizontal (x-direction) and vertical (z-direction) anchoring forces required to hold the elbow in place.

Make sure you draw a detailed free-body diagram with all the forces and their directions! Assume the density of water is ρ=999 kg/m^3.





CONTINUITY $\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho AV = \rho Q$

$$\dot{m} = 999\ kg/m^3 (0.4\ m^3/s) = 399.6\ kg/s$$

$$V_1 = \frac{Q}{A_1} = \frac{0.4\ m^3/s}{\frac{\pi (0.4)^2}{4}} = 3.1831\ m/s$$

$$V_2 = V_1\left(\frac{D_1}{D_2}\right)^2 = 4V_1 = 12.732\ m/s$$

x - DIRECTION

$$\Sigma F_x = p_1A_1 + p_2A_2\cos 45^\circ + \underline{\underline{F_{RX}}} = \dot{m}(u_2 - u_1) = \dot{m}(-V_2\cos 45^\circ - V_1)$$

$$F_{RX} = -p_1A_1 - p_2A_2\cos 45^\circ - \dot{m}(\overbrace{V_2\cos 45^\circ + V_1}^{12.186\ m/s})$$

$$= \underbrace{-150\times10^3 N/m^2 (.1257m^2)}_{-18849\ N} \underbrace{- 90\times10^3 N/m^2 (.031416m^2)\cos 45^\circ}_{-1999\ N} \underbrace{- 399.6\ kg/s (12.732\cos 45^\circ + 3.1831)\ m/s}_{-4869\ N}$$

$F_{RX} = -25718 N$ ie. $F_{RX} = 25.7\ kN \leftarrow$ ANS.

z - DIRECTION: $\Sigma F_z = p_2A_2\sin 45^\circ + \underline{\underline{F_{RZ}}} - W_w - W_N = \dot{m}(w_2 - \cancel{w_1})$

$w_2 = -V_2 \sin 45^\circ$

$$F_{RZ} = -p_2A_2\sin 45^\circ + \overset{\rho \forall g}{W_w} + \overset{mg}{W_N} - \dot{m}V_2\sin 45^\circ$$

$$F_{RZ} = \underbrace{-90\times10^3 N/m^2 (.031416m^2)(.707)}_{-1999.3\ N} + \underbrace{999\ kg/m^3 (0.2m^3)\,9.81\ m/s^2}_{+1960\ N} + \overbrace{12kg(9.81\ m/s^2)}^{+117.7\ N} - \underbrace{399.6(12.73)\sin 45^\circ}_{-3597\ N}$$

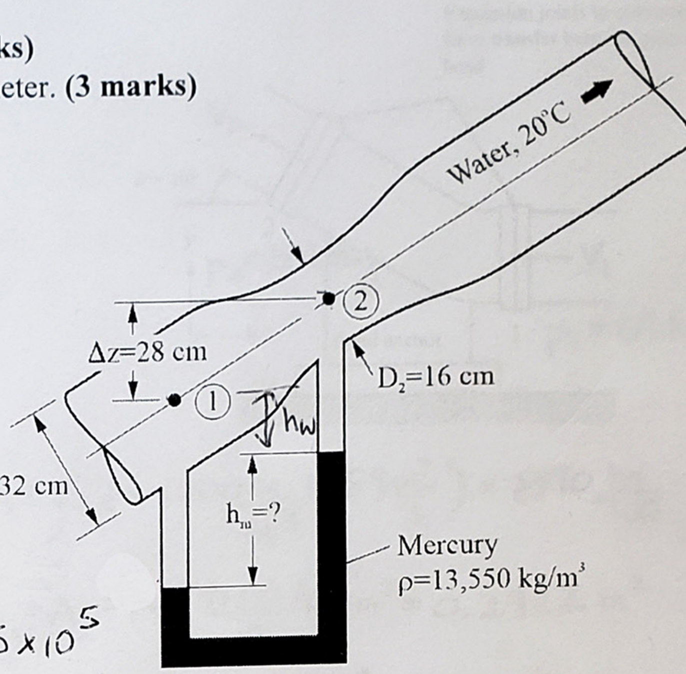
$F_{RZ} = -3519 N$ ie. $F_{RZ} = 3519 N = 3.52\ kN \downarrow$ ANS.

B1. Liquid water at 20°C flows through an inclined Venturi meter at a flow rate of 0.12 m^3/s. The inside pipe diameter is reduced from 32 *cm* at point 1 to 16 *cm* at point 2. Point 1 is 28 *cm* lower than point 2. Assume no head loss from point 1 to point 2. A mercury manometer is used to measure the pressure difference, as shown in the sketch.

(a) Calculate the Reynolds number of the flow in the pipe at point 1. What does this result indicate? **(2 marks)**

(b) Calculate the pressure difference, $p_1 - p_2$. **(5 marks)**

(c) Calculate the height of mercury (h_m) in the manometer. **(3 marks)**



(a) $Re = \frac{\rho V_1 D}{\mu}$

$$V_1 = \frac{Q}{A_1} = \frac{0.12\ m^3/s}{\frac{\pi (.32)^2}{4}\ m^2} = 1.492\ m/s$$

$$Re = \frac{998\ \frac{kg}{m^3} (1.492\ \frac{m}{s})(0.32\ m)}{1.003 \times 10^{-3}\ \frac{kg}{m\,s}} = 4.75 \times 10^5$$

TURBULENT FLOW ANS./

(b)

$$\frac{p_1}{\gamma_w} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_w} + \frac{V_2^2}{2g} + z_2$$

$$p_1 - p_2 = \gamma_w \left[\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1 \right]$$

$$V_2 = \frac{Q}{A_2} = 4V_1 = 5.968\ m/s$$

$$p_1 - p_2 = 998\ \frac{kg}{m^3} (9.81\ \frac{m}{s}) \left[\overbrace{\frac{5.968^2 - 1.492^2\ \frac{m^2}{s^2}}{2(9.81)\ \frac{m}{s^2}}}^{1.702} + 0.28\,m \right]$$

$$= 9790\ \frac{N}{m^3} (1.982\,m) = 19400\ Pa = 19.4\ kPa$$ ANS./

$$p_2 + \Delta z \gamma_w + \cancel{h_w \gamma_w} + h_m \gamma_m = p_1 + \cancel{h_w \gamma_w} + h_m \gamma_w$$

$$h_m(\gamma_m - \gamma_w) = (p_1 - p_2) - \Delta z \gamma_w$$

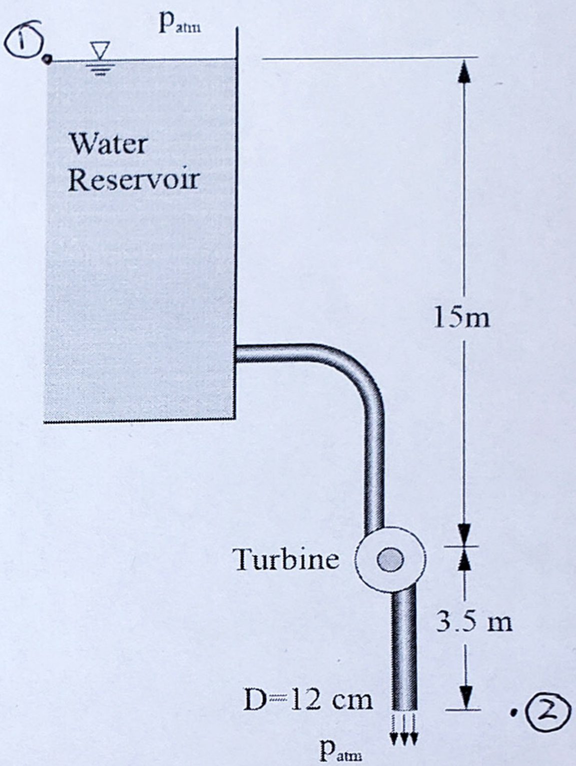
$$h_m = \frac{(p_1 - p_2) - \Delta z \gamma_w}{(\gamma_m - \gamma_w)} = \frac{\overbrace{19400\ N/m^2 - \overbrace{0.28(9790)}^{2741}\ N/m^2}^{16658}}{\underbrace{(13550 - 998)\ \frac{kg}{m^3}(9.81\ m/s^2)}_{123135\ N/m^3}}$$

$h_m = 0.136\ m$ ANS./

5. Water (ρ=998 kg/m^3) flows at a rate of 40 kg/s from a large reservoir to a small hydroelectric turbine, as shown in the sketch. Below the turbine, the water discharges to atmosphere through a pipe with an inside diameter of 12 cm. The total frictional head loss in the piping system is 2.4 m. If the turbine has an efficiency of 78%, calculate the power output of the turbine in kW.

h_ℓ

η



$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 - h_\ell - h_T = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2$$

$$h_T = (z_1 - z_2) - \frac{V_2^2}{2g} - h_\ell$$

$$\dot m_1 = \dot m_2 = \rho A_2 V_2 \qquad V_2 = \frac{\dot m}{\rho A_2}$$

$$V_2 = \frac{40\ \text{kg/s}}{998\ \frac{\text{kg}}{\text{m}^3}\left(\frac{\pi (0.12)^2}{4}\right)\text{m}^2} = 3.544\ \text{m/s}$$

$$h_T = (15\text{m} + 3.5\text{m}) - \frac{(3.544)^2\ \frac{\text{m}^2}{\text{s}^2}}{2(9.81\ \text{m/s}^2)} - 2.4\ \text{m}$$

$$= 18.5 - 0.640\ \text{m} - 2.4\ \text{m} = 15.46\ \text{m}$$

$$P_{TURBINE} = \eta \gamma h_T Q = \eta \overbrace{\rho g Q}^{\dot m} h_T = \eta \dot m g h_T$$

$$= 0.78\left(40\ \frac{\text{kg}}{\text{s}}\right)(9.81\ \text{m/s}^2)\ 15.46\ \text{m} = 4732\ \text{W}$$

$$= 4.73\ \text{kW}$$

ANS.

Question 5 [20 marks total or 10 marks for (a) and 10 marks for (b)].

The pressure drop per unit length of pipe ($\Delta p/L$) depends on the average velocity V, the pipe diameter D, the fluid density ρ, and the fluid viscosity μ. Treat the pressure drop ($\Delta p/L$) as a single variable. There are two parts to this question – part (a) below and part (b) on the next page.

(a) Using the Buckingham Pi method with ρ, V, and D as the repeating variables, show that the two possible Pi groups are given by:

$$\text{(i)}\ \frac{\rho V D}{\mu} \quad \text{and} \quad \text{(ii)}\ \frac{D(\Delta p/L)}{\rho V^2}.$$

$$\frac{\Delta P}{L} = f(V, D, \rho, \mu)$$

V	D	ρ	μ	$\Delta P/L$
$\left\{\frac{L}{T}\right\}$	$\{L\}$	$\left\{\frac{M}{L^3}\right\}$	$\left\{\frac{M}{LT}\right\}$	$\left\{\frac{F}{L^3}\right\} = \left\{\frac{ML}{L^3T^2}\right\} = \left\{\frac{M}{L^2T^2}\right\}$

$$\Pi_1 = \frac{\Delta P}{L} \rho^a V^b D^c$$

$$\left\{\frac{M}{L^2T^2}\right\}\left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c = M^0 L^0 T^0$$

M] $1 + a = 0 \quad a = -1$

T] $-2 - b = 0 \quad b = -2$

L $-2 - 3a + b + c = 0$

$-2 + 3 - 2 + c = 0 \quad c = 1$

$$\Pi_1 = \left(\frac{\Delta P}{L}\right)\frac{D}{\rho V^2} \quad \text{ANS}$$

$$\Pi_2 = \mu \rho^a V^b D^c$$

$$\left\{\frac{M}{LT}\right\}\left\{\frac{M}{L^3}\right\}^a \left\{\frac{L}{T}\right\}^b \{L\}^c = M^0 L^0 T^0$$

M] $1 + a = 0 \quad a = -1$

T] $-1 - b = 0 \quad b = -1$

L] $-1 - 3a + b + c = 0 \quad -1 + 3 - 1 + c = 0 \quad c = -1$

$$\Pi_2 = \frac{\mu}{\rho V D} \quad \left(\frac{1}{Re}\right) \quad \text{OK.}$$

Question 5 (continued).

(b) Kerosene ($\rho = 810$ kg/m³ and $\mu = 0.0019$ N·s/m²) flows through a 0.1-m-diameter pipe with average velocity $V = 2.3$ m/s. The expected pressure drop is $\Delta p = 3800$ Pa over a pipe length of $L = 10$ m. An exact 1/5th scale model pipeline using water ($\rho = 1000$ kg/m³ and $\mu = 0.001$ N·s/m²) is built to test the kerosene system in a safe environment. Assuming dynamically similar conditions, determine for the model pipeline:

i. The pipe diameter D (m) and length L (m),

ii. The average velocity V (m/s), and,

iii. The pressure drop ($\Delta p/L$) (in pascals per metre).

(i) $D_m = \frac{D_p}{5} = \frac{0.1}{5} = .02\,m$ ANS

$L_m = \frac{L}{5} = \frac{10\,m}{5} = 2\,m.$ ANS.

ii) $Re_m = Re_p \qquad \frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$

$$V_m = V_p \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{D_p}{D_m}\right)\frac{\mu_m}{\mu_p}$$

$$= 2.3\,m/s \left(\frac{810 \; \cancel{1000}}{\cancel{810} \; 1000}\right)\left(\frac{5}{1}\right)\left(\frac{0.001}{0.0019}\right) = 4.903\,m/s$$ ANS

iii) $\Pi_{1m} = \Pi_{1p}$

$$\frac{D_m (\Delta P/L)_m}{\rho_m V_m^2} = \frac{D_p (\Delta P/L)_p}{\rho_p V_p^2}$$

$$\left(\frac{\Delta P}{L}\right)_m = \left(\frac{\Delta P}{L}\right)_p \left(\frac{D_p}{D_m}\right)\left(\frac{\rho_m}{\rho_p}\right)\left(\frac{V_m}{V_p}\right)^2$$

$$= \frac{3800\,Pa}{10\,m}\left(\frac{5}{1}\right)\left(\frac{1000}{810}\right)\left(\frac{4.903}{2.3}\right)^2 = 10660\,\frac{Pa}{m}$$