MEC516/BME516: Fluid Mechanics I

Chapter 5: Di mensional Analysis & Similarity
Part 8

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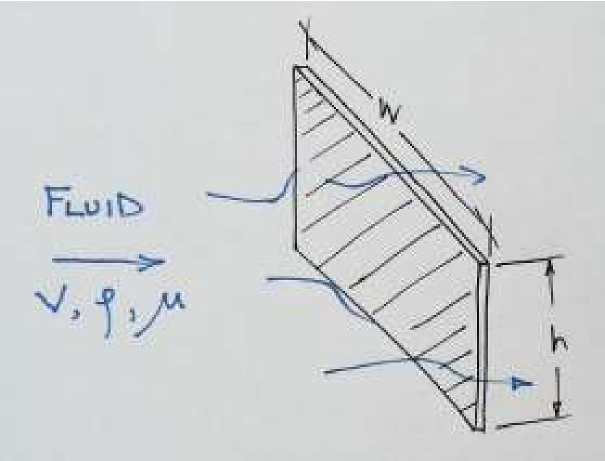
Overview

- The Ipsen Method: An alternate method for determining similarity parameters.



Example 1

Repeat the dimensional analysis of the drag force on a rectangular plate.

Example 2

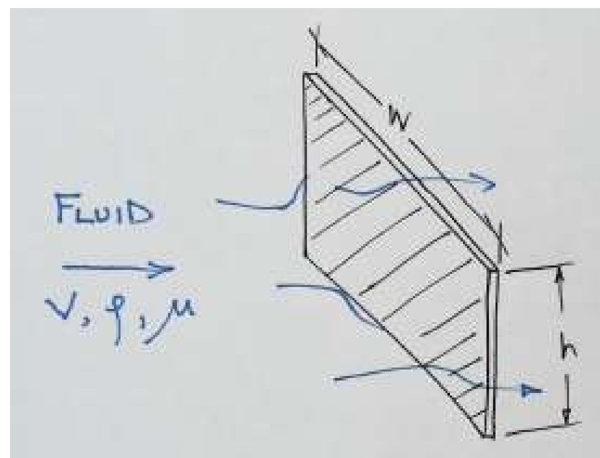
Dimensional analysis of the vortex shedding frequency off a structural component of a bridge.

These examples are not in your text. So, see your textbook for other examples.

The Ipsen Method of Dimensional Analysis

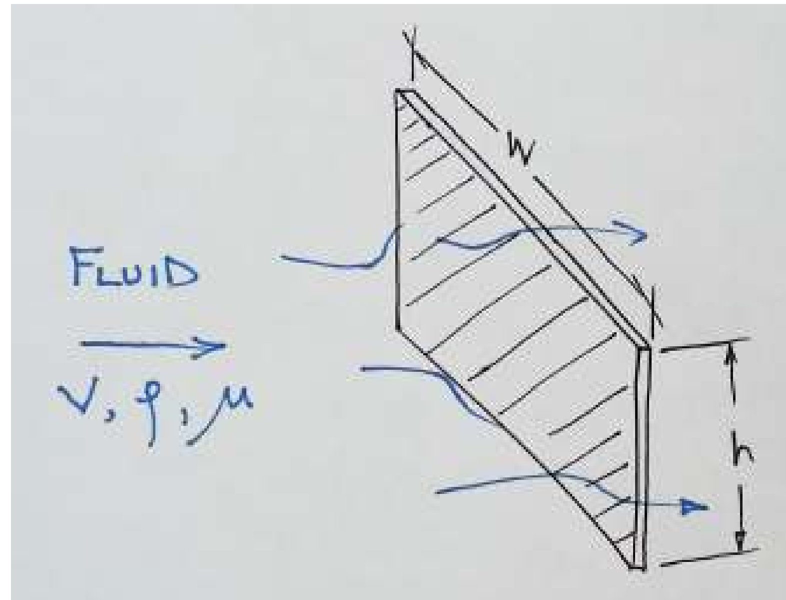
- An alternate approach to the *Method of Repeating Variables* .
- The Ipsen method is easy to implement, but less intuitive (at least to me). Nevertheless, it is worth being aware of this alternative.
- To demonstrate the procedure, we will repeat the analysis of flow over a rectangular plate to find the dimensional parameters that characterise the aerodynamic drag force, F_D. (See video Chapter 5 Part 2)

$$F_D = f_1(w, h, V, \mu, \rho)$$



Example 1: The Ipsen Method

Consider that we want to experimentally characterize the drag force F_D on a rectangular plate produced by a flow of fluid perpendicular to the surface.



The drag force is a function of the flow velocity (V) and fluid properties (ρ and μ) and the dimensions of the rectangular plate. The plate has height h and width w.

Determine the dimensionless parameters needed to conduct the experiment, using the ***Ipsen Method.***

Example 1: The Ipsen Method

fcn stands for some unknown function, which will change during the procedure.

Solution

- Start by writing out the functional relationship with basic dimensions below each quantity:

$$
\begin{array}{cccccc}
F_D = fcn\,(w, & h, & V, & \mu, & \rho) \\
\left\{\frac{ML}{T^2}\right\} & \{L\} & \{L\} & \left\{\frac{L}{T}\right\} & \left\{\frac{M}{L\,T}\right\} & \left\{\frac{M}{L^3}\right\}
\end{array}
$$

- This equation has three dimensions $\{MLT\}$.
- We eliminate these three dimensions, one at a time using the following procedure:
  - Pick a dimension, say {M}. Pick a variable on the right hand side that contains {M}, say ρ.
  - Divide or multiply terms in the contain {M} by the appropriate power of ρ.
  - If a term contained $\{M^2\}$ you would divide by ρ^2 to eliminate {M} in that term.
  - Once this is done, remove ρ from the list of variables on the RHS.
  - Then you repeat this process, eliminating $\{T\}$, then $\{L\}$. (The order does not matter).

Example 1: The Ipsen Method

$$F_D = fcn\,(w, \quad h, \quad V, \quad \mu, \quad \rho)$$

$$\left\{\frac{ML}{T^2}\right\} \quad \{L\} \quad \{L\} \quad \left\{\frac{L}{T}\right\} \quad \left\{\frac{M}{L\,T}\right\} \quad \left\{\frac{M}{L^3}\right\}$$

- We picked dimension {M} and variable that contains {M}, ρ. Divide or multiply terms that contain {M} by the appropriate power of ρ.

$$\frac{F_D}{\rho} = fcn\left(w, \quad h, \quad V, \quad \frac{\mu}{\rho}, \quad \cancel{\rho}\right)$$

$$\left\{\frac{L^4}{T^2}\right\} \quad \{L\} \quad \{L\} \quad \left\{\frac{L}{T}\right\} \quad \left\{\frac{L^2}{T}\right\}$$

Then eliminate the chosen variable from the list

- We did not divide all terms by ρ. We only divided those terms that contained {M}, μ and F_D.

Example 1: The Ipsen Method

$$\frac{F_D}{\rho} = fcn\left(w, \quad h, \quad V, \quad \frac{\mu}{\rho}\right)$$

$$\left\{\frac{L^4}{T^2}\right\} \quad \{L\} \quad \{L\} \quad \left\{\frac{L}{T}\right\} \quad \left\{\frac{L^2}{T}\right\}$$

← re-written without ρ

- Now we repeat this process, eliminating the dimension of time $\{T\}$.
- We pick a RHS variable that contains $\{T\}$, V . We now eliminate {T}, in the terms that contain {T} by dividing or multiplying by appropriate powers of V.

$$\frac{F_D}{\rho\, V^2} = fcn\left(w, \quad h, \quad \cancel{V}, \quad \frac{\mu}{\rho V}\right)$$

$$\{L^2\} \quad \{L\} \quad \{L\} \quad \{L\}$$

Then eliminate
the chosen variable
from the list

Example 1: The Ipsen Method

$$\frac{F_D}{\rho\, V^2} = fcn\left(w, \qquad h, \qquad \frac{\mu}{\rho V}\right)$$

$$\{L^2\} \qquad\qquad \{L\} \qquad \{L\} \qquad \{L\}$$

← re-written without V

- Now we repeat this process, eliminating the dimension of length $\{L\}$.
- We pick a variable that contains $\{L\}, w$. This dimension will be our characteristic length for the problem.
- We now eliminate {L}, in the terms that contain $\{\mathrm{L}\}$ by dividing or multiplying by appropriate powers of w.

$$\frac{F_D}{\rho\, V^2\, w^2} = fcn\left(\cancel{w}, \qquad \frac{h}{w}, \qquad \frac{\mu}{\rho V w}\right)$$

$$\{-\} \qquad\qquad\qquad \{-\} \qquad \{-\}$$

Then eliminate
the chosen variable
from the list

Example 1: The Ipsen Method

- All terms are dimensionless. Thus, the process is complete:

$$\frac{F_D}{\rho\, V^2\, w^2} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$$

← re-written without *w*

- We recognize that the second term is 1/Re. We can invert any term, since the functional relationship is unknown.
- So, we get:

$$\frac{F_D}{\rho\, V^2\, w^2} = fcn\left(\frac{h}{w}, Re\right)$$

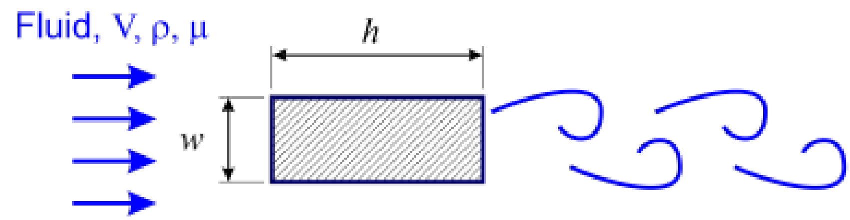
Ans.

- This is *the **same result*** as by the Method of Repeating Variables:

The dimensionless drag force depends only upon the aspect ratio of the plate and the Reynolds number.

Example 2: The Ipsen Method

Consider periodic vortex shedding off a structural component of a bridge, which can create harmful vibrations. Given that the frequency of vortex shedding frequency f is a function of the fluid velocity, V, the fluid properties ρ, μ and the dimensions of the component h, w:



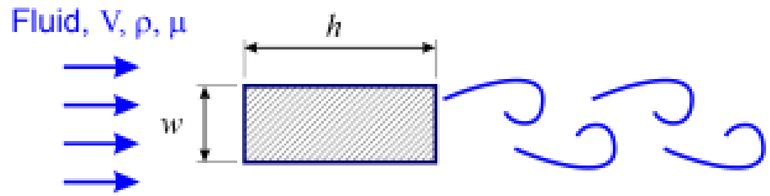
(a) Use the Ipsen method to determine the dimensionless parameters for the vortex shedding frequency.

(b) For a prototype structure the design wind speed is V=14m/s (about 50km/hr) in air 20°C and 100kPa. A 1/5th scale model is tested in a water tunnel at 20°C. The shedding frequency from the model is 50 Hz.

Calculate the required water test speed for the model.

Find the vortex shedding frequency on the prototype.

Example 2: The Ipsen Method



(a)<u>Solution</u>

The problem statement gives that the vortex shedding frequency (Hz or cycles per second) is:

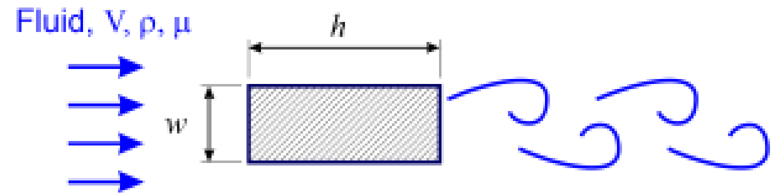
$$
\begin{array}{cccccc}
f & = fcn\,(w, & h, & V, & \mu, & \rho) \\
\left\{\frac{1}{T}\right\} & \{L\} & \{L\} & \left\{\frac{L}{T}\right\} & \left\{\frac{M}{L\,T}\right\} & \left\{\frac{M}{L^3}\right\}
\end{array}
$$

Using the Ipsen's method, pick a variable that contains {M}, ρ. We use ρ to eliminate {M} in the terms that contain the dimension of mass.

$$
\begin{array}{cccccc}
f & = fcn\left(w,\right. & h, & V, & \frac{\mu}{\rho}, & \left.\cancel{\rho}\right) \\
\left\{\frac{1}{T}\right\} & \{L\} & \{L\} & \left\{\frac{L}{T}\right\} & \left\{\frac{L^2}{T}\right\} &
\end{array}
$$

Then eliminate
the chosen variable
from the list

Example 2: The Ipsen Method



$$f \quad = fcn\left(w, \quad h, \quad V, \quad \frac{\mu}{\rho}\right)$$

← re-written without ρ

$$\left\{\frac{1}{T}\right\} \qquad \{L\} \quad \{L\} \quad \left\{\frac{L}{T}\right\} \quad \left\{\frac{L^2}{T}\right\}$$

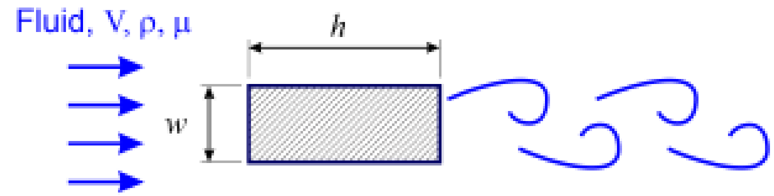
- Now we pick RHS variable that contains {T}, V. We use V to eliminate $\{T\}$ in the terms that contain the dimension of time.

$$\frac{f}{V} \quad = fcn\left(w, \quad h, \quad \cancel{V}, \quad \frac{\mu}{\rho V}\right)$$

Then eliminate
the chosen variable
from the list

$$\left\{\frac{1}{L}\right\} \qquad \{L\} \quad \{L\} \qquad \quad \{L\}$$

Example 2: The Ipsen Method



$$\frac{f}{V} = fcn\left(w, \quad h, \quad \frac{\mu}{\rho V}\right)$$

$$\left\{\frac{1}{L}\right\} \qquad \{L\} \quad \{L\} \quad \{L\}$$

← re-written without V

- Now we pick RHS variable that contains {L}, w. This will be the characteristic dimension. We use w to eliminate $\{L\}$ in the terms that contain the dimension of length.

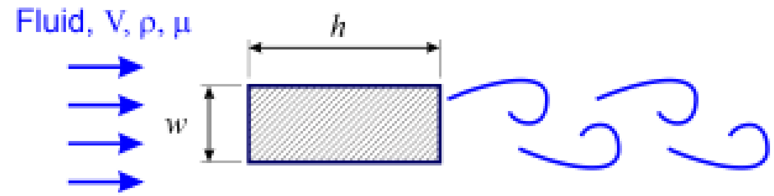
$$\frac{fw}{V} = fcn\left(\cancel{w,} \quad \frac{h}{w}, \quad \frac{\mu}{\rho V w}\right)$$

$$\{-\} \qquad \{-\} \quad \{-\}$$

Then eliminate
the chosen variable
from the list

- All the terms are now dimensionless.

Example 2: The Ipsen Method



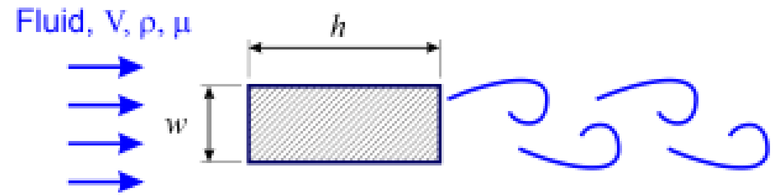
Result: $$\frac{fw}{V} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$$ Ans.

Strouhal number, St — Aspect ratio — Reynolds number, Re

- Strouhal number is a dimensionless frequency, that is only a function of the geometry and Reynolds number of the flow.

- This can be re-written as: $St = fcn(\frac{h}{w}, Re)$

Example 2: The Ipsen Method



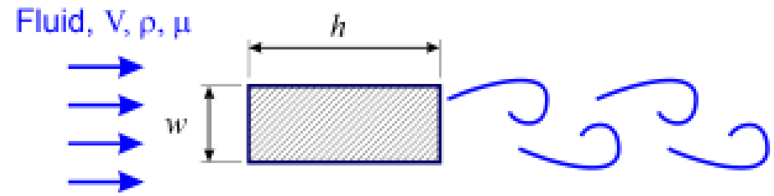
$$\frac{fw}{V} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$$

(b) For a prototype structure the design wind speed is V=14 m/s in air 20°C and 100kPa. A 1/5th scale model is tested in a water tunnel at 20°C. The shedding frequency from the model is 50 Hz. Calculate the required water test speed for the model, V_m. Find the vortex shedding frequency on the prototype, f.

<u>Solution</u>

The model is 1/5th scale. So, $\frac{w}{w_m} = \frac{h}{h_m} = 5$. For similitude, the prototype and model must have the same Reynolds number:

$$\frac{\rho_m V_m w_m}{\mu_m} = \frac{\rho V w}{\mu} \qquad V_m = \frac{\rho}{\rho_m}\frac{\mu_m}{\mu}\frac{w}{w_m}V$$



$$\frac{fw}{V} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$$

Example 2: The Ipsen Method

- We have: $V_m = \frac{\rho}{\rho_m} \frac{\mu_m}{\mu} \frac{w}{w_m} V$

- The prototype properties correspond to air (20 ºC, 100kPa):

$$\rho = \frac{P}{RT} = \frac{100x10^3 Pa}{287 \frac{J}{kgK}(293K)} = 1.19 \frac{kg}{m^3} \quad \mu = 1.8x10^{-5} \frac{Ns}{m^2} \quad \text{(Table A.2)}$$

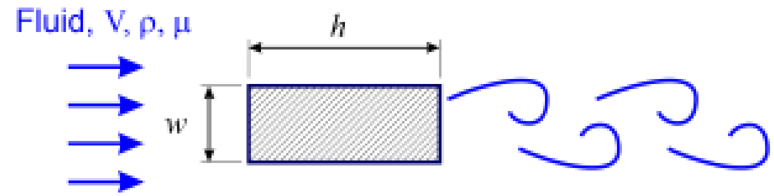
- The model properties are water at 20 ºC (Table A.1): $\rho_m = 998 \frac{kg}{m^3} \quad \mu_m = 1.003x10^{-3} \frac{Ns}{m^2}$
- So, the required water velocity for the water flow is:

$$V_m = \frac{\rho}{\rho_m} \frac{\mu_m}{\mu} \frac{w}{w_m} V = \frac{1.19}{998} \frac{1.003\ E{-3}}{1.8E{-5}} \left(\frac{5}{1}\right) 14.0 \frac{m}{s} = 4.65 \frac{m}{s}$$

- Water has lower kinematic viscosity ($\upsilon = \frac{\mu}{\rho}$) than air. So, the velocity for the same Re is lower than for air.

(If the model was tested in a wind tunnel (air): V=5(14m/s)= 250 km/hr! Very difficult to achieve.)

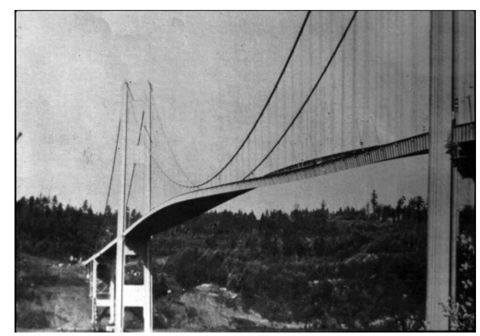
Example 2: The Ipsen Method



$$\frac{fw}{V} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$$

- In part (a) we showed that: $\frac{fw}{V} = fcn\left(\frac{h}{w}, \frac{\mu}{\rho V w}\right)$
- So, our prediction equation is: $\frac{f_m w_m}{V_m} = \frac{fw}{V}$
- So, the vortex shedding frequency for the prototype is:

$$f = \frac{V}{V_m}\frac{w_m}{w} f_m = \frac{14.0\frac{m}{s}}{4.65\frac{m}{s}}\left(\frac{1}{5}\right) 50\ Hz = 30.1\ Hz$$ Ans.



- This unsteady flow produces weak periodic forces on the structure. Even weak forces near the resonant frequency can produce large amplitude flow induced vibration. So, this forcing frequency would be compared to that natural frequencies from a vibration analysis, to avoid possible resonance.
- An interesting aside: Note that the values of h and w were not needed, only that the model was 1/5th scale.

- This is the last video in the course! I hope you enjoyed these video lectures.
- Feel free to send me any helpful comments or constructive suggestions.
- Good luck on the final exam.

END NOTES

Presentation prepared and delivered by Dr. David Naylor.