MEC516/BME516: Fluid Mechanics I

Chapter 5: Dimensional Analysis & Similarity
Part 7

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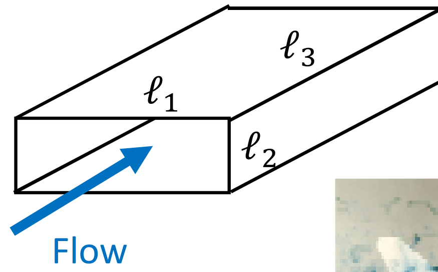
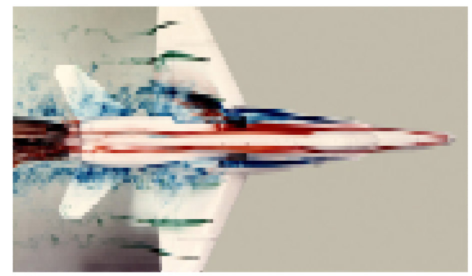
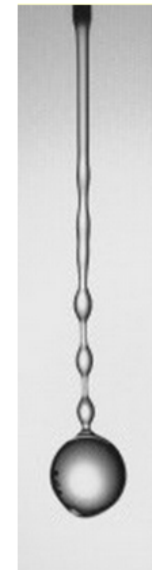
Overview

- Some Typical Model Studies in Engineering
  - Internal flows in pipes and ducts
  - External flow over immersed objects
  - Flows with a free surface

- Distorted Models & the Limitations of Model Testing
  - e.g. Open channel flows







Solved Example

Modelling a hydraulic dam spillway, including time scale issues.

End Slide: Use of models in motion pictures.

Source: http://nu.libguides.com

Typical Model Studies

1. Internal Incompressible Flow

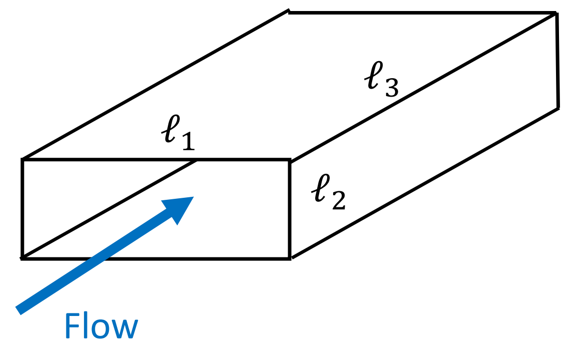
- flows in pipes, ducts, valves and fittings
- liquid flows, gases flows with Ma<0.3

The dependent Pi term: $\Pi_1 = f(\frac{\ell_i}{\ell},\ \frac{\epsilon}{\ell},\ \frac{\rho V \ell}{\mu})$

geometry

relative roughness

Reynolds number

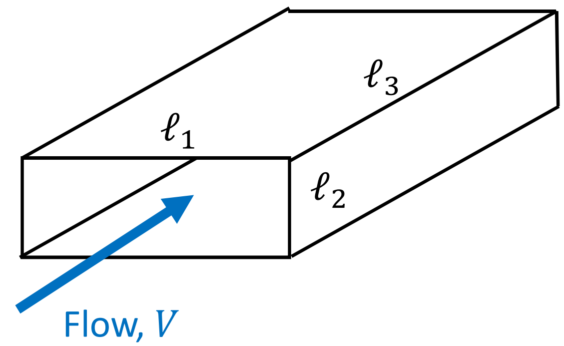


- Similitude requires perfect geometric scaling including surface roughness, and the model flow must have the same Reynolds number

Typical Model Studies

- For example, consider pressure losses Δp in a duct.
- Dimensional analysis gives:

$$\Pi_1 = \frac{\Delta p}{\rho V^2}$$



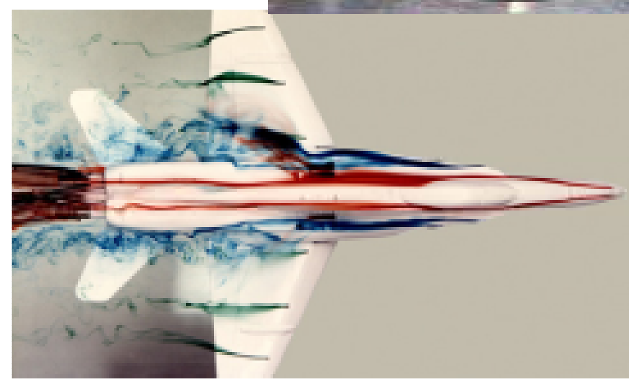
- So, the pressure losses in the prototype are obtained from the prediction equation:

$$\Pi_1 = \Pi_{1,m} \qquad \frac{\Delta p}{\rho V^2} = \frac{\Delta p_m}{\rho_m V_m^2} \rightarrow \Delta p = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2 \Delta p_m$$

Typical Model Studies

2. External Incompressible Flow over Immersed Objects

- flows around models of aircraft, buildings, automobiles, etc.
- liquid flows, gases flows with Ma<0.3

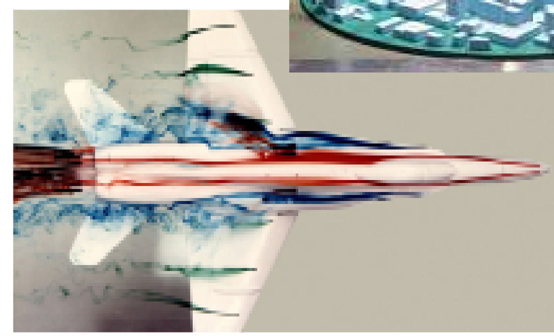


The dependent Pi term:

$$\Pi_1 = f\Big(\underbrace{\frac{\ell_i}{\ell}}_{\text{geometry}},\ \underbrace{\frac{\epsilon}{\ell}}_{\text{relative roughness}},\ \underbrace{\frac{\rho V \ell}{\mu}}_{\text{Reynolds number}}\Big)$$

- As with internal flow, similarity requires perfect geometric scaling including surface roughness, and the model flow must have the same Reynolds number.

Typical Model Studies



- For example, consider the aerodynamics drag force F_D on an object.
- We have shown that dimensional analysis gives:

$$\Pi_1 = \frac{F_D}{\rho V^2\, \ell^2}$$

- So, the drag force on the prototype are obtained from the prediction equation:

$$\Pi_1 = \Pi_{1,m} \qquad \frac{F_D}{\rho V^2\, \ell^2} = \frac{F_{D\,m}}{\rho_m V_m^2\, \ell_m^2} \;\rightarrow\; F_D = \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2\left(\frac{\ell}{\ell_m}\right)^2 F_{D\,m}$$

Typical Model Studies

3. Flows with a Free Surface

- large scale flows in canals, rivers, dams and breakwaters
- small scale flow such as liquid sprays and liquid droplet motion.

- The dependent Pi term:

Weber number, We

$$\Pi_1 = f\left(\frac{\ell_i}{\ell},\ \frac{\epsilon}{\ell},\ \frac{\rho V \ell}{\mu},\ \frac{\rho V^2 \ell}{\Upsilon},\ \frac{V^2}{g\ell}\right)$$

geometry

relative roughness

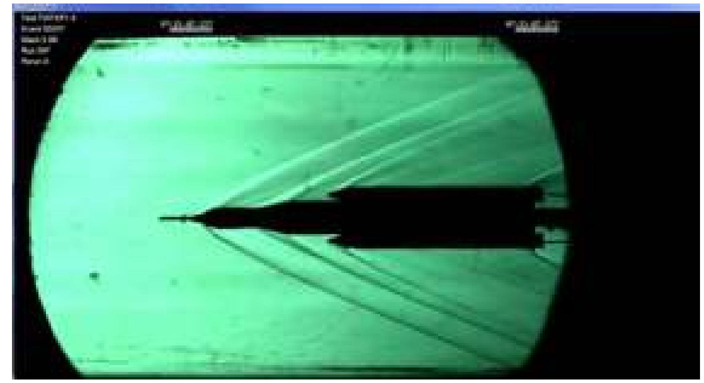
Reynolds number, Re

Froude number, Fr

- For large models, the effect of surface tension is usually small and can be neglected. We number is not important.
- Re number plays a role in surface shear stresses (i.e., skin friction) and can be important.
- When all the Pi parameters are the same, there is perfect dynamic similarity. However, it is often difficult, if not impossible, to achieved perfect similarity with a model.

I purposely skipped this slide in the video. It is included here in care you are interested in compressible flows.

Typical Model Studies



Source: www.nasa.gov

4. High Speed Gas Flows

- aerospace applications to high speed flight, rockets, shell ballistics.

- The dependent Pi term:

Mach number

$$\Pi_1 = f\left(\frac{\ell_i}{\ell},\ \frac{\epsilon}{\ell},\ \frac{\rho V \ell}{\mu},\ \frac{V}{c},\ \frac{c_p}{c_v}\right)$$

geometry

relative roughness

Reynolds number

Specific heat ratio, k

- When all the Pi parameters are the same, there is perfect dynamics similarity. With wind tunnel models involving high speed flow it is very difficult to achieved perfect similarity.
- For models, the similarity of the Mach number is satisfied but Re is not, resulting in a so-called “distorted model”. (Distorted model are discussed next.).

Distorted Models

- It is often not possible to perfectly satisfy all the similarity requirements when using a scaled model.
- In this case, engineering judgement and experience is required to assess the most important Pi parameters.
- Here is a common engineering example:

Models of Open Channel Flows

If we neglect small surface tension effects (and assume we can build a perfect scaled model) similarity requires that the model and prototype have the same Reynolds and Froude numbers.

It will be shown that these two requirements are difficult to achieve at the same time.

Distorted Models

Froude number similarity requires: $\frac{V_m^2}{g_m \ell_m} = \frac{V^2}{g\ell}$

Note that $g_m = g$. So, the model velocity is: $\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$

Equate

Reynolds number similarity requires: $\frac{\rho_m V_m \ell_m}{\mu_m} = \frac{\rho V \ell}{\mu}$ $\rightarrow$ $\frac{V_m}{V} = \frac{\mu_m}{\mu} \frac{\rho}{\rho_m} \frac{\ell}{\ell_m}$

Equating the velocity scales, we get the requirement for similarity: $\frac{\upsilon_m}{\upsilon} = (\frac{\ell_m}{\ell})^{3/2}$

where $\upsilon = \frac{\mu}{\rho}$ is the kinematic viscosity. Thus, different fluids are required.

Distorted Models

Open channel flow similarity requires: $\frac{\upsilon_m}{\upsilon} = \ (\frac{\ell_m}{\ell})^{3/2}$

- Consider a 1:100 scale model of a dam: $(1/100)^{1.5}=10^{-3}$. Thus, to maintain perfect similarity the model test requires working fluid with a kinematic viscosity one thousand times less than that of water!
- Does such a working fluid (liquid) exist? Have a look in the property tables. Probably not.
- How is this resolved? Hydraulic models of dams are usually based on similarity of the Froude number ***with water***. The Re number will be lower than for the prototype → A distorted model.
- So, the prediction equation, $\Pi_1 = \Pi_{1,m}$ will be approximate, and less accurate. Corrections can be applied for the effect of the low Reynolds number (beyond the scope of this course).
- An example of a "scale up" error. Remember: Extrapolation of data from scaled models to the full scale prototype is often not perfect.

Example: Modelling of a Dam Spillway

A 1:65 scale model of the Bluestone Lake Dam (West Virginia) was constructed to study the flow in the spillway of a dam. The spillway is designed to carry a flow of 125 m^3/s.

The effects of viscosity and surface tension can be neglected. So, Reynolds and Weber number effects are unimportant (i.e. the judicious use of a distorted model).

The model is to be tested using water at the same Froude number as the prototype. Calculate:

(a) the required volume flow rate for the model.

(b) the time scale of the model. What operating time corresponds to 1 day in the prototype?

Example: Modelling of a Dam Spillway

(a) Solution

Similarity requires that the Froude number for the model and prototype to be equal.

$$\frac{V_m^2}{g_m \ell_m} = \frac{V^2}{g\ell}$$

(Note some books define Fr=$\frac{V}{\sqrt{g\ell}}$. This does not change the solution or numerical answer).

Note that $g_m = g$ So, the model velocity is:

$$\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$$

The volume flow rate is $Q = VA,$ where $A = w\ h$

Example: Modelling of a Dam Spillway

The model velocity is: $\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$ $\quad Q = VA,$ where $A = w\ h$

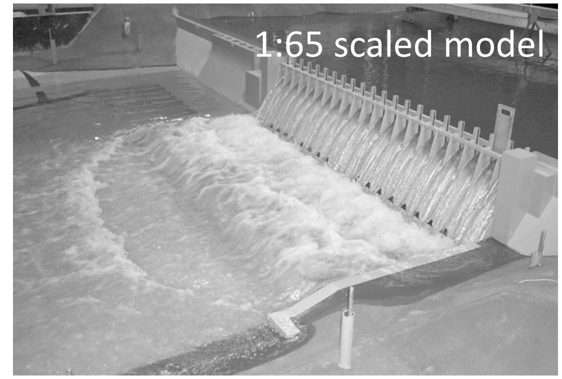
It is required that all the geometric features of the model dam, including surface roughness have a 1:65 scale relative to the prototype. Thus,

$$\frac{w_m}{w} = \frac{h_m}{h} = \frac{\ell_m}{\ell} = \frac{1}{65}$$

$$\frac{Q_m}{Q} = \frac{V_m w_m h_m}{V\ w\ h} = \sqrt{\frac{\ell_m}{\ell}}\left(\frac{\ell_m}{\ell}\right)\left(\frac{\ell_m}{\ell}\right) = (\frac{\ell_m}{\ell})^{5/2}$$

$$Q_m = Q(\frac{\ell_m}{\ell})^{5/2} = 125\frac{m^3}{s}\ (\frac{1}{65})^{5/2} = 3.67x10^{-3}\frac{m^3}{s}$$ Ans.

Example: Modelling of a Dam Spillway



(b) Solution

The time scale between the model and prototype can be obtained from the velocity scale. Velocity is distance over time $V = \frac{\ell}{t} \rightarrow t = \ell/V$

We showed that the velocity ratio is: $\frac{V_m}{V} = \sqrt{\frac{\ell_m}{\ell}}$

Thus, the time scale is: $\frac{t_m}{t} = \left(\frac{\ell_m}{\ell}\right) \frac{V}{V_m} = \left(\frac{\ell_m}{\ell}\right) \left(\frac{\ell}{\ell_m}\right)^{\frac{1}{2}} = \sqrt{\frac{\ell_m}{\ell}} = \sqrt{\frac{1}{65}} = 0.124$

So, one day of flow on the prototype takes only 0.124 day (3.0 hrs) on the model. Ans.

An advantage of using a hydraulic models is that they speed up events.

A model sequence from "Raise the Titanic", a 1980 movie. The footage is slowed down to try to match the time scale of the actual process. Despite the fact that a large model of the ship (1/18th scale) was used, the visual effect was relatively poor because the dynamic similarity was not achieved. Source: https://www.youtube.com/watch?v=KRhf2H3MLzg

END NOTES

Presentation prepared and delivered by Dr. David Naylor.