MEC516/BME516: Fluid Mechanics I

Chapter 5: Dimensional Analysis & Similarity Part 6

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Overview

- Theory of Models and "Scale-up"

  e.g. data from small models in wind and water tunnels

www.nasa.gov

http://www.blwtl.uwo.ca/

Numerical Example

Predicting the forces on billboard sign using measurements on a model in a water tunnel.

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Theory of Models

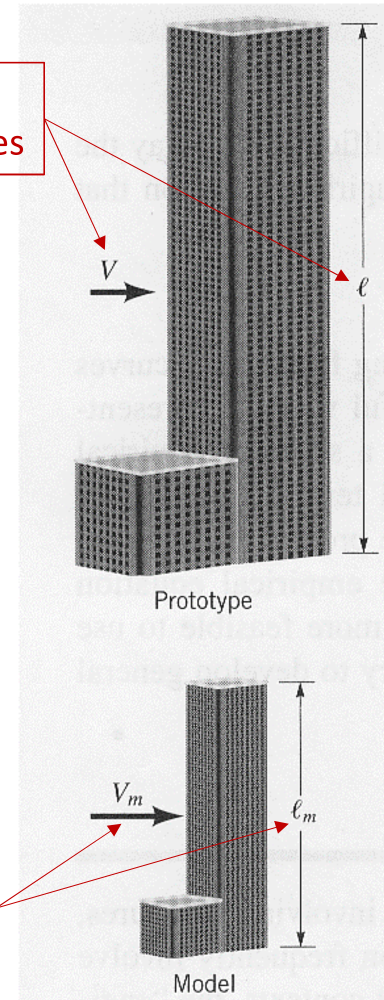
Terminology: Model versus Prototype

- Physical models (usually smaller than the full scale prototype) are commonly used in fluid mechanics testing.

Model: a physical model used to predict the behaviour of the full scale system.

Prototype: The full scale system for which predictions are being made.

no subscripts for prototype variables



subscript *m* for model variables

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Theory of Models

- We have shown that a given problem can be described in terms of a set of Pi terms:

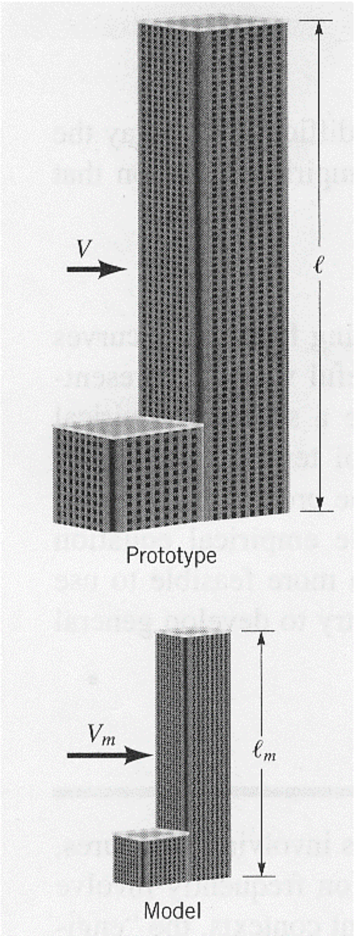
$$\Pi_1 = f(\Pi_2, \Pi_3, \dots, \Pi_k)$$ (for the prototype)

- A similar relationship can be written for the scaled model:

$$\Pi_{1m} = f(\Pi_{2m}, \Pi_{3m}, \dots, \Pi_{km})$$ (for the model)

where the subscript m refers to the model. The unknown function f will be the same for the model and prototype.

Recall that the Π_1 term contains the dependent variable -- the variable to be predicted.



Theory of Models

www.nasa.gov

- For similarity, the model is designed and operated at conditions such that:

$$\Pi_2 = \Pi_{2m}, \Pi_3 = \Pi_{3m}, \ \dots, \ \Pi_k = \Pi_{km}$$

Then it follows that: $\Pi_1 = \Pi_{1m}$ ← The prediction equation (contains dependent variable)

- So, the model conditions must correspond to the same Pi terms (Reynolds number, Froude number, Mach number, etc) as the prototype.
- Complete geometric similarity between the model and prototype is a requirement of similarity, i.e. a perfectly scaled model including all the fine details. e.g. wingtip winglets.
- Geometric similarity should extend to surface roughness, e.g. If the height of the surface roughness of a prototype wing is 1000th of its span, this must be same ratio for the model of the wing.
- So-called “relative roughness” must be the same (influences the development of turbulence).

Example

It is desired to predict the drag force on a billboard sign. The full scale (prototype) billboard has dimensions of 2.0m x 1.3m.

A 1/20th scale model of the billboard is tested in a ***water tunnel*** at 20°C. Using a load cell, the drag force on the model is measure to be $F_{D,m} = 1.3$ kN at water speed of $V_m = 15$ m/s.

At 20°C and 100 kPa, what is the corresponding wind speed for similarity. What is the predicted drag force on the prototype in air?

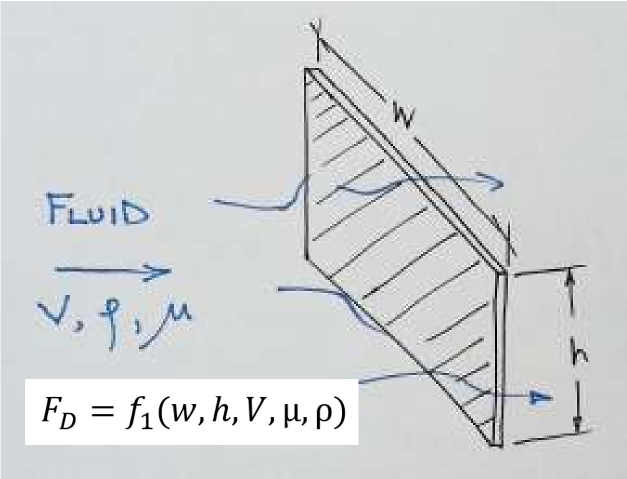
Water tunnel at U. of Massachusetts
Source: http://www.umass.edu/fsi/facilities.htm

Example

Solution

We considered this problem in a previous video (Chapter 5 Part 2). The drag force on a rectangular plate was:

$$F_D = f_1(w, h, V, \mu, \rho)$$



Using dimensional analysis we showed that this problem reduced to 3 Pi parameters:

$$\underbrace{\frac{F_D}{w^2 \rho V^2}}_{\Pi_1} = f_3\Big(\underbrace{\frac{h}{w}}_{\Pi_2}, \underbrace{\frac{\rho V w}{\mu}}_{\Pi_3}\Big)$$

Dimensionless drag force depends only on geometry and Reynolds number.

This relationship also applies to the model: $$\frac{F_{D,m}}{w_m^2 \rho_m V_m^2} = f_3\Big(\frac{h_m}{w_m}, \frac{\rho_m V_m w_m}{\mu_m}\Big)$$

where the subscript m refers to the model. The non-subscripted variables refer to the prototype.

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Example

Similarity requires $\Pi_2 = \Pi_{2,m}$ and $\Pi_3 = \Pi_{3,m}$:

$$\frac{h_m}{w_m} = \frac{h}{w} \quad and \quad \frac{\rho_m V_m w_m}{\mu_m} = \frac{\rho V\, w}{\mu}$$

The model scale is 20:1. Thus, from geometric similarity: $\frac{h}{h_m} = \frac{w}{w_m} = 20.$ To have the same Reynolds number the corresponding velocity for the prototype is:

$$\mathrm{V} = \frac{\mu}{\mu_m}\frac{\rho_m}{\rho}\frac{w_m}{w}\, V_m$$

To get the drag force on the prototype, we use the prediction equation $(\Pi_1 = \Pi_{1,m})$. The dimensionless drag forces and the model and prototype will be equal:

$$\Pi_{1m} = \Pi_1$$

$$\frac{F_{D,m}}{w_m^2 \rho_m V_m^2} = \frac{F_D}{w^2\, \rho\, V^2} \quad \rightarrow \quad F_D = (\frac{w}{w_m})^2 \left(\frac{\rho}{\rho_m}\right) (\frac{V}{V_m})^2\ F_{D,m}$$

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Example

$$\frac{h}{h_m} = \frac{w}{w_m} = 20 \qquad \mathrm{V} = \frac{\mu}{\mu_m}\frac{\rho_m}{\rho}\frac{w_m}{w}\,V_m \qquad F_D = \left(\frac{w}{w_m}\right)^2 \left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2 F_{D,m}$$

From problem statement: Force on model $F_{D,m} = 1.3$ kN, Water speed $V_m = 15$ m/s

Model test performed in water at 20°C: $\mu_m = 1.003 \times 10^{-3}$ Ns/m², $\rho_m = 998$ kg/m³

The prototype will be in air at 20°C, 100kPa: $\mu = 1.8 \times 10^{-5}$ Ns/m²

$$\rho = \frac{p}{RT} = \frac{100\ kPa}{0.287\frac{kJ}{kgK}(293K)} = 1.19\ \text{kg/m}^3$$

So, the corresponding wind (air) speed is:

$$\mathrm{V} = \frac{\mu}{\mu_m}\frac{\rho_m}{\rho}\frac{w_m}{w}\,V_m = \left(\frac{1.8\times 10^{-5}}{1.003\times 10^{-3}}\right)\frac{998}{1.19}\left(\frac{1}{20}\right)15\,\frac{m}{s} = \ 11.3\ m/s$$

(for same Re)

Ans.

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Example

The drag force on the prototype is:

$$F_D = \left(\frac{w}{w_m}\right)^2 \left(\frac{\rho}{\rho_m}\right) \left(\frac{V}{V_m}\right)^2 F_{D,m} = \left(\frac{20}{1}\right)^2 \left(\frac{1.19}{998}\right) \left(\frac{11.3}{15}\right)^2 1.3\ kN = 0.35\ kN$$

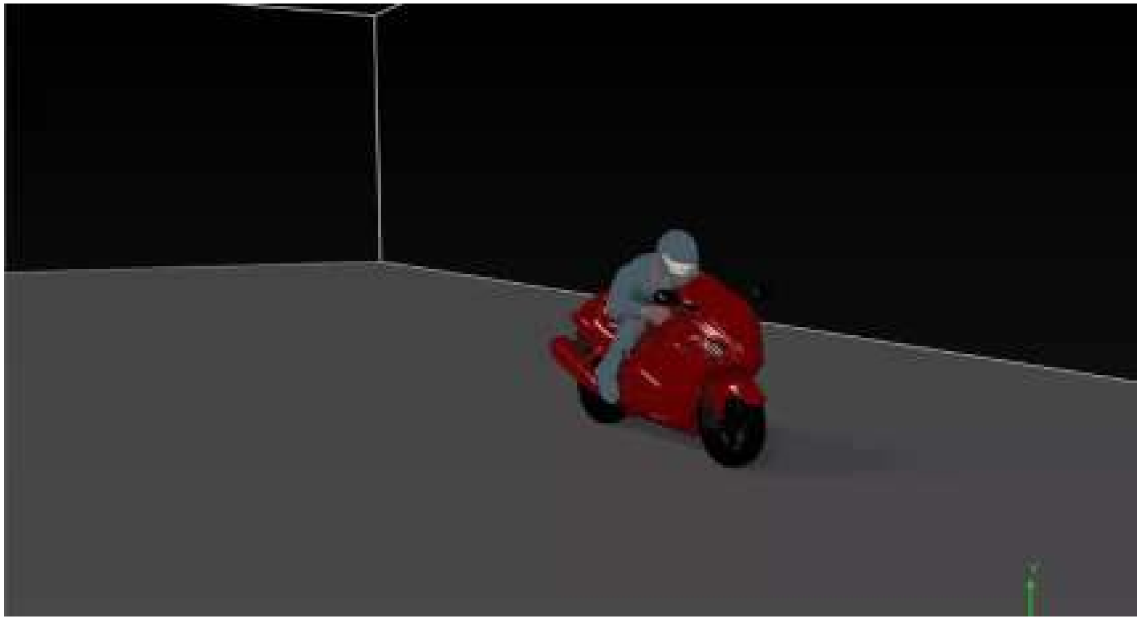
Ans.

Comments

- Water has a kinematic viscosity ($\nu = \mu/\rho$) about 15 times lower than air, and Re can expressed as:

$$Re = \frac{\rho V w}{\mu} = \frac{V w}{\nu}$$

- So, water can be used to achieve relatively high Reynolds numbers even for the small length scales associated with scaled models.
- To achieve the same Reynolds number as the prototype in a wind tunnel (i.e. $\nu = \nu_m$) very high wind speeds would be needed. Testing in air at the same temperature and pressure requires: $V_m w_m = V w$. (20 times higher air speed for this example, 300 m/s!)



Computation fluid dynamics (CFD) simulation of an accelerating motorcycle.
Source: www.xflow-cfd.com

END NOTES

Presentation prepared and delivered by Dr. David Naylor.

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