MEC516/BME516: Fluid Mechanics I

Chapter 5: Dimensional Analysis & Similarity Part 5

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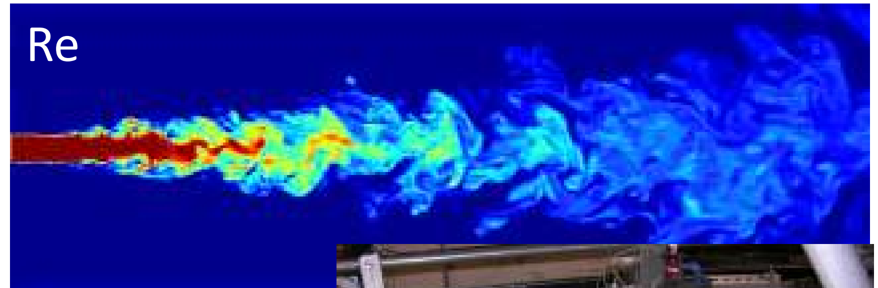
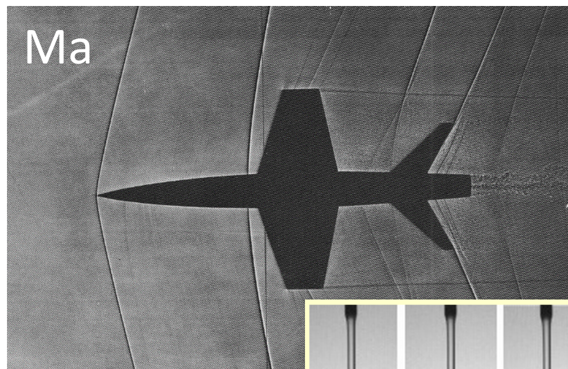
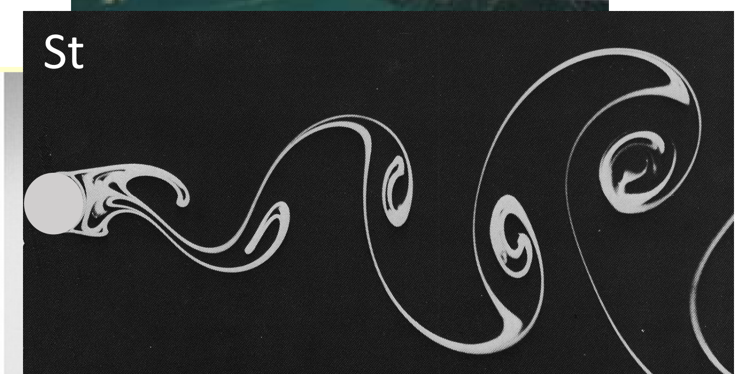
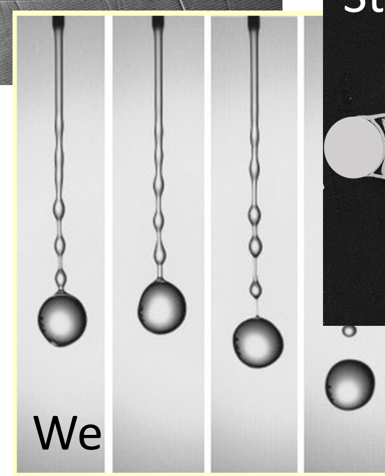
Overview

- Discussion of some common dimensionless parameters in fluid mechanics:
  - Reynolds number, Re
  - Mach number, Ma
  - Froude number, Fr
  - Weber number, We
  - Strouhal number, St
- Used to characterise laminar/turbulent flow, compressible flow, open channel flow, flows with strong surface tension effects, oscillating flows.









Common Dimensionless Parameters in Fluid Mechanics

- **Reynolds number**

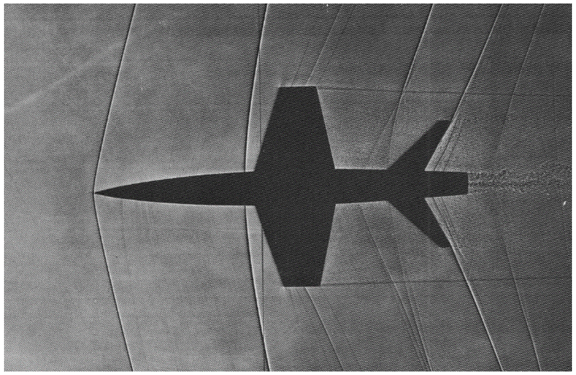
$$Re = \frac{\rho V \ell}{\mu} = \frac{inertia\ \ force}{viscous\ force}$$

  - Widely applicable in fluid dynamic problems
    - Re<<1, viscous forces dominate and forces due to acceleration are small (Stokes flow).
    - Re determines transition from laminar to turbulent flow (Lab #2)

- **Mach number**

$$Ma = \frac{V}{c} = \frac{flow\ speed}{speed\ of\ sound}$$

  - Important in compressible flows $(Ma > 0.3)$
  - Applications: supersonic flight, rocket ballistics.



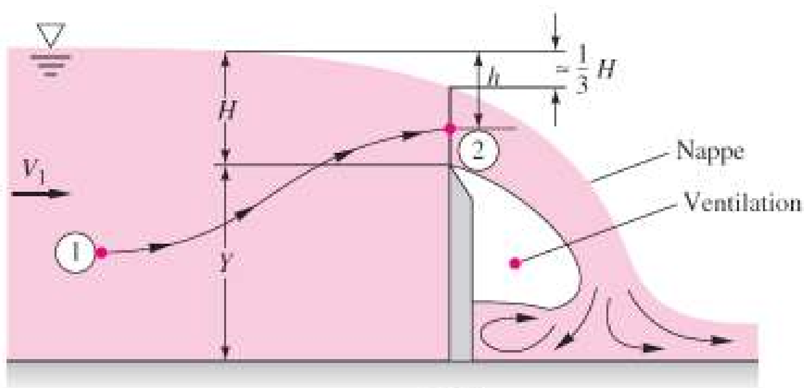
Shock waves from a model launched at $Ma = 1.1$
Source: Album of Fluid Motion

Common Dimensionless Parameters in Fluid Mechanics

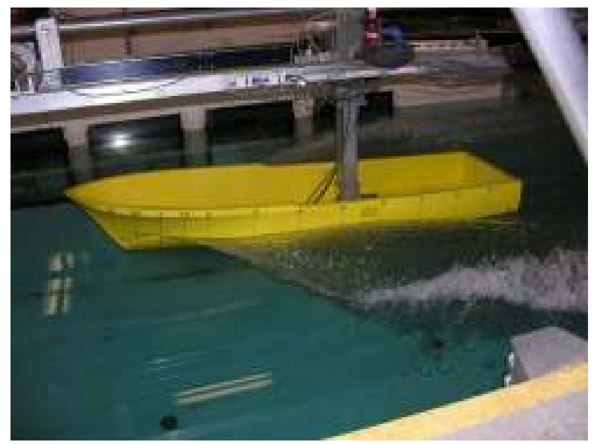
- **Froude number**

$$Fr = \frac{V^2}{g\,\ell} = \frac{inertia\ \ force}{gravity\ force}$$

  - Important in open channel flows, flows with a free surface.
  - Applications: Weirs, modelling river flow, flood & tsunamis prediction, wave drag on hulls.



Measurement of flow using a weir (see Chapter 10)



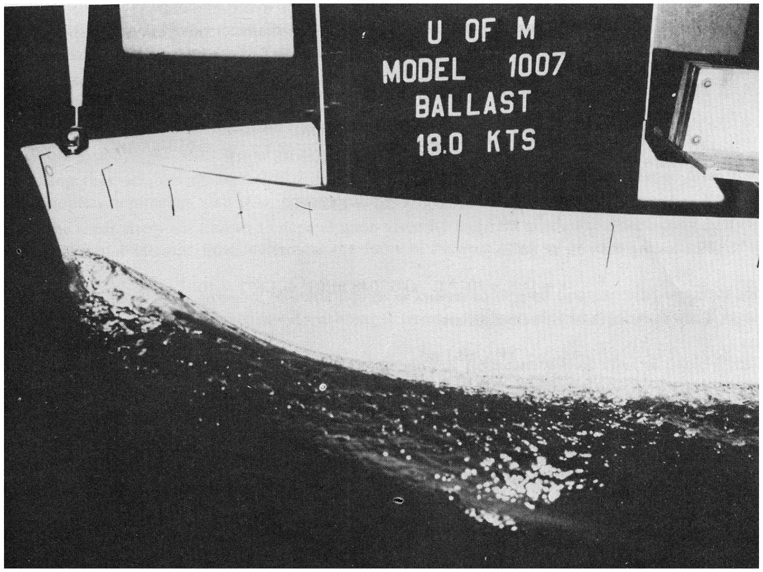
Measurement of drag on a model boat in a tow tank.

Common Dimensionless Parameters in Fluid Mechanics

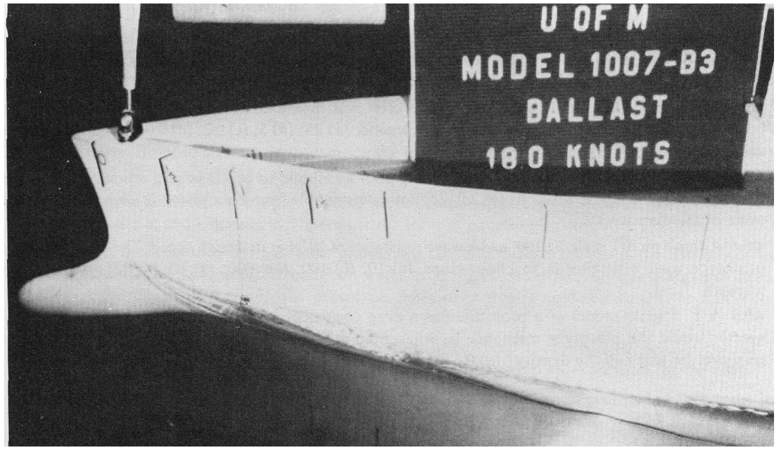
- **Froude number**

$$Fr = \frac{V^2}{g\,\ell} = \frac{inertia\ \ force}{gravity\ force}$$

  - Ship are towed such that they have the same Froude number as the full scale ship.



Hull without a bulbous bow.



Hull with a bulbous bow. Notice the large reduction in the wake of the bow. Can result in as much as 10-15% reduction in wave drag and ship fuel consumption.

Source: Fluid Mechanics, Streeter and Wylie, 1979

Common Dimensionless Parameters in Fluid Mechanics

- **Weber number**

$$We = \frac{\rho V^2 \ell}{\Upsilon} = \frac{inertia\ \ force}{surface\ tension\ force}$$

  - Important in flows with an interface between two fluids, e.g. water and air.
  - Applications: Multi-phase flows where liquid droplet and capillary effects are important e.g. spray painting, inkjet printing.

Source: http://www.youtube.com/watch?v=zyfqL4sgVuc

Experiments with water jets in microgravity (near zero "g") via parabolic arc flight.

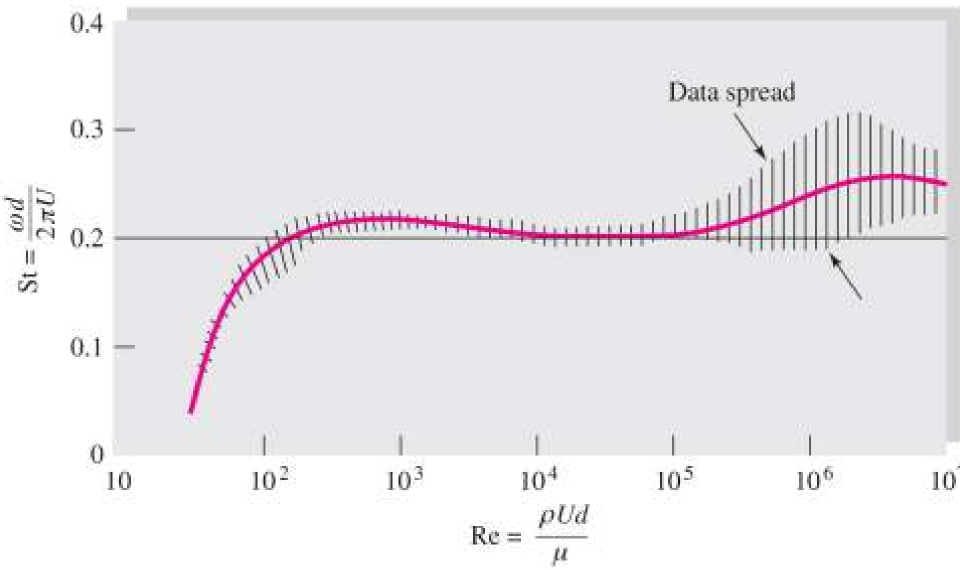
Common Dimensionless Parameters in Fluid Mechanics

- **Strouhal number**

$$St = \frac{(\omega/2\pi)\,\ell}{V} = \frac{flow\ oscillation\ speed}{mean\ flow\ speed}$$

  - A dimensionless frequency, important in unsteady oscillating flows e.g. Karman vortex shedding.
  - Applications: Flow induced vibration of structures and hydro wires. Avoid *St* values near resonance.

Vortex shedding behind a cylinder.
http://www.youtube.com/watch?v=JI0M1gVNhbw



Common Dimensionless Parameters in Fluid Mechanics

Table 5.2

Parameter	Definition	Qualitative ratio of effects	Importance
Reynolds number	$\mathrm{Re} = \frac{\rho U L}{\mu}$	$\frac{\text{Inertia}}{\text{Viscosity}}$	Almost always
Mach number	$\mathrm{Ma} = \frac{U}{a}$	$\frac{\text{Flow speed}}{\text{Sound speed}}$	Compressible flow
Froude number	$\mathrm{Fr} = \frac{U^2}{gL}$	$\frac{\text{Inertia}}{\text{Gravity}}$	Free-surface flow
Weber number	$\mathrm{We} = \frac{\rho U^2 L}{\Upsilon}$	$\frac{\text{Inertia}}{\text{Surface tension}}$	Free-surface flow
Rossby number	$\mathrm{Ro} = \frac{U}{\Omega_{\text{earth}} L}$	$\frac{\text{Flow velocity}}{\text{Coriolis effect}}$	Geophysical flows
Cavitation number (Euler number)	$\mathrm{Ca} = \frac{p - p_v}{\rho U^2}$	$\frac{\text{Pressure}}{\text{Inertia}}$	Cavitation
Prandtl number	$\mathrm{Pr} = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$\mathrm{Ec} = \frac{U^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific-heat ratio	$k = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Strouhal number	$\mathrm{St} = \frac{\omega L}{U}$	$\frac{\text{Oscillation}}{\text{Mean speed}}$	Oscillating flow

Common Dimensionless Parameters in Fluid Mechanics

Table 5.2 continued...

(This is just a partial list.)

Roughness ratio	$\frac{\epsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent, rough walls
Grashof number	$\text{Gr} = \frac{\beta \Delta T g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Rayleigh number	$\text{Ra} = \frac{\beta \Delta T g L^3 \rho^2 c_p}{\mu k}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural convection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Aerodynamics, hydrodynamics
Lift coefficient	$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Drag coefficient	$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Aerodynamics, hydrodynamics
Friction factor	$f = \frac{h_f}{(V^2/2g)(L/d)}$	$\frac{\text{Friction head loss}}{\text{Velocity head}}$	Pipe flow
Skin friction coefficient	$c_f = \frac{\tau_{\text{wall}}}{\rho V^2/2}$	$\frac{\text{Wall shear stress}}{\text{Dynamic pressure}}$	Boundary layer flow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END NOTES
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