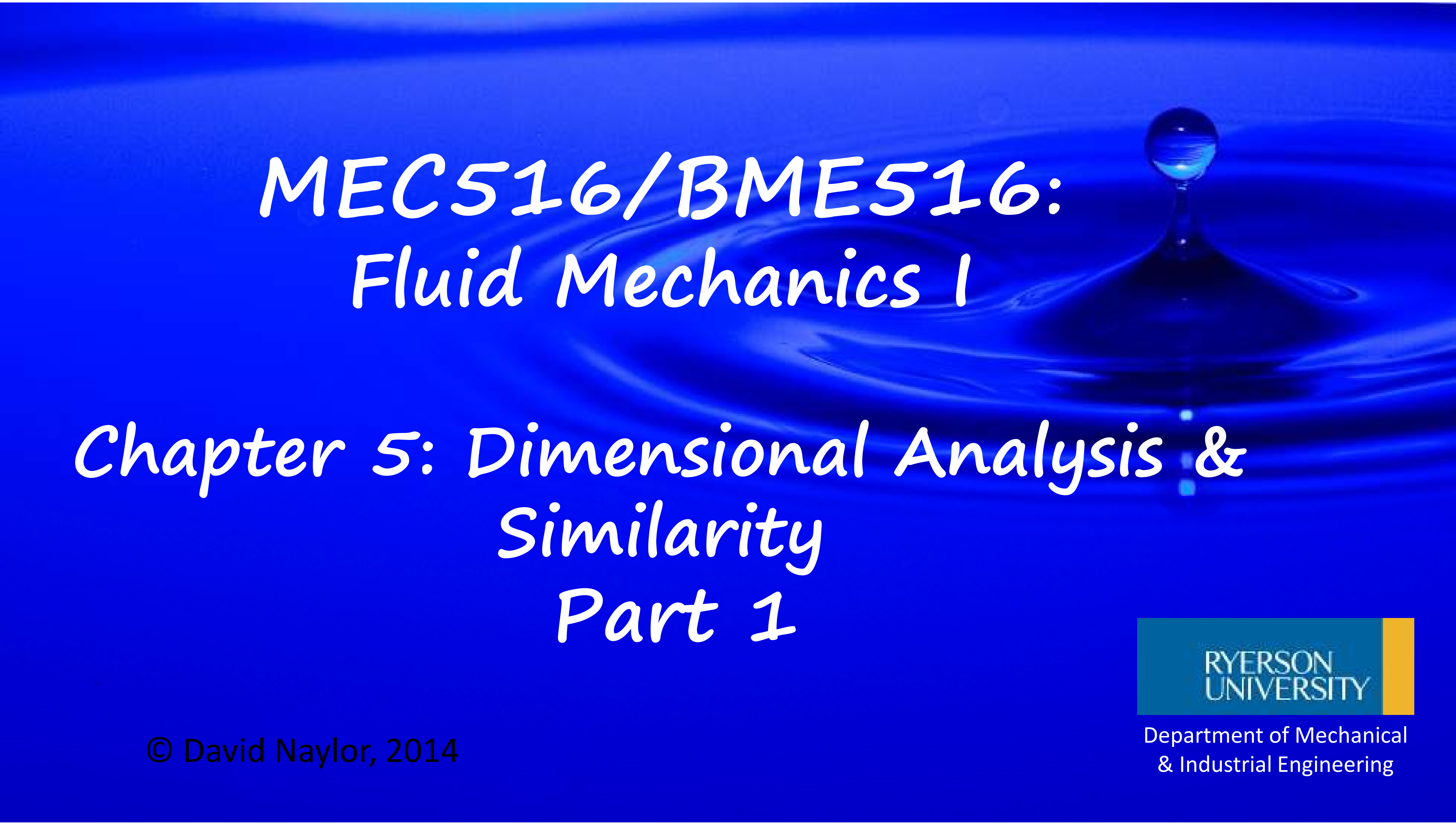


MEC516/BME516: Fluid Mechanics I

Chapter 5: Dimensional Analysis & Similarity

Part 1

RYERSON UNIVERSITY

Department of Mechanical
& Industrial Engineering

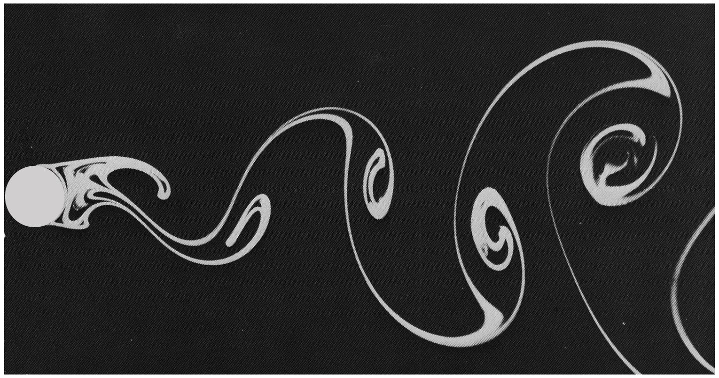
Overview

An Introduction to Similarity & Dimensional Analysis

- First encounter: The Reynolds number

- An visual example of similarity:
  - Flow regimes for a cylinder in cross flow

- A practical example of similarity:
  - Aerodynamic drag force on a cylinder



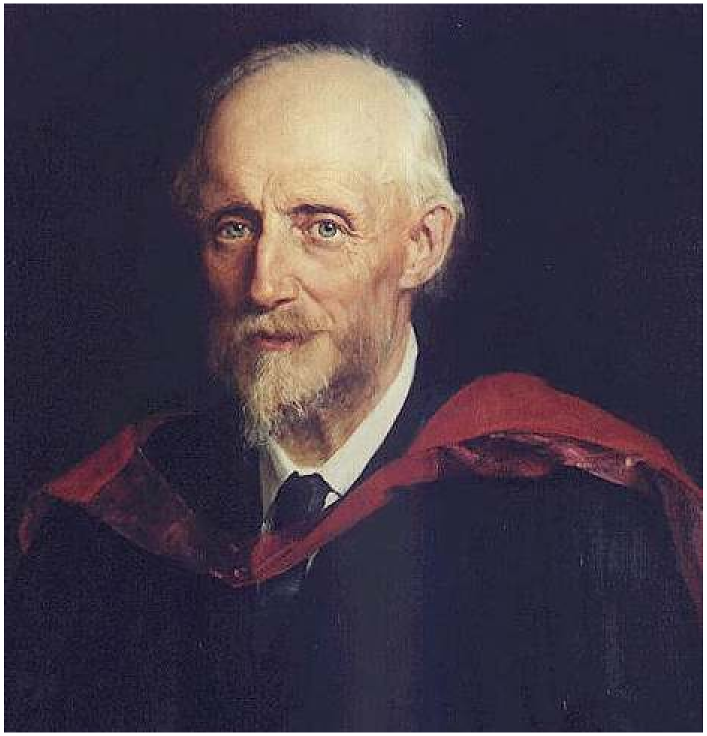
The Reynolds Number

- Flow regimes depend upon a dimensionless parameter called the ***Reynolds number (Re):***

$$Re = \frac{\rho V D}{\mu}$$ (e.g. Re<2300 laminar pipe flow)

ρ $fluid\ density$
μ $fluid\ viscosity$
V $fluid\ velocity$
D $length\ scale$ $(\text{e.g. pipe diameter})$

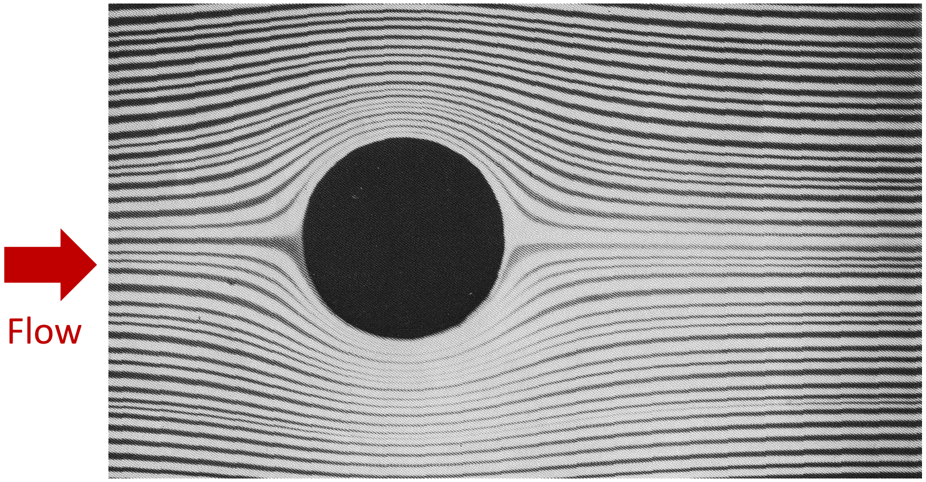
- This chapter explains where this important result comes from.



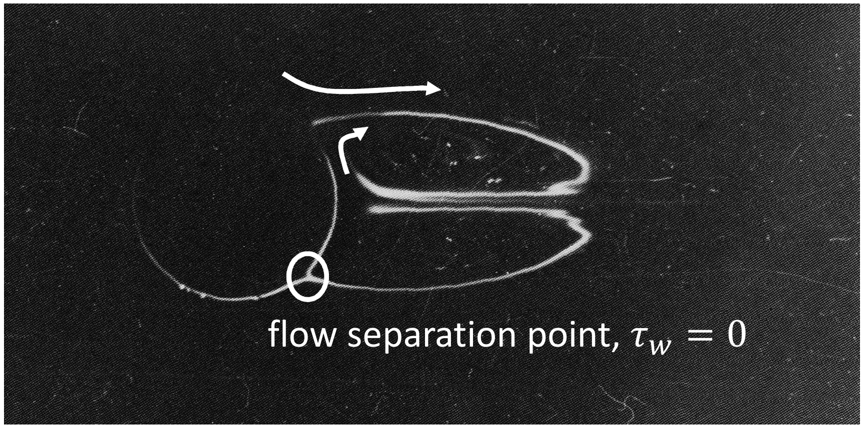
Osborne Reynolds (1842-1912) was a pioneer in fluid dynamics.

A Visual Example of Similarity

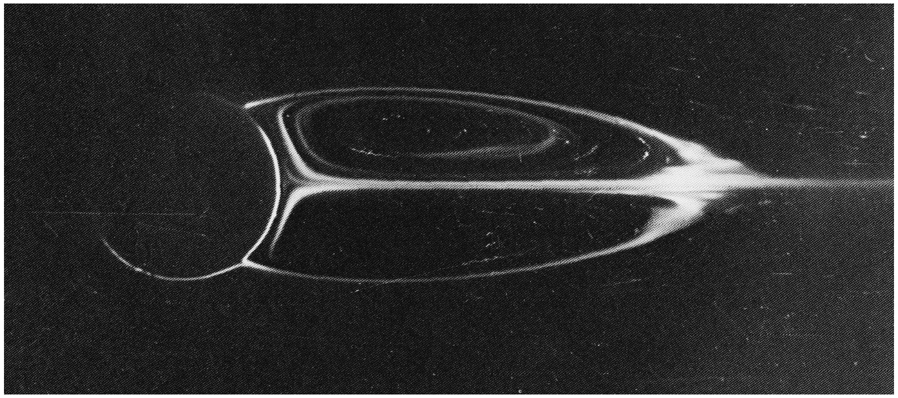
Flow Regimes for a Cylinder in Cross Flow



Re < 1. Stokes Flow. Steady, laminar, no flow separation.



Re = 28. Steady, laminar, flow separation with flow recirculation in wake region.

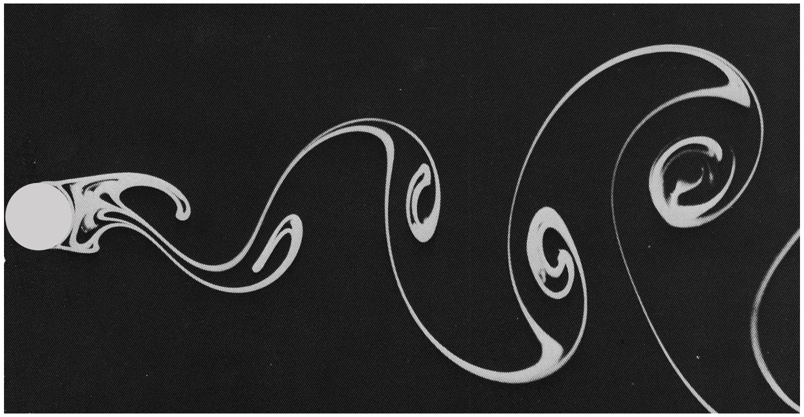


Re = 41. Steady, laminar, separation with longer recirculating flow region . Upper limit for steady flow.

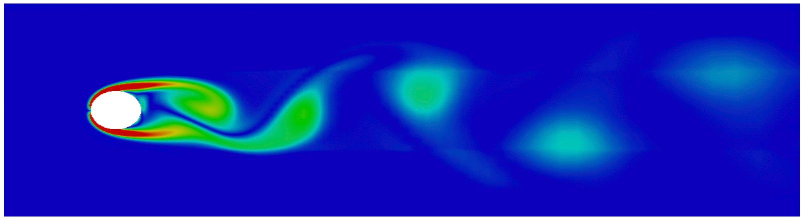
A Visual Example of Similarity

Flow Regimes for a Cylinder in Cross Flow

Flow visualization experiment



Computational fluid dynamics (CFD) prediction

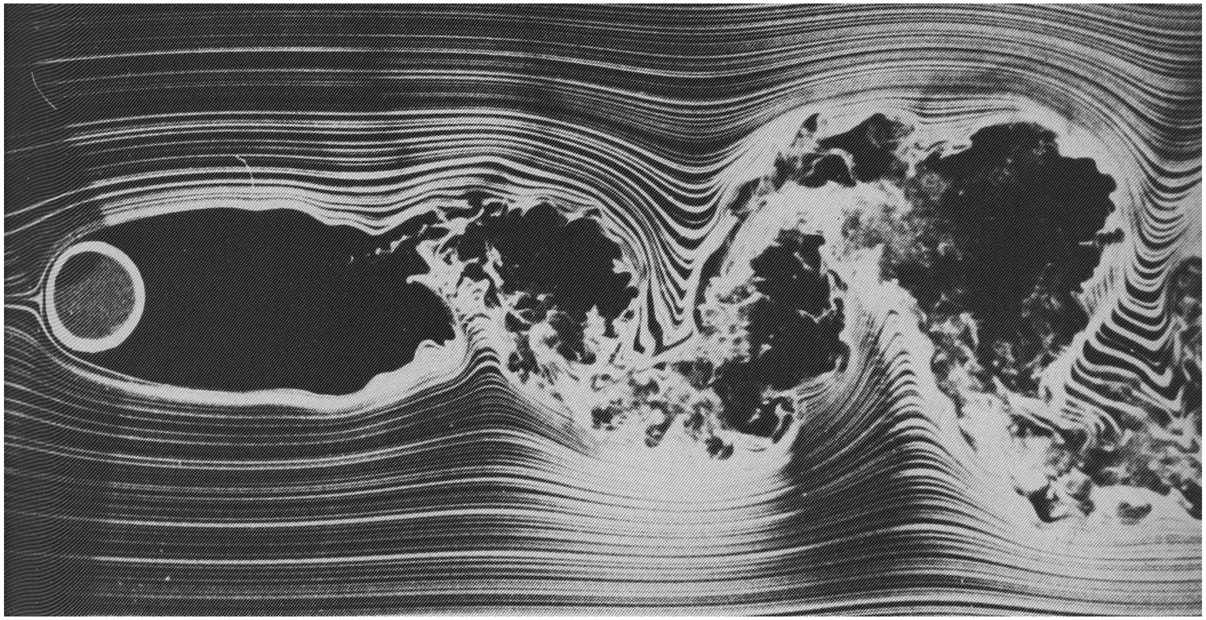


Re ≈140. Unsteady, laminar, flow. Separated flow with alternating vortex shedding (called a "von Karman Vortex street").

The reason for spiral vortex breakers on industrial stacks

A Visual Example of Similarity

Flow Regimes for a Cylinder in Cross Flow



Re =10,000. Unsteady flow. Separated flow with alternating vortex shedding. Turbulent flow (random velocity fluctuations) in the wake region.

A Visual Example of Similarity

Flow Regimes for a Cylinder in Cross Flow

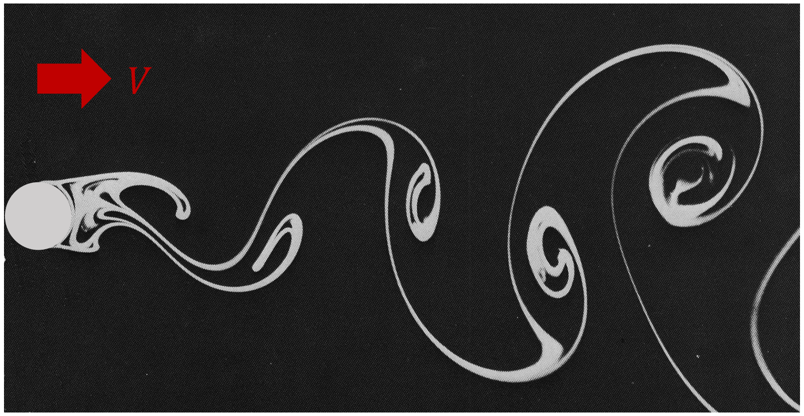
- So far no mention of the "specifics".
- For a smooth cylinder, the type of flow depends on four variables:

  Fluid velocity, V
  Cylinder diameter, D
  Fluid density, ρ
  Fluid dynamic viscosity, μ

- Amazingly, this can be reduced to one dimensionless variable, the Reynolds number:

$$Re = \frac{\rho V D}{\mu}$$

- Only the ratio of parameters matter.
- In this chapter we will discuss how this similarity parameter is obtained, using *Dimensional Analysis*.



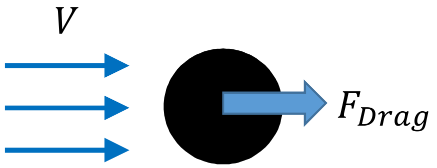
Re =140. Unsteady, laminar, flow. Separated flow with alternating vortex shedding.

Table: Various conditions corresponding to Re=140.

Fluid (at 20 °C)	D	V
Water	1.0 cm	0.014m/s
Oil SAE 10W50	1.0 cm	2.7 m/s
Air	1.0 cm	0.21 m/s
Air	0.1 cm	2.1 m/s

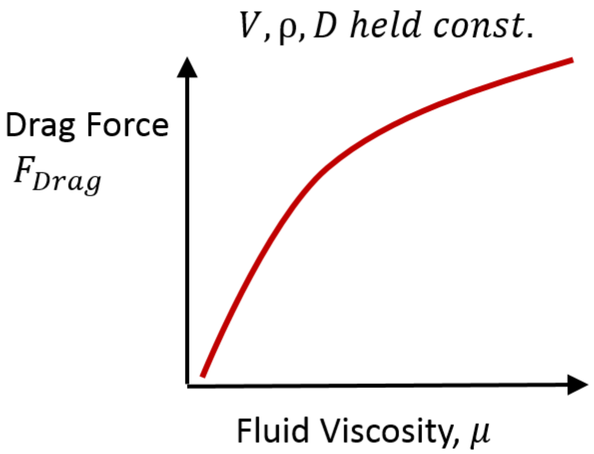
Aerodynamic Drag on a Cylinder

- Imagine that you want to determine the aerodynamic drag force on a cylinder, F_{Drag}
- The drag force on a smooth cylinder depends upon:



Fluid velocity, V
Cylinder diameter, D
Fluid density, ρ
Fluid dynamic viscosity, μ

- How could you characterize the drag force?
- One method is to run many experiments, sequentially holding one variable constant, while varying the others.
- Expensive. Time consuming. Sometimes difficult.
- How do you put the results together in a useful way?
- Luckily there is a better way, called *Dimensional Analysis*.

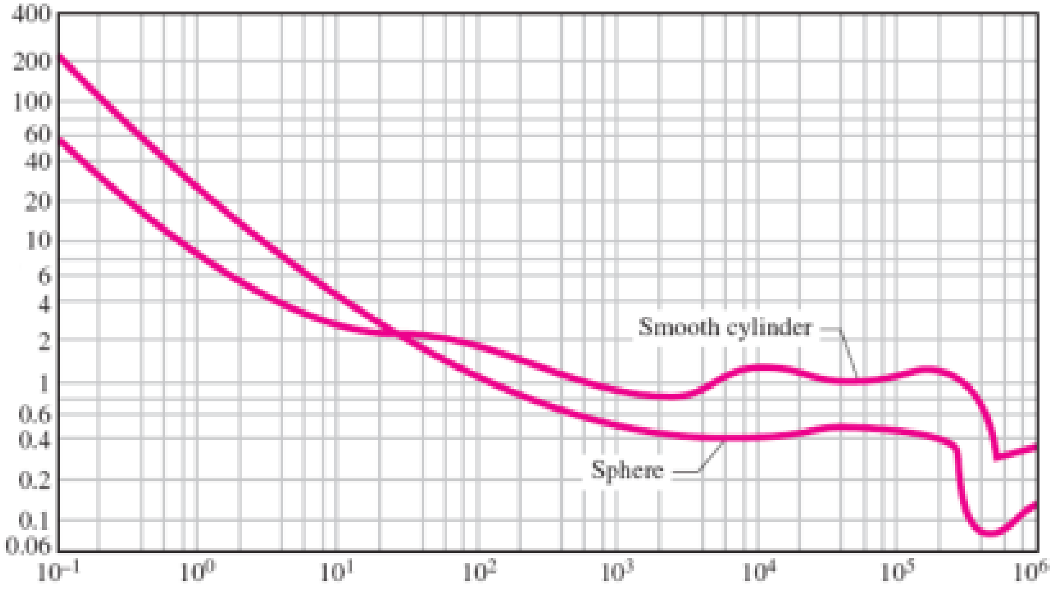


Aerodynamic Drag on Cylinders and Spheres

- To show the power of this method, this is a preview of the result (which will be demonstrated in the upcoming videos).
- Using *dimensional analysis*, the dimensionless drag force can be shown to be ONLY a function of the Reynolds number, Re.

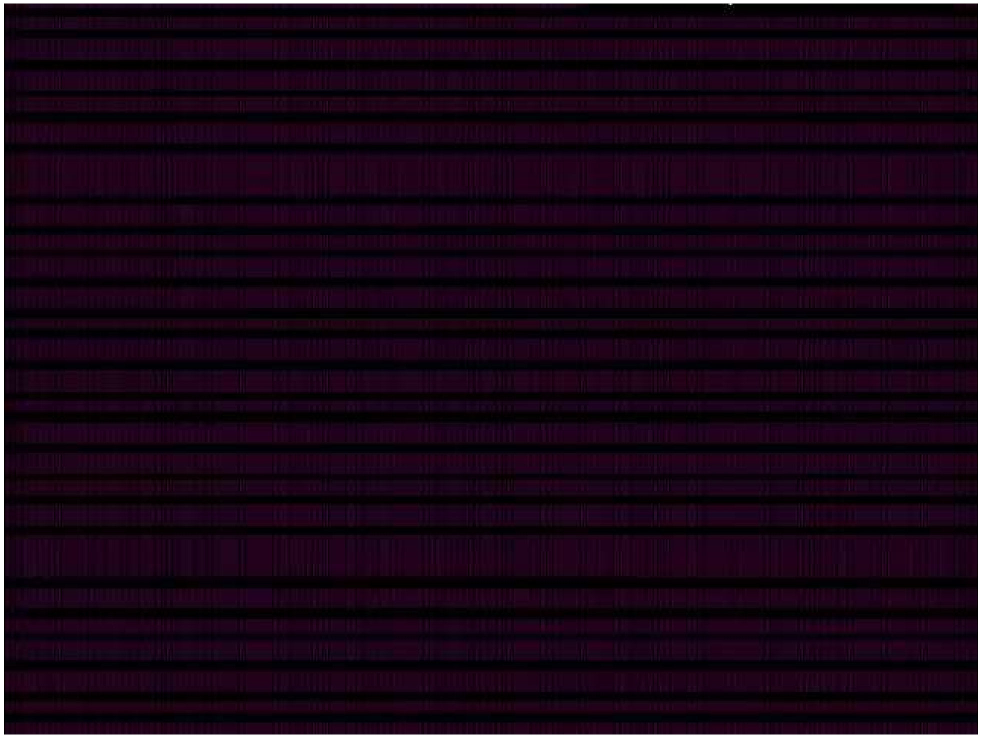
A single curve for any fluid, any velocity, and any size cylinder!

$$\frac{F_{Drag}}{A\,\rho V^2/2}$$



$$\text{Re} = \frac{\rho V D}{\mu}$$

Drag force non-dimensionalized using the stagnation pressure $\rho V^2/2$ times the frontal area, $A = DL$.



A real-time CFD WebGL Javascript

https://haxiomic.github.io/GPU-Fluid-Experiments/html5/

END NOTES

Presentation prepared and delivered by Dr. David Naylor.