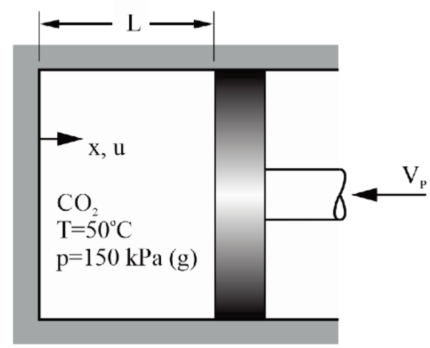
Last Updated: June 2021

MEC516/BME516
Fluid Mechanics I

Chapter 4
Recommended Problem Set

I strongly recommended that you work through the details of these problems, with pen and paper. ***Be sure that you can solve these problems without looking at the solutions.***

1. Carbon dioxide gas is compressed in a piston, as shown in the sketch below. When the piston is located at L=15.0 cm the gas has a temperature of 50°C and a gauge pressure of 150 kPa. The local atmospheric pressure is 98 kPa. The piston is compressing the gas at a speed of V_p= 1.5 m/s. The gas flow inside the piston can be approximated as one dimensional:



$$u = -\frac{x}{L}V_p, \quad v = 0, \quad w = 0$$

Starting from the appropriate form of the continuity equation, calculate the instantaneous rate change of the density of the carbon dioxide gas, $\partial\rho/\partial t$. (Note: The speed of the compression is low compared to the speed of sound. Thus, the gas density does not vary spatially i.e., $\rho \neq f(x,y,z)$.)

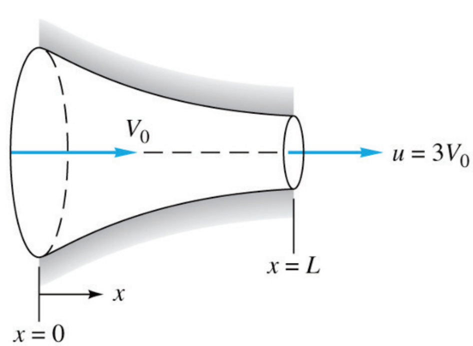
2. Consider an idealized velocity field is given by the formula

$$\boldsymbol{V} = 4tx\,\boldsymbol{i} - 2t^2y\,\boldsymbol{j} + 4xz\,\boldsymbol{k}$$

(a) Is this flow field steady or unsteady? Is it two- or three-dimensional? (b) Derive and expression for the acceleration vector. Evaluate the acceleration vector at the point (x, y, z) = (–1, +1, 0).

3. A steady incompressible flow passes through the converging nozzle shown in the sketch. The velocity field can be approximated as one-dimensional:

$$u = V_o\left(1 + \frac{2x}{L}\right), \quad v = 0, \qquad w = 0$$



Derive a general expression for the fluid acceleration in the nozzle. For the specific case of $V_o = 10$ ft/s and L= 6.0 in, compute the fluid acceleration at the nozzle entrance and exit.

4. An inviscid (frictionless) incompressible two-dimensional steady flow is given by:

$$\boldsymbol{V} = 2xy\,\boldsymbol{i} - y^2\,\boldsymbol{j}$$

The x-component of gravity is g_x. Obtain an expression for the pressure gradient in the x-direction ($\partial p/\partial x$).

5. A two-dimensional incompressible flow has the velocity components:

$$u = 4y \quad v = 2x$$

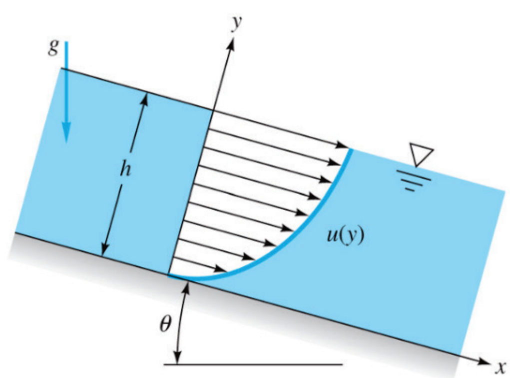
Neglect the effects of gravity: $g_x = g_y = 0$. (a) Check that this flow satisfies conservation of mass. (b) Check that this flow satisfies conservation of momentum. (c) Derive an expression of the pressure field $p(x,y)$, if the pressure at the origin ($x=y=0$) is p_o.

6. A film of viscous liquid with constant thickness h flows down a plate inclined at angle θ as shown in the sketch. The laminar steady velocity profile is:

$$u = Cy(2h - y), \quad v = 0, \quad w = 0$$

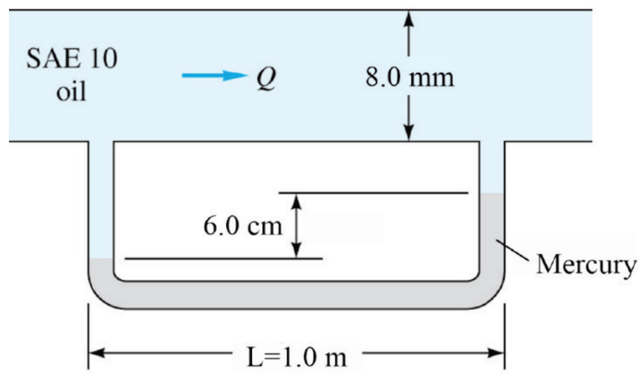
(a) Apply the Navier-Stokes equation in the x-direction to evaluate constant C in terms of the fluid density, viscosity and angle θ. You can set the pressure gradient in the flow direction to zero because the pressure is constant (atmospheric pressure) along the free surface.

(b) Integrate the velocity profile to obtain an expression for the volume flow rate Q per unit width (into the page).



7. Consider fully developed laminar flow of oil between two parallel plates 8 mm apart, as shown in the sketch. The fluid is SAE 10W oil at 20°C. A mercury manometer with wall pressure taps L= 1.0m apart is used to measure the pressure gradient in the pipe. The manometer vertical deflection is 6.0 cm.

(a) Calculate the flow rate of oil.
(b) Calculate the Reynolds number based on the hydraulic diameter.



Want more solved problems? Some past final exam problems ***with full solutions*** are posted on D2L.