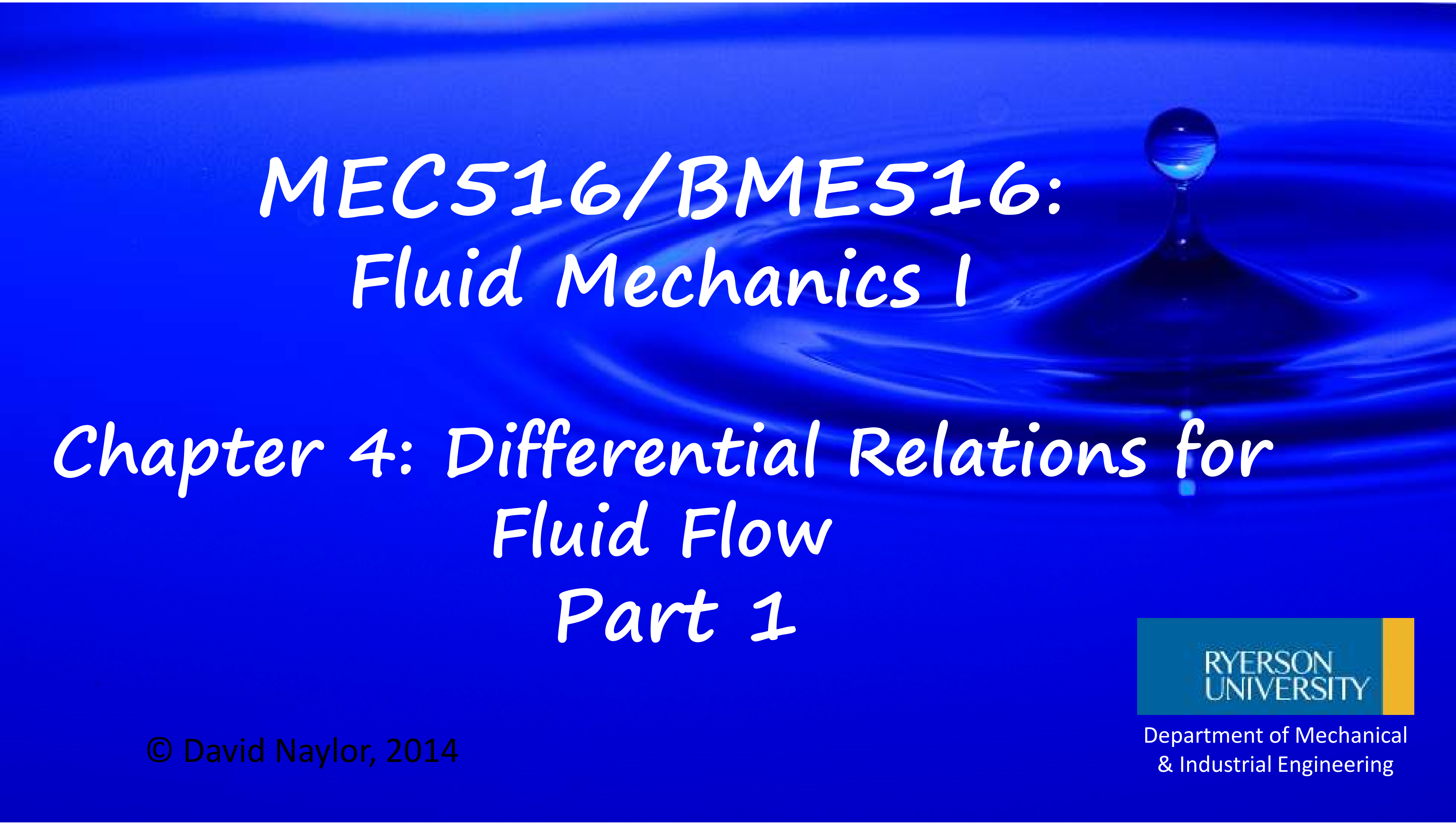

MEC516/BME516: Fluid Mechanics I

Chapter 4: Differential Relations for Fluid Flow

Part 1

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Department of Mechanical
& Industrial Engineering

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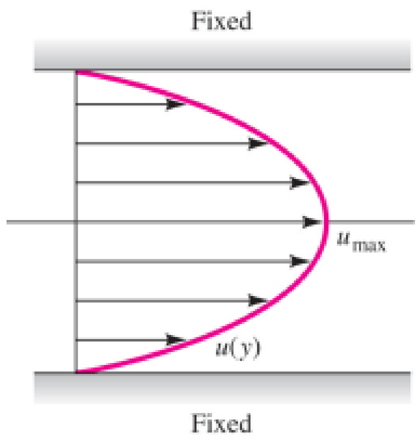
Overview

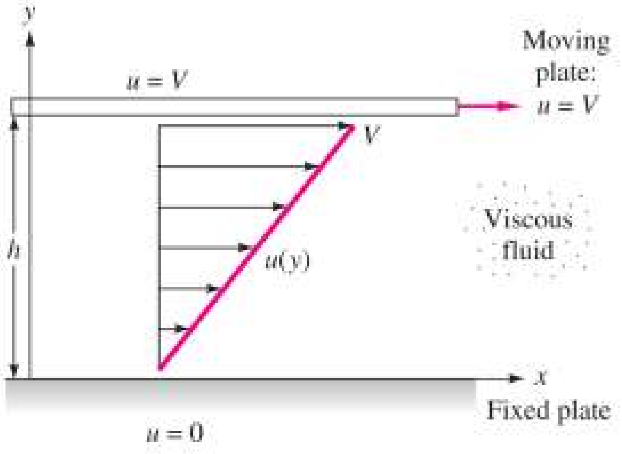
Introduction to Chapter 4:

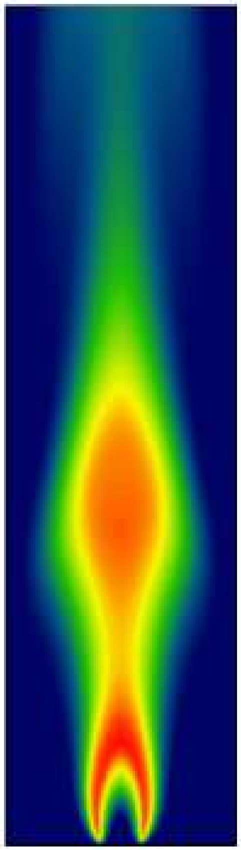
- **Governing Equations for Fluid Flow**
  - Continuity equation
  - Navier-Stokes equations
  - Energy equation

- **Some Classical Exact Solutions**
  - Couette flow
  - Poiseuille flow
  - Hagen-Poiseuille flow

- **Computational Fluid Dynamics (CFD)**
  - Brief introduction
  - Engineering applications







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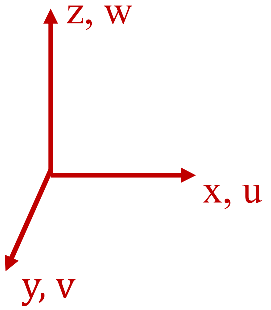
Governing Equations (Newtonian fluid with constant properties)



- Continuity Equation (Conservation of Mass):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

- Navier-Stokes Equations (Conservation of Momentum):

x-momentum: $$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

y-momentum: $$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

z-momentum: $$\rho(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z$$

- Control volume analysis (Chpt.3) gave gross average effects (flow rate, induced force, etc.)
- Solutions to these equations of fluid motion give *point-by-point details* of the flow.
- 4 unknowns (u, v, w, p) & 4 equations . Thus, we have mathematical "closure".

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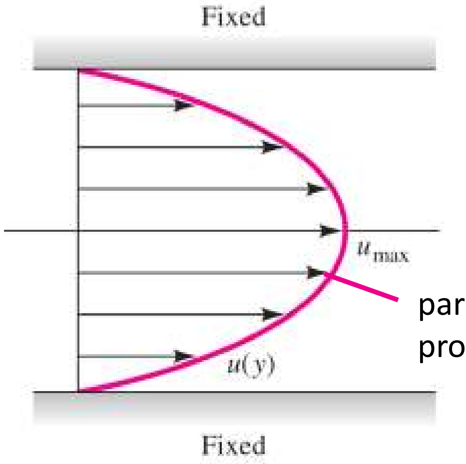
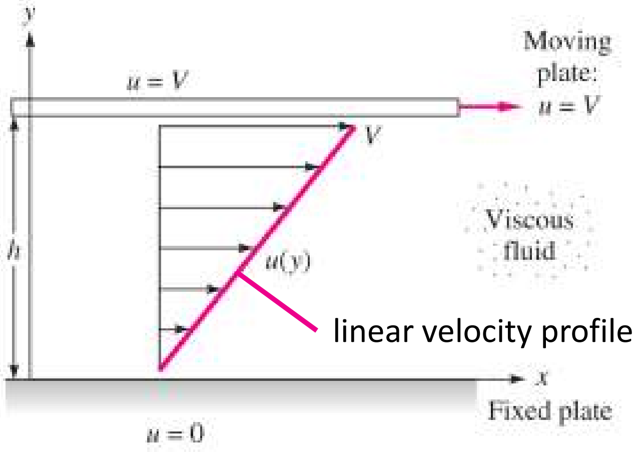
Some Classical Exact Solutions

- Nevertheless, analytical solution of the Navier-Stokes/continuity equations is extremely difficult.
- Navier-Stokes are a set of 2nd order non-linear partial differential equations.
  (Which terms are non-linear?)
- Exact solutions only exist for a few simple (but important) laminar flows. We will discuss:



parabolic velocity profile

Laminar incompressible fully developed flow between in a parallel plates: *Poiseuille Flow.*
In a round pipe: *Hagen-Poiseuille Flow*



linear velocity profile

Laminar incompressible flow between a moving and fixed plate: *Couette Flow*

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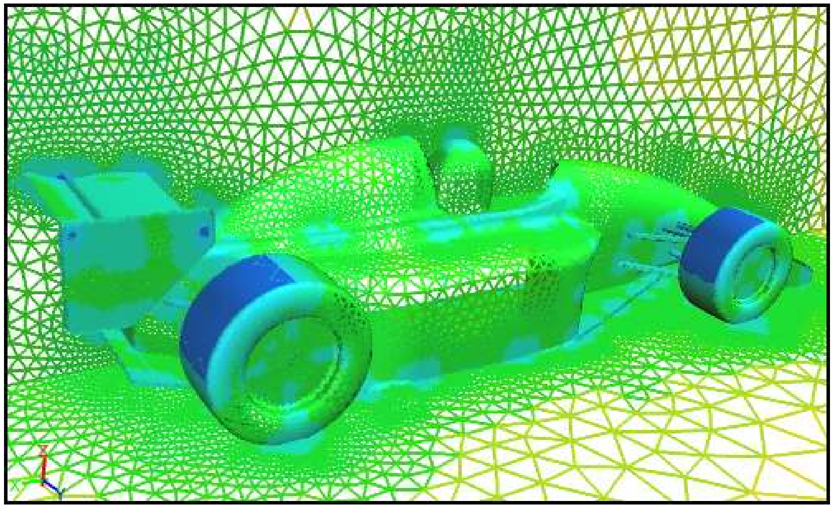
Computational Fluid Dynamics (CFD)

- For real-world flows in complex geometries, solutions to the Navier-Stokes and Continuity are obtained numerically.
- In CFD we *discretize* the volume of interest into fine control volumes, called a *grid* or *mesh*. Mass and momentum is conserved for each small volume.
- The solution gives the velocity $\mathbf{V}$ (u, v, w) and pressure in each control volume. This is a discontinuous solution.
- But the discretized problem approaches the continuous solution for a fine enough grid.
- Partial differential equations are converted into algebraic equations.

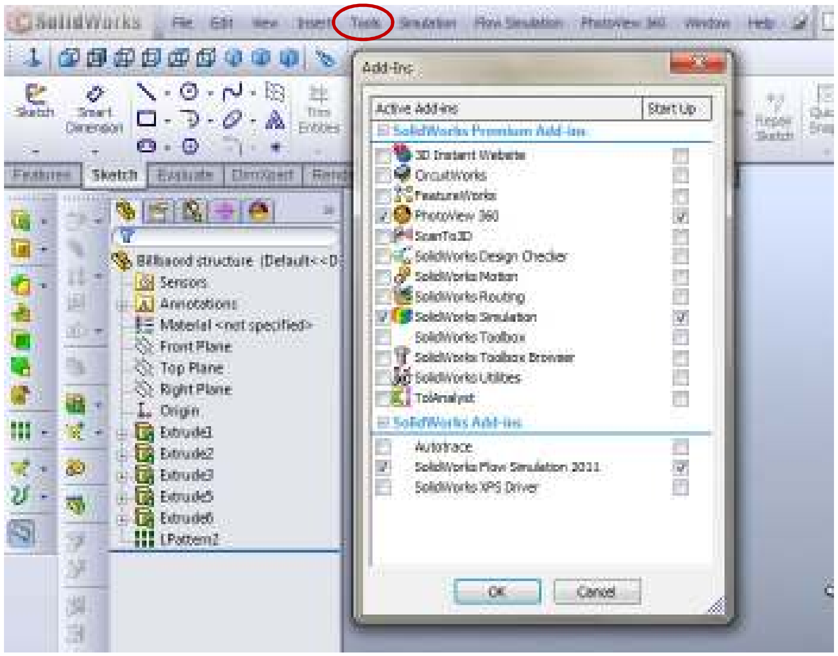
  e.g. 500,000 control volumes with unknowns u, v, w, p
  yields 2,000,000 equations in 2,000,000 unknowns!
- Matrix of simultaneous equations solved iteratively (iteration needed because of non-linearity)
- CFD requires enormous computing power: large RAM, multi-processors with parallel computing.



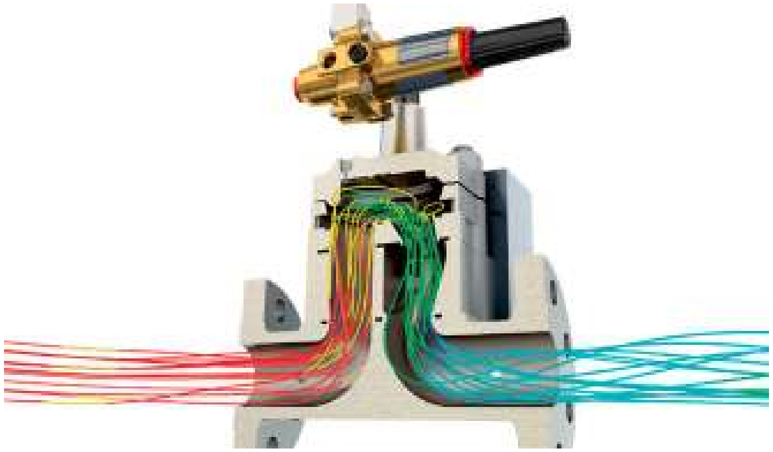
http://www.symscape.com/

Sample CFD Applications

- From the *SolidWorks Flow Simulation* (available in the Mech. Eng. Computer Labs).
- “Add-Ins” under “Tools” menu
- Most user-friendly CFD software on the market. Great for CFD novices.
- Interfaces seamlessly with the 3D drawing package & FEA stress analysis module.
- Consider for 4th year “Capstone Design Project”



Active Add-ins	Start Up
SolidWorks Premium Add-ins	
☐ 3D Instant Website	☐
☐ CircuitWorks	☐
☐ FeatureWorks	☐
☑ PhotoView 360	☑
☐ ScanTo3D	☐
☐ SolidWorks Design Checker	☐
☐ SolidWorks Motion	☐
☐ SolidWorks Routing	☐
☑ SolidWorks Simulation	☑
☐ SolidWorks Toolbox	☐
☐ SolidWorks Toolbox Browser	☐
☐ SolidWorks Utilities	☐
☐ TolAnalyst	☐
SolidWorks Add-ins	
☐ Autotrace	☐
☑ SolidWorks Flow Simulation 2011	☑
☐ SolidWorks XPS Driver	☐



Analysis of pressure losses across a valve

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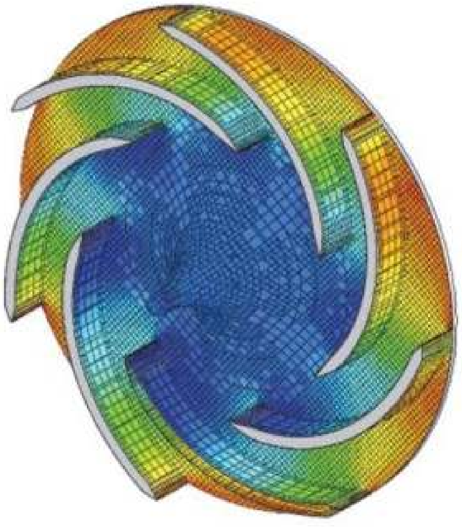
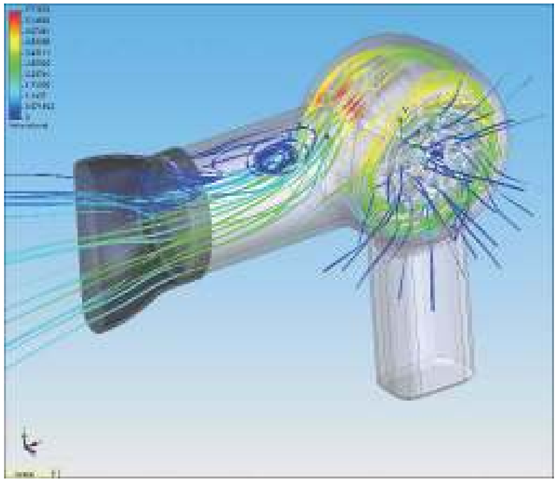
Sample CFD Applications

From the SolidWorks Flow Simulation





Analysis of flow in devices rotating parts; pumps and turbines.

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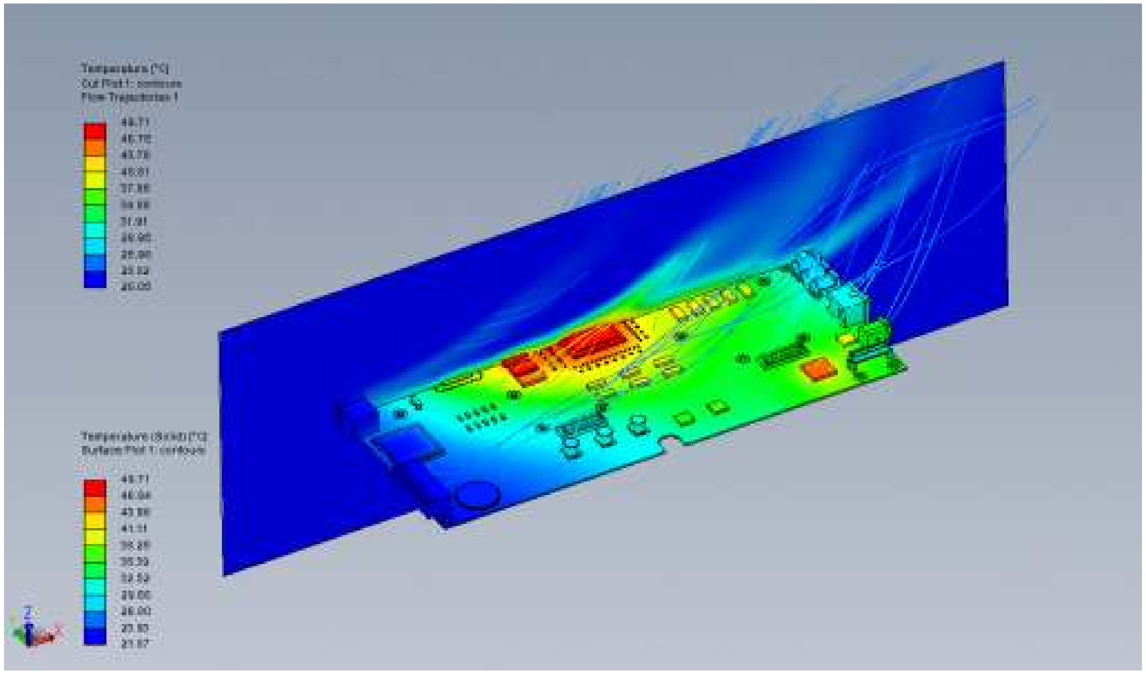
Sample CFD Applications

From the SolidWorks Flow Simulation



Analysis of convective heat transfer from a computer circuit board

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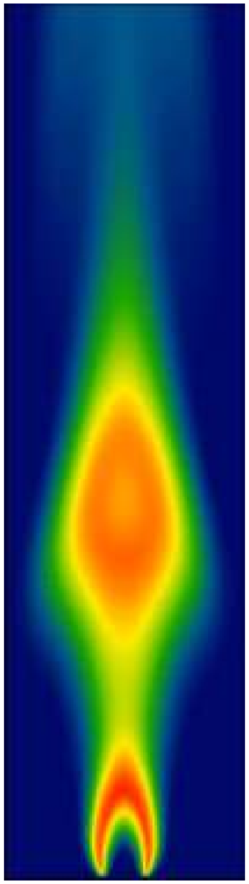
Combustion Research at Ryerson

Dr. Seth Dworkin's research group.

- Air pollution prediction and control.
- Prediction of soot formation in industrial flames; reduction of particulate air pollution.
- Transient solutions involve hundreds of parallel CPUs.
- Runs can take several days on some of the highest performance computers in Canada.



Ethylene Flame
Actual elapsed time 0.05s

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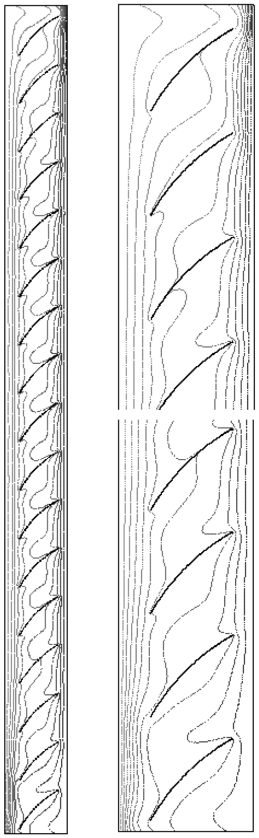
Advanced Fenestration Research at Ryerson

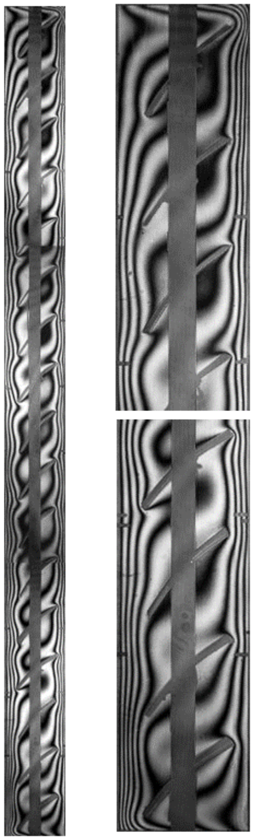
- Dr. David Naylor's research group.
- Optimization of the thermal performance of windows using CFD.
- Experimental validation by laser interferometry.

…happy to give you a quick lab tour.

www.pella.com



CFD Prediction



Laser Measurements

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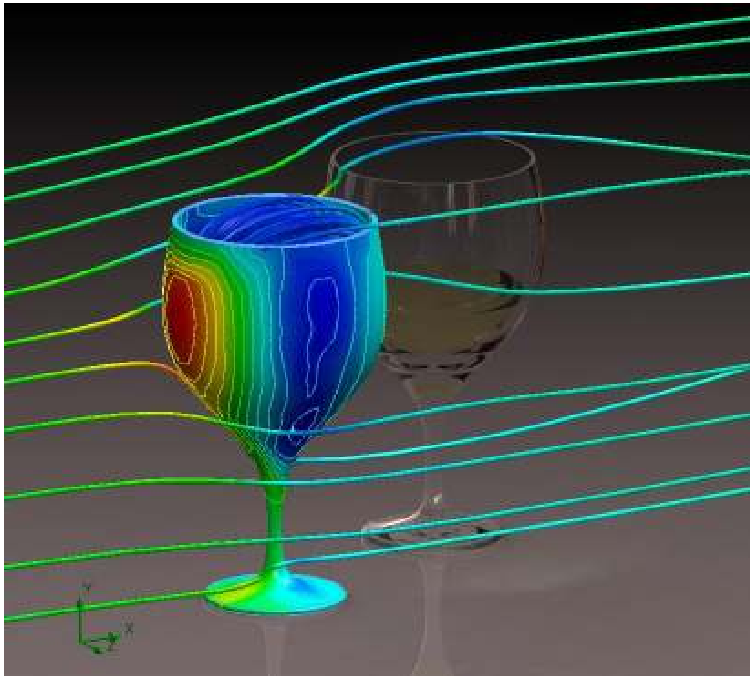

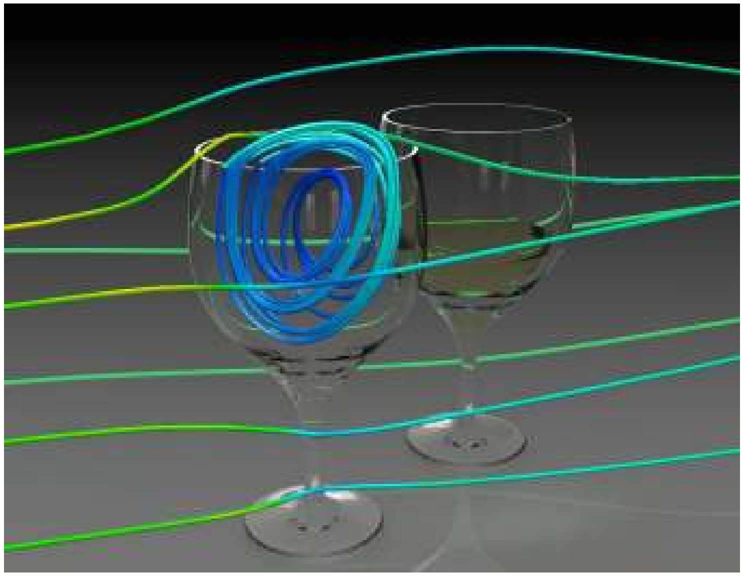

Pathlines/Streamlines/Streaklines/
over Wine Glasses using SolidWorks
Flow Simulation (by D. Naylor)

END NOTES

Presentation prepared and delivered by Dr. David Naylor.

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