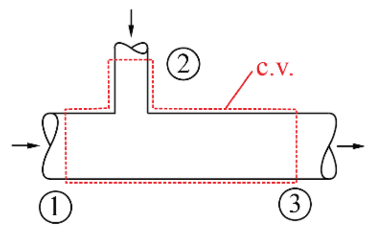
MEC516/BME516
Fluid Mechanics I

Chapter 3
Recommended Problem Set

Caution: Reading solutions can be deceiving. Solutions that look obvious at a glance can be difficult to reproduce later. Consult these solutions only after making an honest independent attempt at each problem.

1. The density of water at 90°C (Table A.2): $\rho = 965\frac{kg}{m^3}$. Consider the control volume (c.v.) shown below:



There is no change of mass inside the control volume with time:

$$\sum_{in}\dot{m}_i - \sum_{out}\dot{m}_i = \frac{dm_{cv}}{dt} = 0$$

$$\dot{m}_1 + \dot{m}_2 - \dot{m}_3 = 0 \quad \rho_1 V_1 A_1 + \rho_2 V_2 A_2 = \rho_3 V_3 A_3$$

This is an incompressible flow, i.e. the density of liquid is a constant: $\rho_1 = \rho_2 = \rho_3$. Thus,

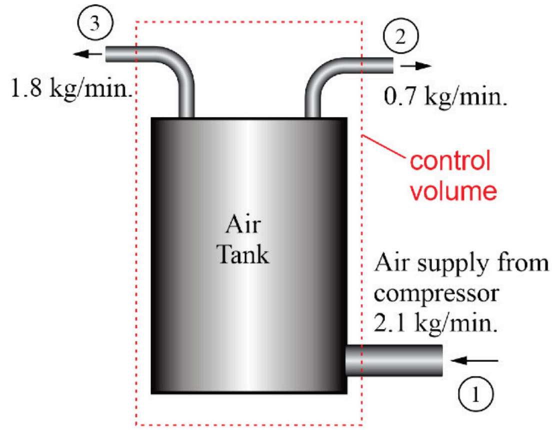
$$Q_1 + Q_2 = Q_3 \qquad V_1 A_1 + V_2 A_2 = Q_3$$

$$Q_3 = V_1\frac{\pi D_1^2}{4} + V_2\frac{\pi D_2^2}{4} = \left(1.2\frac{m}{s}\right)\frac{\pi(0.15m)^2}{4} + \left(2.1\frac{m}{s}\right)\frac{\pi(0.05m)^2}{4} = 0.0253\frac{m^3}{s}$$

$$V_3 = \frac{Q_3}{A_3} = \frac{4Q_3}{\pi D_3^2} = \frac{4(0.0253\frac{m^3}{s})}{\pi(0.15m)^2} = 1.43\frac{m}{s}$$

$$\dot{m}_3 = \rho Q_3 = 965\frac{kg}{m^3}\left(0.0253\frac{m^3}{s}\right) = 24.4\frac{kg}{s}$$

2. (a) Air is being extracted from the tank at a greater rate than it is being supplied. This is an *unsteady* problem because the mass of fluid inside the control volume is changing with time. So, consider the unsteady conservation of mass equation applied to the control volume shown in the sketch.



The rate of change of mass inside the control volume:

$$\sum_{in} \dot{m}_i - \sum_{out} \dot{m}_i = \frac{dm_{cv}}{dt} = \int_{CV} \frac{\partial \rho}{\partial t}\, d\forall = \forall \frac{\partial \rho}{\partial t}$$

$$\dot{m}_1 - \dot{m}_2 - \dot{m}_3 = \frac{dm_{cv}}{dt}$$

$$\frac{dm_{cv}}{dt} = 2.1 \frac{kg}{min}\left(\frac{1\ min}{60s}\right) - 1.80 \frac{kg}{min}\left(\frac{1\ min}{60s}\right) - 0.70 \frac{kg}{min}\left(\frac{1\ min}{60s}\right) = -6.67x10^{-3} \frac{kg}{s}$$

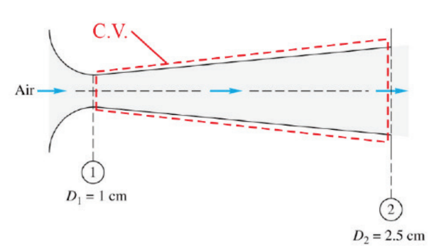
(a) The rate of change of density wihin the the control volume:

$$\frac{dm_{cv}}{dt} = \int_{CV} \frac{\partial \rho}{\partial t}\, d\forall = \forall \frac{\partial \rho}{\partial t}$$

$$-6.67x10^{-3} \frac{kg}{s} = (4.5\ m^3) \frac{\partial \rho}{\partial t}$$

The air density in the tank is decreasing at the rate: $\frac{\partial \rho}{\partial t} = -1.48x10^{-3} \frac{kg}{m^3 s}$

3. (a) Air densities from the ideal gas equation of state:



$$\rho_1 = \frac{p_1}{RT_1} = \frac{284{,}000 \frac{N}{m^2}}{287 \frac{Nm}{kgK}(392 + 273)K} = 1.488 \frac{kg}{m^3}$$

$$\rho_2 = \frac{8{,}000 \frac{N}{m^2}}{287 \frac{Nm}{kgK}(-33 + 273)K} = 0.1161 \frac{kg}{m^3}$$

For steady flow with one inlet and one outlet:

$$\sum_{in} \dot{m}_i - \sum_{out} \dot{m}_i = \frac{dm_{cv}}{dt} = 0 \;\rightarrow\; \dot{m}_1 = \dot{m}_2 = \dot{m} \qquad \dot{m} = \rho_1 V_1 A_1$$

$$\dot{m} = \rho_1 V_1 A_1 = 1.488 \frac{kg}{m^3}\left(517 \frac{m}{s}\right)\frac{\pi(0.01m)^2}{4} = 0.0604 \frac{kg}{s}$$

(b) Nozzel exit velocity:

$$\dot{m} = \rho_2 V_2 A_2 \quad V_2 = \frac{\dot{m}}{\rho_2 A_2} = \frac{0.0604 \frac{kg}{s}}{0.1161 \frac{kg}{m^3}\left(\frac{\pi(0.025m)^2}{4}\right)} = 1060 \frac{m}{s}$$

(c) The local speed of sound in a gas is: $c = \sqrt{kRT}$ where k is the specific heat ratio, R is the gas constant and T is the absolute temperature. From Table A.4, for air k =1.40.

$$c_1 = \sqrt{kRT_1} = \sqrt{1.40\left(287 \frac{Nm}{kgK}\right)665K} = 517 \frac{m}{s} \quad Ma_1 = \frac{V_1}{c_1} = \frac{517 \frac{m}{s}}{517 \frac{m}{s}} = 1.0$$

$$c_2 = \sqrt{kRT_2} = \sqrt{1.40\left(287 \frac{Nm}{kgK}\right)240K} = 311 \frac{m}{s} \quad Ma_2 = \frac{V_2}{c_2} = \frac{1060 \frac{m}{s}}{311 \frac{m}{s}} = 3.41$$

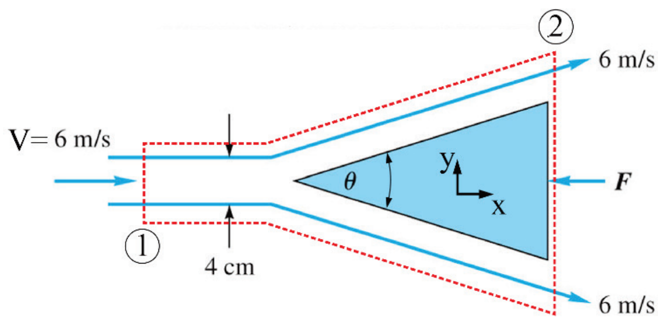
This is compressible flow. Recall: The rule of thumb for a gas is that compressibility effects need to be considered for Mach numbers Ma>0.3.

4. The volume flow rate (per unit depth) is found by integrating the velocity profile:

$$Q = \int_0^{\delta} u\, dy = U_\infty \int_0^{\delta} \left(\frac{y}{\delta}\right)^{1/7} dy = \frac{U_\infty}{\delta^{1/7}}\left[\frac{7}{8} y^{8/7}\right]_0^{\delta} = \frac{7}{8} U_\infty \delta$$

5. Consider the control volume shown in the sketch. For a steady flow, there is no storage of momentum in the control volume. So, conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum (\dot{m}_i \boldsymbol{V}_i)_{out} - \sum (\dot{m}_i \boldsymbol{V}_i)_{in}$$



The only force on the control volume is force F, which acts in the negative x-direction. Applying this equation in the x-direction, and noting that for a steady flow $\dot{m}_1 = \dot{m}_2 = \dot{m}$:

$$\sum F_x = -F = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(u_2 - u_1) = \dot{m}\left(V\cos\left(\frac{\theta}{2}\right) - V\right)$$

The total mass flow rate of the water sheet (per unit length into the page) is:

$$\dot{m} = \rho V_1 A_1 = 998\frac{kg}{m^3}\left(6.0\frac{m}{s}\right)(0.04m)(1.0m) = 239.5\frac{kg}{s}$$

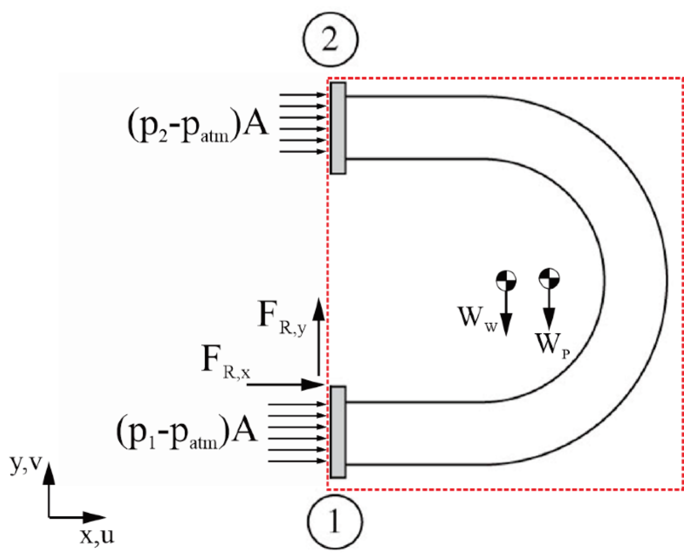
Applying conservation of momentum:

$$-124\ N = 239.5\frac{kg}{s}\left(6.0\frac{m}{s}\right)\left(\cos\left(\frac{\theta}{2}\right) - 1\right)$$

$$\cos\left(\frac{\theta}{2}\right) = 0.9137 \quad \theta = 48.0^0$$

6. See the free body diagram below showing all the forces on the control volume: the pressure forces at the flanges (normal to the flanges, acting **inwards** regardless of the flow direction!), the weights of the U-bend pipe and water, and the external reaction forces. The reaction forces, $F_{R,x}$ and $F_{R,y}$, are the external forces at the flanges required to hold the pipe in place. (These forces will be split between the bolts at the two flanges and must be combined for this analysis.)

Note that the pressures at the flanges must be expressed as **gauge pressures**. This enables the force due to the atmospheric pressure on the outside of the pipe to be eliminated. This is a critical point. See the lecture notes for a full explanation.



The flow speed and mass flow rates are:

$$V = \frac{Q}{A} = \frac{4(0.0235\frac{m^3}{s})}{\pi(0.05m)^2} = 11.97\frac{m}{s} \quad \dot{m} = \rho Q = 998\frac{kg}{m^3}(0.0235\frac{m^3}{s}) = 23.45\frac{kg}{s}$$

For a steady flow, there is no storage of momentum in the control volume. Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum (\dot{m}_i \boldsymbol{V}_i)_{out} - \sum (\dot{m}_i \boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction, and noting that for a steady flow $\dot{m}_2 = \dot{m}_1 = \dot{m}$:

$$F_{R,x} + (p_1 - p_{atm})A + (p_2 - p_{atm})A = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(-V - V) = -2\dot{m}V$$

Solving for the x reaction force:

$$F_{R,x} = -(p_1 - p_{atm})A - (p_2 - p_{atm})A - 2\dot{m}V$$

$$F_{R,x} = -\left(64{,}000\frac{N}{m^2}\right)\frac{\pi(0.05m)^2}{4} - \left(33{,}000\frac{N}{m^2}\right)\frac{\pi(0.05m)^2}{4} - 2\left(23.45\frac{kg}{s}\right)11.97\frac{m}{s} = -752\ N$$

$$F_{R,x} = +752\ N\ \leftarrow$$

Noting the y-component of the velocity vector is zero at the inlet and outlet ($v_1 = v_2 = 0$), applying conservation of momentum in the y-direction gives:

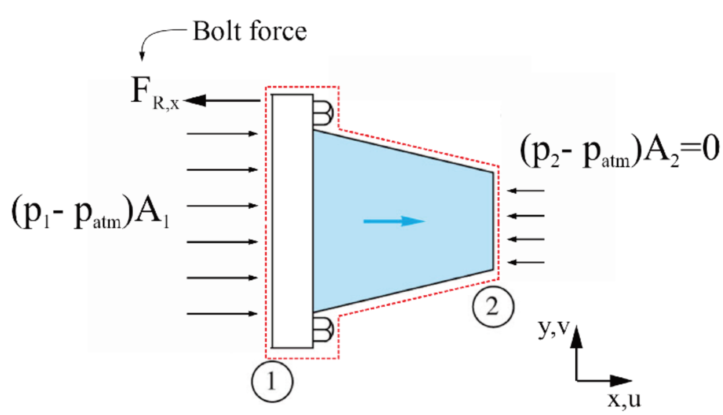
$$F_{R,y} - W_p - W_w = 0$$

The weight of the water in the pipe is:

$$W_w = \gamma\forall = 9790\frac{N}{m^3}\left(\frac{\pi(0.05m)^2}{4}\right)0.75m = 14.4\ N$$

$$F_{R,y} = W_p + W_w = 14.4N + 21.5N = 35.9\ N\ \uparrow$$

7. See the free body diagram showing all the forces on the control volume. The reaction force, $F_{R,x}$ is the external force in all the bolts required to hold the pipe in place. Again, the pressures at the flange and nozzle discharge **must** be expressed as gauge pressures.



The mass flow rate is:

$$\dot{m} = \dot{m}_2 = \dot{m}_1 = \rho V_2 A_2 = 1.937\frac{slug}{ft^3}\left(56\frac{ft}{s}\right)\frac{\pi(0.5\ ft)^2}{4} = 21.30\frac{slug}{s}$$

Since the flow is incompressible, the inlet water velocity is:

$$Q = V_1 A_1 = V_2 A_2 \quad V_1 = V_2\ \left(\frac{D_2}{D_1}\right)^2 = 56\frac{ft}{s}\left(\frac{6}{12}\right)^2 = 14.0\ \frac{ft}{s}$$

For a steady flow, there is no storage of momentum in the control volume. Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum(\dot{m}_i \boldsymbol{V}_i)_{out} - \sum(\dot{m}_i \boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction:

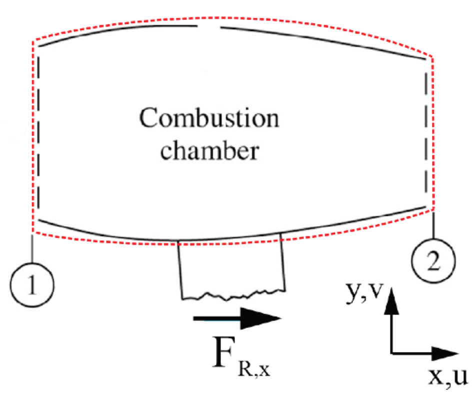
$$-F_{R,x} + (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(V_2 - V_1)$$

$$F_{R,x} = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 - \dot{m}(V_2 - V_1)$$

$$F_{R,x} = (38 - 15)\frac{lb}{in^2}\left(\frac{\pi(12\ in)^2}{4}\right) - 0\ lbs\ - 21.30\frac{slug}{s}(56 - 14)\frac{ft}{s} = 1710\ lb\ \longleftarrow$$

This is a combined force carried by all the flange bolts.

8. The jet engine inlet and outlet are at atmospheric pressure (zero gauge). So, the only force on the control volume is reaction force of the strut, $F_{R,x}$.



The air density at the inlet density and mass flow rate are:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101{,}300\frac{N}{m^2}}{287\frac{Nm}{kgK}(20 + 273)K} = 1.205\frac{kg}{m^3}$$

$$\dot{m}_1 = \rho_1 V_1 A_1 = 1.205\frac{kg}{m^3}\left(250\frac{m}{s}\right)0.5\ m^2 = 150.6\frac{kg}{s}$$

The fuel/air mass ratio is 30:1. Applying continuity equation for a steady state system the outlet mass flow rate is:

$$\dot{m}_2 = \dot{m}_1 + \dot{m}_{fuel} = \dot{m}_1 + \frac{1}{30}\,\dot{m}_1$$

$$\dot{m}_2 = \dot{m}_1\,\left(1 + \frac{1}{30}\right) = 150.6\frac{kg}{s}\left(1 + \frac{1}{30}\right) = 155.6\frac{kg}{s}$$

For a steady flow, there is no storage of momentum in the control volume. Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum (\dot{m}_i \boldsymbol{V}_i)_{out} - \sum (\dot{m}_i \boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction:

$$F_{R,x} = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}_2 V_2 - \dot{m}_1 V_1$$

$$F_{R,x} = 155.6\frac{kg}{s}\left(900\frac{m}{s}\right) - 150.6\frac{kg}{s}\left(250\frac{m}{s}\right) = 102{,}000\ N = 102\ kN\ \rightarrow$$

9. The mercury manometer deflection is used to get the gauge pressure at point 1. (This analysis was covered in Chapter 2). Make sure you can show that:

$$p_1 - p_{atm} = (\gamma_m - \gamma_w)h = \left(132{,}900\frac{N}{m^3} - 9{,}790\frac{N}{m^3}\right)0.58\,m = 71{,}400\ Pa$$

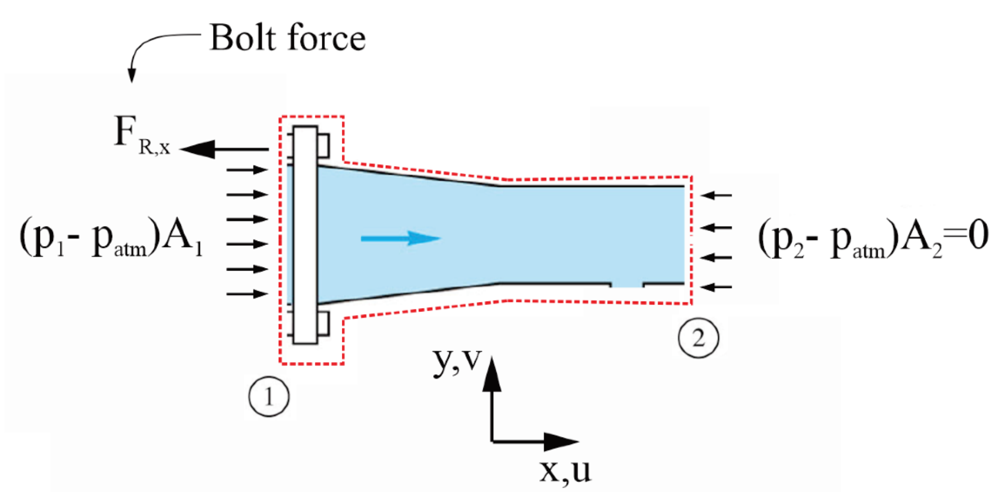
The mass flow rate is:

$$\dot{m} = \dot{m}_2 = \dot{m}_1 = \rho V_1 A_1 = 998\frac{kg}{m^3}\left(5.0\frac{m}{s}\right)\frac{\pi(0.08\,m)^2}{4} = 25.08\frac{kg}{s}$$

Since the flow is incompressible, the inlet water velocity is:

$$Q = V_1 A_1 = V_2 A_2 \quad V_2 = V_1\,\left(\frac{D_1}{D_2}\right)^2 = 5.0\frac{m}{s}\left(\frac{8.0}{5.9}\right)^2 = 12.8\,\frac{m}{s}$$

Again, the pressure forces must be expressed in gauge pressures. See the free body diagram below:



For a steady flow, there is no storage of momentum in the control volume. Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum (\dot{m}_i \boldsymbol{V}_i)_{out} - \sum (\dot{m}_i \boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction:

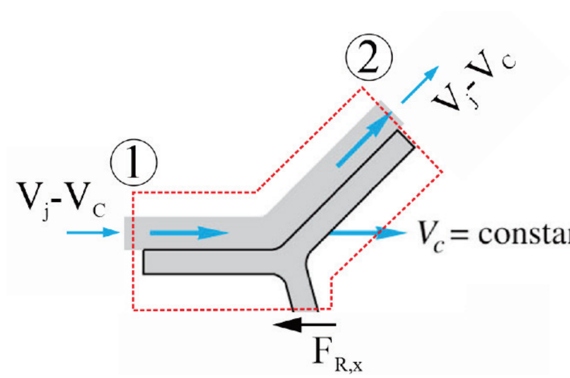
$$-F_{R,x} + (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(V_2 - V_1)$$

$$F_{R,x} = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 - \dot{m}(V_2 - V_1)$$

$$F_{R,x} = 71{,}400 \frac{N}{m^2}\left(\frac{\pi(0.08\ m)^2}{4}\right) - 0\ N\ - 25.08\frac{kg}{s}(12.8 - 5.0)\frac{m}{s} = 163\ N\ \leftarrow$$

This is a combined force carried by all the bolts in the flange.

10. See the free body diagram below. Note that the control volume (in red) is moving at contant speed with the cart. Normally, you should only put forces on a free body diagram. However, for illustration, I have added the speed of the jet **relative to the control volume.** The cart is moving away from the jet. So, the speed at which the jet crosses the moving control volume is $V_j - V_c$. Thus, the analysis is the same as for a stationary vane, but using these relative speeds.



For a steady flow, there is no storage of momentum in the control volume. The boundary of the control volume is at atmospheric pressure. So, the only force on the control volume is the reaction force, $F_{R.x}$.

Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum (\dot{m}_i \boldsymbol{V}_i)_{out} - \sum (\dot{m}_i \boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction, using the velocities relative to the control volume:

$$-F_{R,x} = \dot{m}_2 u_{2,rel} - \dot{m}_1 u_{1,rel} = \dot{m}[(V_j - V_c)\cos\theta - (V_j - V_c)]$$

The rate at which mass flows through the control volume is:

$$\dot{m} = \rho(V_j - V_c)A_j$$

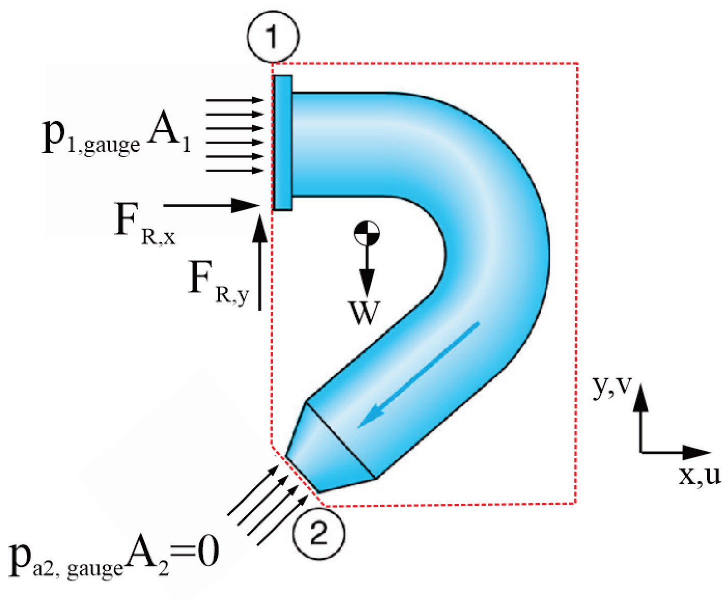
So, the external force required to keep the cart from accelerating is:

$$F_{R,x} = \rho(V_j - V_c)A_j[(V_j - V_c) - (V_j - V_c)\cos\theta]$$

$$F_{R,x} = \rho(V_j - V_c)^2 A_j (1 - \cos\theta)$$

The key point for analysing a moving control volume is that the velocities and mass flow rates need to be expressed relative to the moving control volume.

11. See the freebody diagram below:



The flow speeds and mass flow rates are:

$$V_1 = \frac{Q}{A_1} = \frac{4\left(0.0153\frac{m^3}{s}\right)}{\pi(0.1m)^2} = 1.948\ \frac{m}{s} \quad V_2 = \frac{Q}{A_2} = \frac{4\left(0.0153\frac{m^3}{s}\right)}{\pi(0.03m)^2} = 21.65\ \frac{m}{s}$$

$$\dot{m} = \rho Q = 998\frac{kg}{m^3}(0.0153\frac{m^3}{s}) = 15.27\frac{kg}{s}$$

For a steady flow, there is no storage of momentum in the control volume. Conservation of linear momentum for the control volume is:

$$\sum \boldsymbol{F} = \sum(\dot{m}_i\boldsymbol{V}_i)_{out} - \sum(\dot{m}_i\boldsymbol{V}_i)_{in}$$

Applying this equation in the x-direction, and noting that for a steady flow $\dot{m}_2 = \dot{m}_1 = \dot{m}$:

$$F_{R,x} + (p_1 - p_{atm})A_1 = \dot{m}_2 u_2 - \dot{m}_1 u_1 = \dot{m}(-V_2\cos\theta - V_1)$$

Solving for the x reaction force:

$$F_{R,x} = -(p_1 - p_{atm})A_1 - \dot{m}(V_2 \cos\theta + V_1)$$

$$F_{R,x} = -\left(233{,}000 \frac{N}{m^2}\right) \frac{\pi(0.1m)^2}{4} - \left(15.27 \frac{kg}{s}\right)(21.65 \frac{m}{s} \cos 40^o + 1.948 \frac{m}{s}) = -2110\ N$$

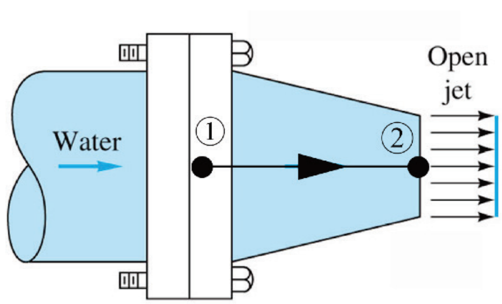
$$F_{R,x} = +2110\ N \ \longleftarrow$$

Applying this conservation of momentum equation in the y-direction:

$$F_{R,y} - W = \dot{m}_2 v_2 - \dot{m}_1 v_1 = \dot{m}(-V_2 \sin\theta - 0)$$

$$F_{R,y} = W - \dot{m}V_2 \sin\theta = 250N - \left(15.27 \frac{kg}{s}\right) 21.65 \frac{m}{s} \sin 40^o = 37.5\ N \ \uparrow$$

12. (a) I recommend using the form of Bernoulli's equation where each term has the units of head (feet in this problem). Using this form makes the transition to the general energy equation less error prone. Applying Bernoulli between points 1 and 2 on the centre line of the pipe:



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

Noting that $z_1 = z_2$, the pressure a point 1 is:

$$p_1 = p_2 + \gamma \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right)$$

Using continuity for an incompressible flow, the inlet velocity is:

$$Q = V_1 A_1 = V_2 A_2 \quad V_1 = V_2 \left(\frac{D_2}{D_1}\right)^2 = 56 \frac{ft}{s} \left(\frac{6\ in}{12\ in}\right)^2 = 14.0\ \frac{ft}{s}$$

Subtituting these values:

$$p_1 = 15 \frac{lb}{in^2} 144 \frac{in^2}{ft^2} + 62.4 \frac{lb}{ft^3} \left(\frac{\left(56 \frac{ft}{s}\right)^2}{2\left(32.2 \frac{ft}{s^2}\right)} - \frac{\left(14 \frac{ft}{s}\right)^2}{2\left(32.2 \frac{ft}{s^2}\right)} \right) = 5000\ \frac{lb}{ft^2}$$

$$p_1 = 5000\ \frac{lb}{ft^2} \left(\frac{1}{144} \frac{ft^2}{in^2}\right) = 34.8\ psia$$

The value of p_1 predicted by Bernoulli's equation will be lower than the actual value because of frictional head losses in the nozzle.

(b) The actual pressure at the nozzle inlet is $p_1 = 38$ psia=5472 lb/ft². Also, $p_2 = 15$ psia=2160 lb/ft². The energy at point 1 minus the head loss, equals the energy at point 2. Writing the steady flow energy equation from 1 to 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} - h_{friction} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}$$

$$h_f = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g}$$

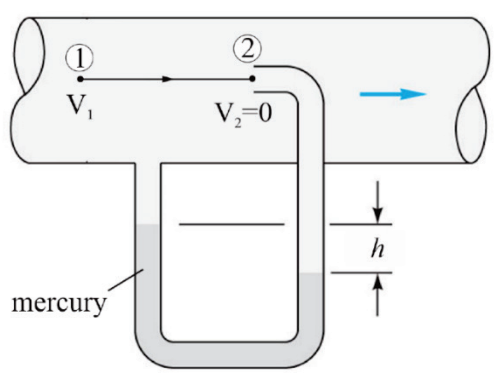
$$h_{friction} = \frac{5472\,\frac{lb}{ft^2}}{62.4\frac{lb}{ft^3}} - \frac{2160\,\frac{lb}{ft^2}}{62.4\frac{lb}{ft^3}} + \frac{\left(14\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} - \frac{\left(56\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 7.42\,ft$$

13. As shown below, apply Bernoulli's equation along a streamline from 1 to 2, noting that that $z_1 = z_2$:

$$\frac{p_1}{\gamma_f} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma_f} + \frac{V_2^2}{2g} = \frac{p_2}{\gamma_f} + 0$$

where γ_f is the specific weight of the fluid in the pipe. The fluid flow comes to rest at the nose of the Pitot tube, i.e., $V_2 = 0$. This is called a stagnation point. Thus, the fluid velocity on the centre line of the pipe can be expressed as:

$$V_1 = \sqrt{\frac{2g(p_2 - p_1)}{\gamma_f}}$$



(a) Table A.3: Gasoline, γ_f =6670 N/m³, mercury gauge fluid, γ_m =132,900 N/m³

The pressure difference $p_2 - p_1$ is calculated using manometry analysis (from Chapter 2):

$$p_2 - p_1 = h(\gamma_m - \gamma_f) = 0.0254m\left(132{,}900\frac{\text{N}}{m^3} - 6670\frac{\text{N}}{m^3}\right) = 3210\ \text{Pa}$$

$$V_1 = \sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)3210Pa}{6670\frac{\text{N}}{m^3}}} = 3.07\frac{m}{s}$$

(b) For air as the fluid in the pipe:

$$\rho_f = \frac{p_1}{RT_1} = \frac{100{,}000\frac{N}{m^2}}{287\frac{Nm}{kgK}(20+273)K} = 1.19\frac{kg}{m^3}, \qquad \gamma_f = 11.7\frac{\text{N}}{m^3}$$

For air, the specific weight of the fluid (air) is negligible compared to the gauge fluid:

$$p_2 - p_1 = h\gamma_m = 0.0254m\left(132{,}900\frac{\text{N}}{m^3}\right) = 3376 \text{ Pa}$$

$$V_1 = \sqrt{\frac{2\left(9.81\frac{m}{s^2}\right)3376Pa}{11.7\frac{\text{N}}{m^3}}} = 75.3\frac{m}{s}$$

14. Applying Bernoulli between points 1 and 2 on the centre line of the pipe:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$p_1 = \left(p_2 + \frac{1}{2g}(V_2^2 - V_1^2) + (z_2 - z_1)\right)\gamma$$

The density of gasoline (Table A.3): $\rho = 680$ kg/m^3, $\gamma = 6670$ N/m^3. The fluid velocity at points 1 and 2:

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{4\left(12.2\frac{kg}{s}\right)}{680\frac{kg}{m^3}\pi(0.08m)^2} = 3.57\frac{m}{s}$$

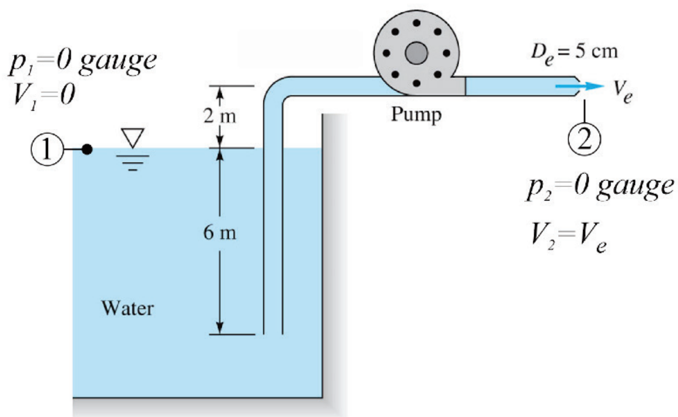
$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{4\left(12.2\frac{kg}{s}\right)}{680\frac{kg}{m^3}\pi(0.05m)^2} = 9.14\frac{m}{s}$$

$$p_1 = \left(0\,kPa + \frac{1}{2\left(9.81\frac{m}{s^2}\right)}\left(\left(9.14\frac{m}{s}\right)^2 - \left(3.57\frac{m}{s}\right)^2\right) + 12.0m\right)6670\frac{N}{m^3} = 104\,kPa\,(g)$$

This is the **gauge pressure** at point 1. Note that p_1 is a gauge pressure because $p_2 = 0$ is the gauge pressure where the fluid discharges to atmospheric pressure. If p_2 was the absolute pressure (i.e. 99 kPa), then the computed value of p_1 would be absolute pressure.

15. The general energy equation is applied from point 1 on the surface of the reservior to point 2 at the exit of the nozzle:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_{pump}-h_{friction}=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$



The fluid velocity at point 1 is approximately zero ($V_1 \approx 0$) because it is far from the pipe inlet. The pressure at point 1 and point 2 is local atmospheric pressure i.e., $p_1 = p_2 = 0$ gauge. Substituting values into the general energy equation:

$$0+0+z_1+h_{pump}-h_{friction}=0+\frac{V_2^2}{2g}+z_2$$

$$h_{pump}=\frac{V_2^2}{2g}+(z_2-z_1)+h_{friction}$$

From continuity, the velocity at the discharge is:

$$V_2=\frac{Q}{A_2}=\frac{4\left(\frac{220}{3600}\frac{m^3}{s}\right)}{\pi(0.05m)^2}=31.12\frac{m}{s}$$

Substituting values:

$$h_{pump}=\frac{(31.12\frac{m}{s})^2}{2\,(9.81\frac{m}{s^2})}+2.0m+6.5m=57.86\ m$$

The pump shaft power is:

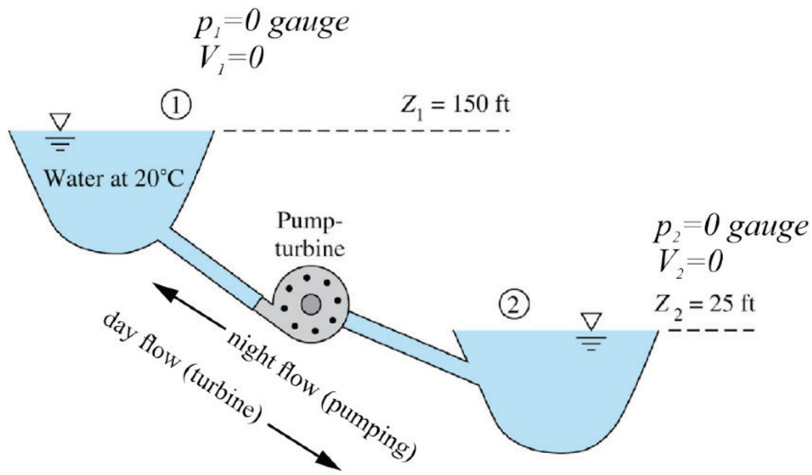
$$P=\frac{\gamma Q h_{pump}}{\eta}=\frac{9790\frac{N}{m^3}\left(\frac{220}{3600}\frac{m^3}{s}\right)57.86\ m}{0.75}=46.2\ kW$$

Note that the pump efficiency (η) is in the denominator of the power equation. Why? The process by which the spinning impeller of the pump adds energy to the water flow has losses. So **more power** must be added at the pump's shaft than gets transfered to the fluid.

16. (a) During the daytime the water flows through the turbine from point 1 to point 2 in order to generate electrical power for the city. The energy at point 1 minus the head loss, minus the energy extracted by the turbine, equals the energy at point 2. The general energy equation is:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 - h_{turbine} - h_{friction} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

The fluid velocity at point 1 and point 2 is approximately zero ($V_1 \approx 0,\ V_2 \approx 0$) because they are far from the pipe (i.e., the penstock and draft tube). The pressure at point 1 and point 2 is local atmospheric pressure i.e., $p_1 = p_2 = 0$ gauge.



Sustituing these values into the energy equation for daytime conditions:

$$0 + 0 + z_1 - h_{turbine} - h_{friction} = 0 + \ 0 + z_2$$

$$h_{turbine} = (z_1 - z_2) - h_{friction} = (150ft - 25ft) - 17\ ft = 108\ ft$$

The turbine power delivered to the generator is:

$$P_{turbine} = \eta\gamma Q h_{turbine} = (0.82)62.4\ \frac{lb}{ft^3}\left(33.4\frac{ft^3}{s}\right)\ 108ft = 185{,}000\frac{ftlb}{s} = 336\ hp$$

Note that the turbine efficiency is in the numerator of the power equation. Why? The process by which the spinning impeller of the turbine extracts energy from the water flow has losses. Less power is extracted from the flow than is available.

(b) At night water flows through a pump from point 2 to point 1 in order to store gravitational potential energy for the next day's electricity production. The energy at point 2 plus the energy added by the pump, minus the head loss, equals the energy at point 1. The general energy equation is:

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{pump} - h_{friction} = \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1$$

Again, $V_1 \approx 0,\ V_2 \approx 0$ and $p_1 = p_2 = 0$ gauge. Subtituting these values:

$$0 + 0 + z_2 + h_{pump} - h_{friction} = 0 + \ 0 + z_1$$

$$h_{pump} = +(z_1 - z_2) + h_{friction} = (150ft - 25ft) + 17\,ft = 142\,ft$$

$$P_{pump} = \frac{\gamma Q h_{pump}}{\eta} = \frac{62.4\,\frac{lb}{ft^3}\left(33.4\frac{ft^3}{s}\right)\,142\,ft}{0.70} = 423{,}000\frac{ft\,lb}{s} = 769\,hp$$

Notice the effect of the $h_{friction}$ term in parts (a) and (b). The frictional head works against the system in both directions, requiring additional power input at night and permitting less power extraction during the day.

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