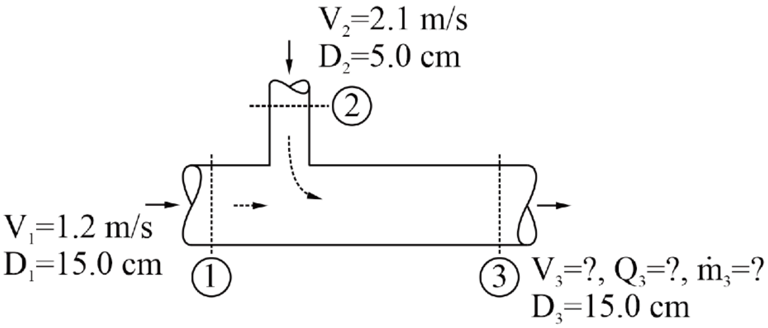
Last Updated: July 2020

MEC516/BME516
Fluid Mechanics I

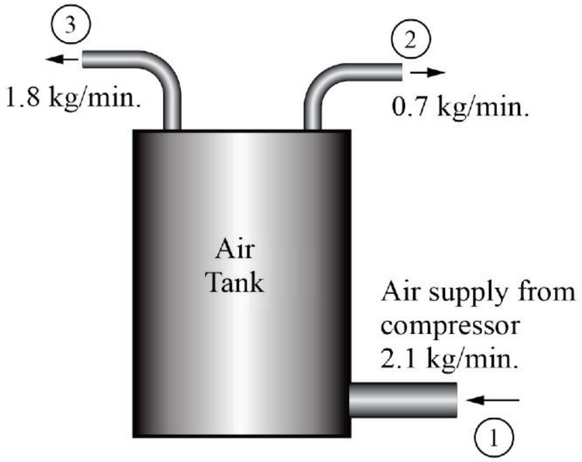
Chapter 3
Recommended Problem Set

I strongly recommended that you work through the details of these problems, with pen and paper. ***Be sure that you can solve these problems without looking at the solutions.***

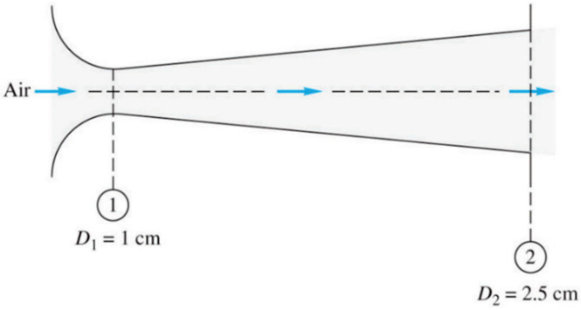
1. Liquid water at 90°C flows in a piping system with the mean flow velocities and internal pipe diameters shown in the sketch. At section 3, calculate the mean water velocity V_3, the volume flow rate Q_3, and the mass flow rate $\dot{m}_3$.



2. An air tank with an internal volume of $\forall$=4.5 m^3 receives air from a compressor at a mass flow rate of 2.1 kg/min. The tank supplies high pressure air to a building via two pipes. At an instant in time, one pipe extracts air at 1.8 kg/min. and the other pipe extracts air at 0.70 kg/min. Calculate (a) the rate of change of mass of air in the tank and (b) the rate of change of the air density in the tank.



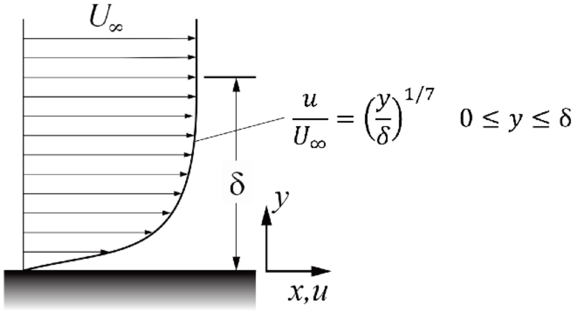
3. Mechanical engineering students will learn about supersonic nozzle flow in *MEC514 Applied Thermodynamics*. (This *is* Rocket Science!) The converging-diverging nozzle shown in the sketch expands and accelerates dry air to supersonic speeds at the exit, where the absolute pressure is $p_2 = 8.0\ kPa$ and $T_2 = -33^oC$. At the nozzle throat, the absolute pressure is $p_1 = 284\ kPa$ and $T_1 = 392^oC$, and $V_1 = 517$ m/s. Assuming steady compressible flow of an ideal gas, estimate: (a) the mass flow $\dot{m}$, (b) the exit velocity V_2, and (c) the Mach numbers at the throat and exit.



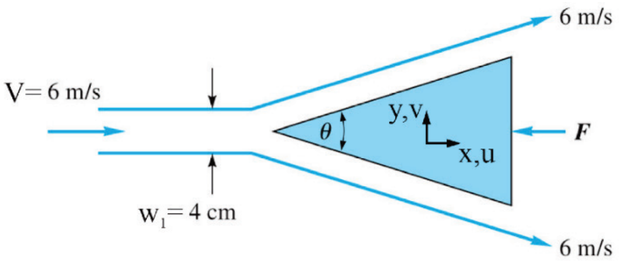
4. Consider flow in a turbulent boundary layer adjacent to a wall. If the x-component of velocity profile is:

$$\frac{u}{U_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \quad 0 \le y \le \delta$$

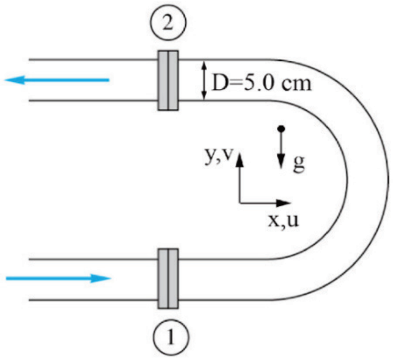
Derive an expression for the volume flow rate Q in the boundary layer (from y=0 to y=δ) per unit depth into the page.



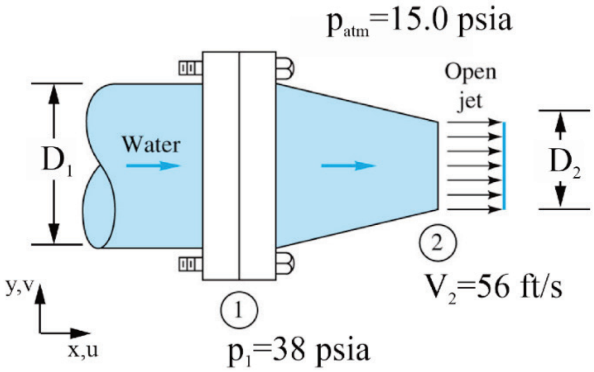
5. A wedge splits a sheet of liquid water (at 20°C), as shown in the sketch. Both wedge and water sheet are very long into the page. The speed of the water flow is steady at V=6.0 m/s. If the force required to hold the wedge stationary is F = 124 N per meter of depth into the paper, what is the angle θ of the wedge? Neglect the viscous flow resistance such that the water speed remains constant (at 6.0 m/s) as it passes over the wedge. (Note: This is similar to the experiment in Lab 3.)



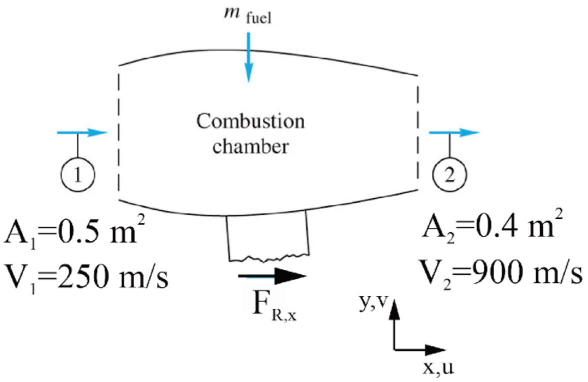
6. Water at 20°C flows through a pipe which has a 180° vertical bend, as shown in the sketch. The pipe's inside diameter is 5.0 cm and the total length of pipe between flanges is 75 cm. The volume flow rate is 23.5 ℓ/s. The absolute pressures at the flanges are p_1 =163 kPa, and p_2 =132 kPa. The local atmospheric pressure is p_{atm} =99 kPa. The weight of the steel U-bend is $W_P = 21.5\ N$. Determine the forces at the flanges in the x- and y-directions required to hold the pipe in place. Clearly indicate the direction of these forces.



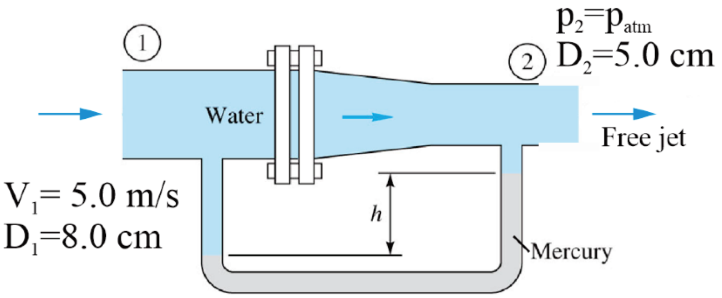
7. As shown in the sketch, a horizontal nozzle has inlet and outlet diameters D_1= 12.0 inches and D_2 =6.0 inches. The absolute pressure at the nozzle inlet is p_1 = 38 psia. The local atmospheric pressure is 15 psia. The water jet exits at a speed of V_2 = 56.0 ft/s. For a water flow at 68°F, find the force in the x-direction provided by the flange bolts to hold the nozzle in place.



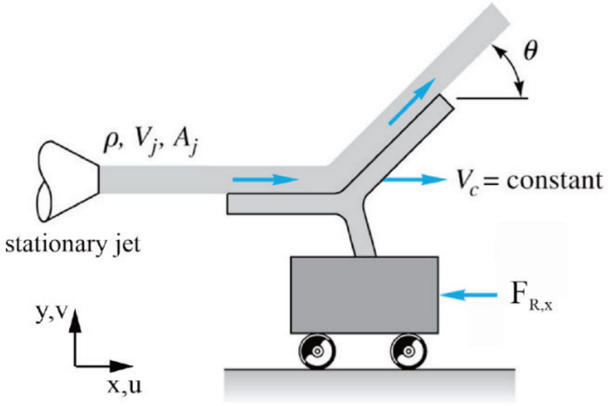
8. Jet engines provide thrust by accelerating the air that passes through the engine. A jet engine is mounted on a fixed test stand as shown in the sketch. The engine admits air at 20°C and 101.3 kPa, where A_1 =0.5 m^2 and V_1 =250 m/s. Fuel is added to the incoming air stream at a fuel/air mass ratio of 1:30. The air leaves section 2 at 101.3 kPa, V_2 =900 m/s, and A_2 = 0.4 m^2. The flow is steady. Calculate the stand support reaction force $F_{R,x}$ required to hold the engine in place, i.e., the thrust produced by the jet engine.



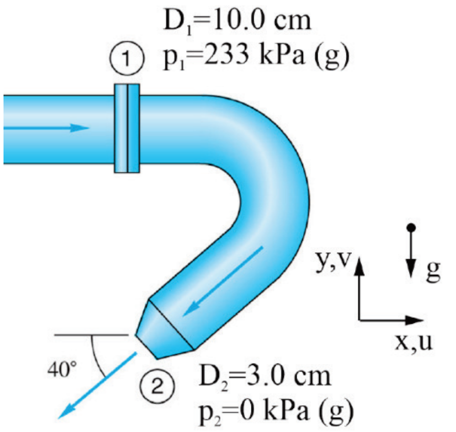
9. A reducing section is bolted to a horizontal pipe, as shown in the sketch. The pipe has an inside diameter of $D_1 = 8.0$ cm. The pipe is reduced to a diameter of $D_2 = 5.0$ cm, where the pipe discharges to atmosphere. Water at 20°C flows in the pipe at $V_1 = 5.0$ m/s. The mercury manometer deflection is $h =$ 58 cm. Calculate the total horizontal force provided by the flange bolts to hold the reducing section in place. The forces in the manometer tubing are neglegible.



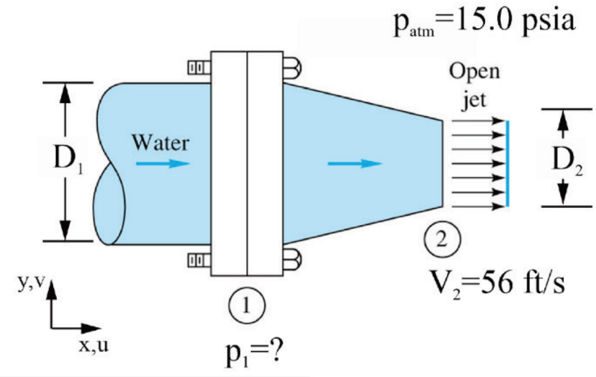
10. As shown in the sketch, a liquid jet strikes a vane which moves to the right at constant velocity V_c on a frictionless cart. Derive an expression for the force $F_{R,x}$ required to prevent the cart from accelerating. Neglect the viscous flow resistance, such that the jet speed relative to the cart remains constant.



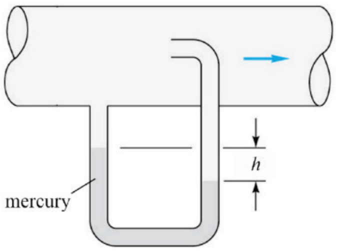
11. As shown in the sketch, water at 20°C flows through the vertical elbow and discharges to the atmosphere at an angle of θ=40° from horizontal. The internal pipe diameter is $D_1 = 10.0$ cm and the nozzle exit has internal diameter D_2 =3.0 cm. For a volume flow rate of 15.3 ℓ/s, the pressure at point 1 is p_1=233 kPa (gauge). The combined weight of steel elbow and the water in the elbow is 250 N. Calculate the forces in the flange bolts in the x- and y-directions (at section 1) required to hold the pipe in place.



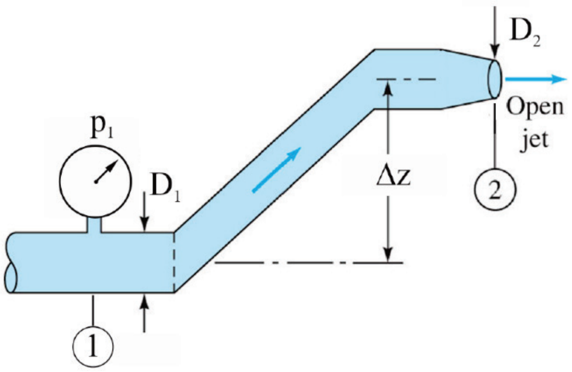
12. For the flow conditions in Problem 7: (a) calculate absolute pressure at the nozzle inlet p_1 using Bernoulli's equation, i.e. assuming no frictional head losses in the nozzle. (b) If the actual absolute pressure at point 1 is 38 psia, calculate the frictional head loss in the nozzle. (Recall from Problem 6: Horizontal nozzle with diameters D_1= 12.0 in, D_2 =6.0 in. The water jet is at 68°F and is discharged at a speed of V_2 = 56.0 ft/s.)



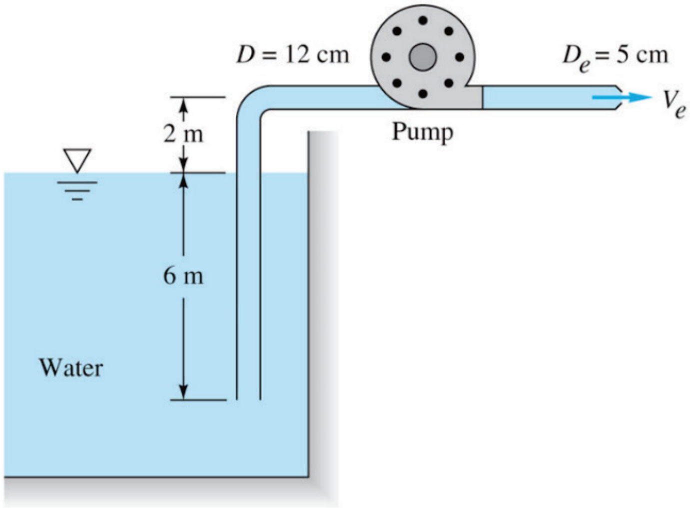
13. The sketch below illustrates the use of a *Pitot Tube* to measure the fluid velocity in a horizontal pipe. The deflection of the mercury gauge fluid in the manometer is h =25.4 mm. Calculate the fluid velocity on the centre line of the pipe if the fluid in the pipe is: (a) gasoline at 20°C and (b) air at 20°C and an abolute pressure of 100 kPa.



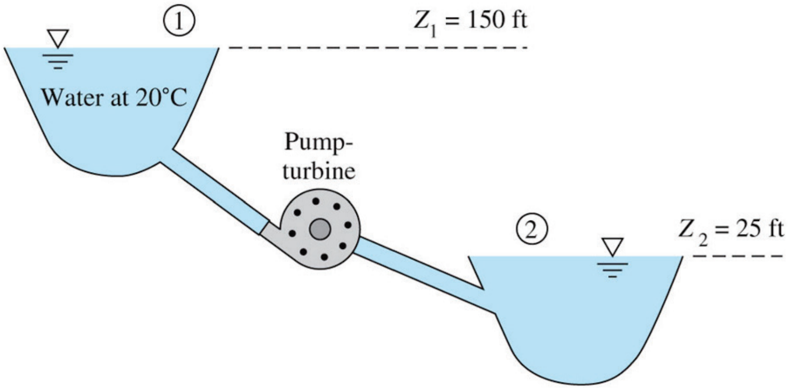
14. Gasoline at 20°C flows at mass flow rate of 12.2 kg/s in the piping system shown in the sketch. The flow discharges to atmosphere at point 2 from a nozzle with internal diameter D_2 =5.0 cm. The internal pipe diameter at point 1 is D_1 =8.0 cm. Point 2 is 12.0 m higher than point 1. Assuming no frictional head losses in the pipe, calculate the pressure reading on the Bourdon gauge at section 1. The local atmospheric pressure is 99.0 kPa.



15. As shown in the sketch, a pump draws 220 m^3/h of water at 20°C from a reservoir. The flow discharges through a nozzle to the atmosphere. The diameter of the discharge nozzle is 5.0 cm. The total friction head loss from the reservoir to the discharge point is $h_{friction} = 6.5$ m. For a pump efficiency of η=75%, calculate the power input to the pump's shaft required to deliver this flow of water.



16. As shown in the sketch, in the daytime water runs from an upper reservoir through a hydraulic turbine to produce electrical power for a city. At night, water is pumped from lower reservoir to the upper reservoir to restore the situation. (This is a common method for temporary energy storage for use during periods of high electrical demand, called *pumped storage.*) For a design flow rate of 33.4 ft^3/s in either direction, the friction head loss is 17.0 ft. The centrifugal pump has an efficiency of 70%. The hydraulic turbine has an efficiency of 82%. The surface of the upper reservoir is Δz=125 ft higher than the surface of lower reservior. At the steady design flow rate, calculate: (a) The horse power that the turbine shaft supplies to the electrical generator during the day. (b) The horse power delivered to the pump's shaft at night. Note: $1\ hp = 550$ ft-lb/s



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