MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis

Part 6.1: Example Problem

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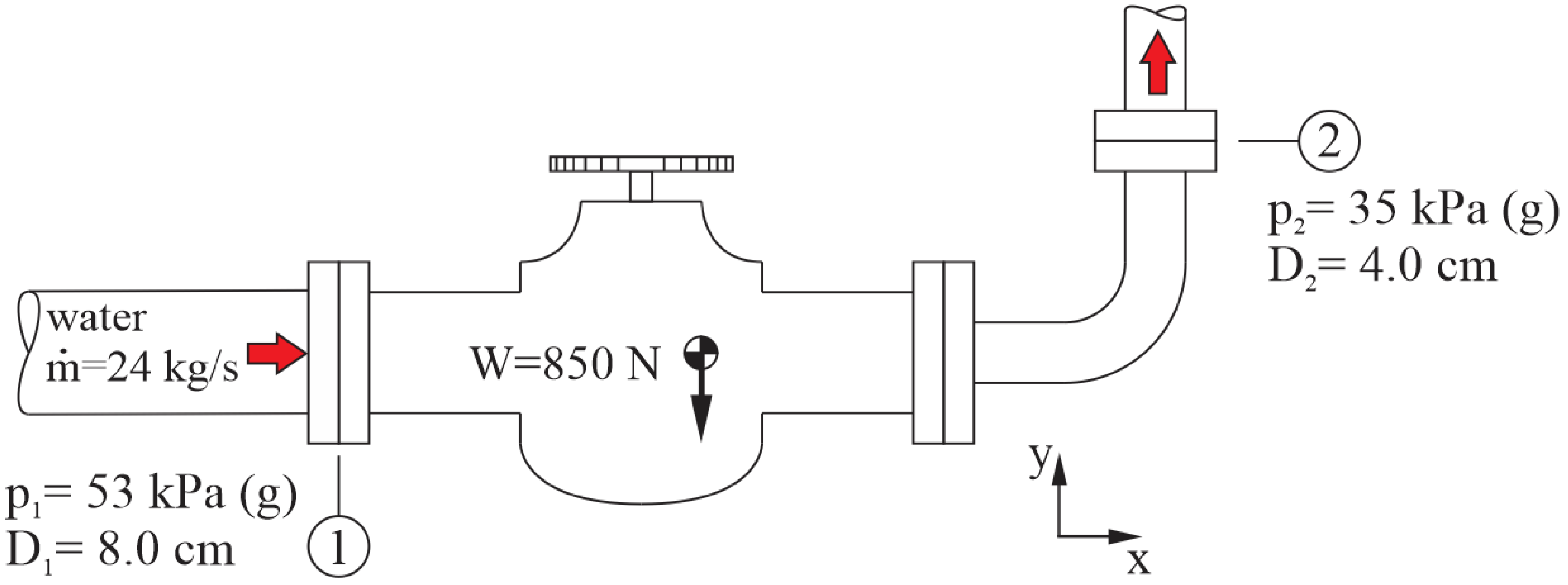
Example: Linear Momentum (Makeup Final Exam 2017)

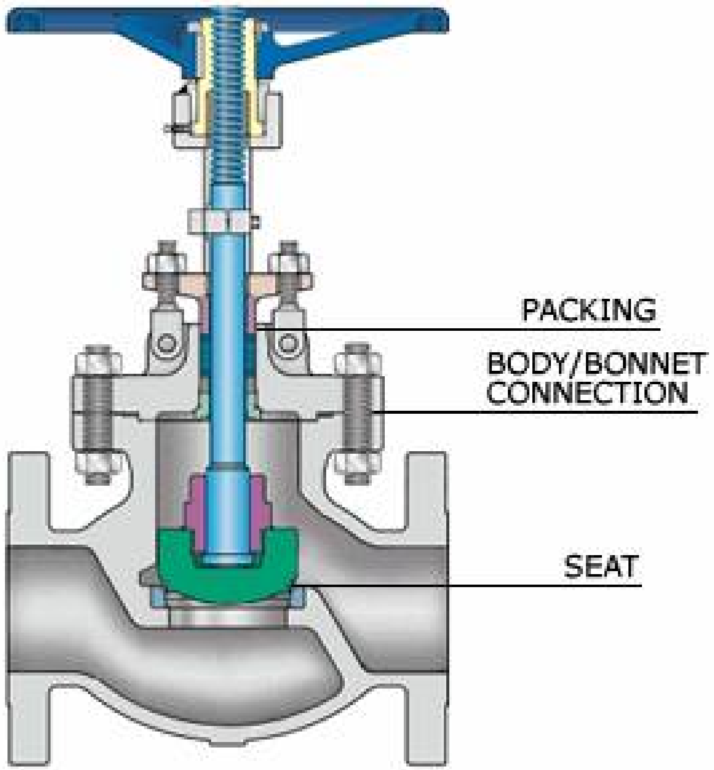
Water (ρ = 998 kg/m³) enters a valve at a mass flow rate of 24 kg/s. A *globe valve* reduces the pressure from 53 kPa (gauge) at the inlet to 35 kPa (gauge) at the outlet. The weight of the valve, pipe elbow, and the water from ① to ② is 850 N.

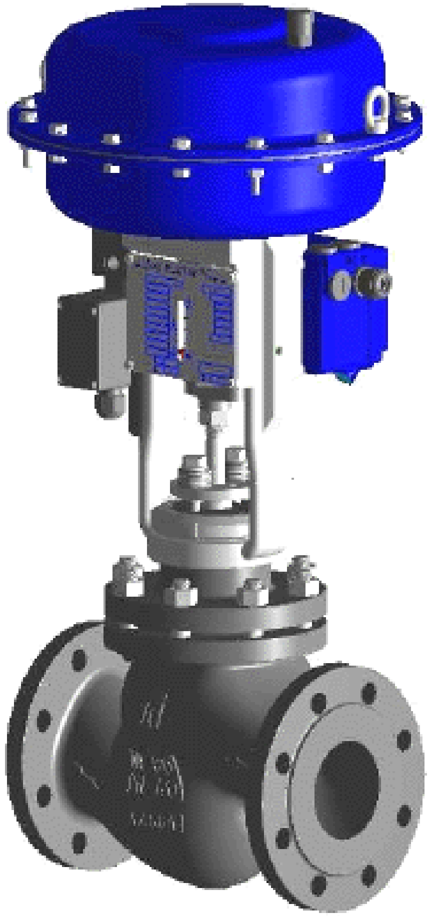
Calculate the forces in the x-direction (R_X) and y-direction (R_Y) required to hold this valve and piping system in place.



An Aside: What is a Globe Valve?

- Used for flow control in industrial piping systems
  - Hand wheel or pneumatic actuator for remote flow adjustment from control room

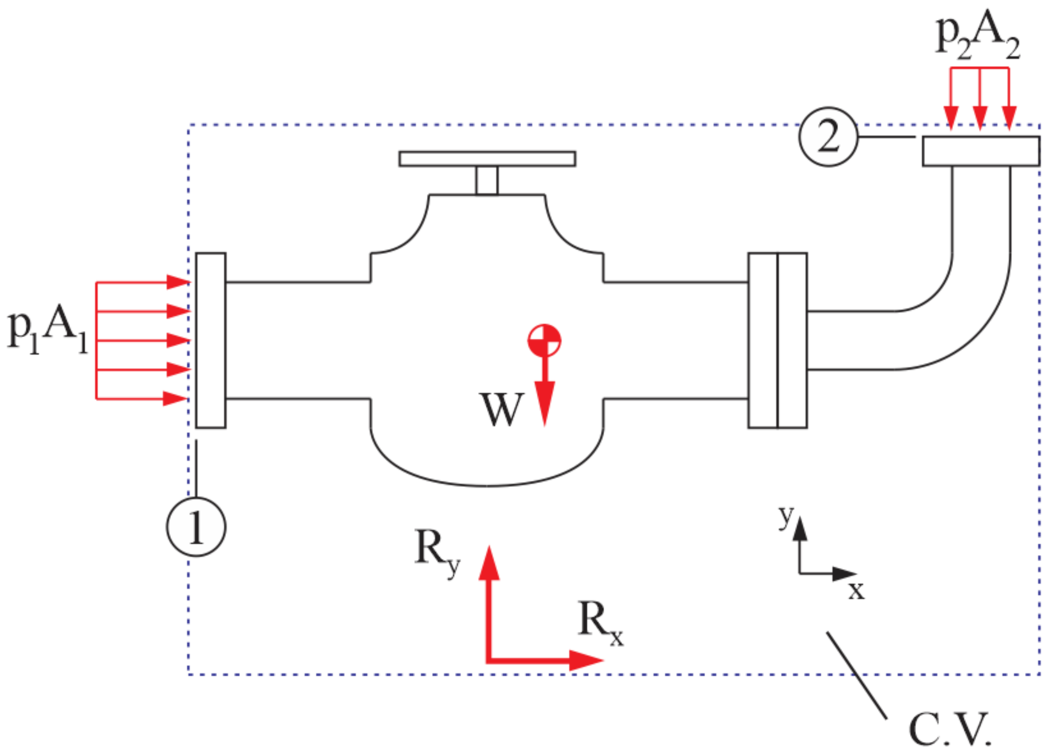




Example: Linear Momentum

Free Body Diagram

- Draw the isolated section. Work in symbols!
- All forces applied to C.V.:
  - Unknown reaction forces (R_x, R_y)
  - Weight(s)
  - Pressure forces, acting inward & normal to surface
  - Use ***gauge*** pressures. (Why?)



- Conservation of linear momentum. Steady flow, control volume with one inlet and one outlet:

Continuity $\dot{m}_1 = \dot{m}_2 = \dot{m}$

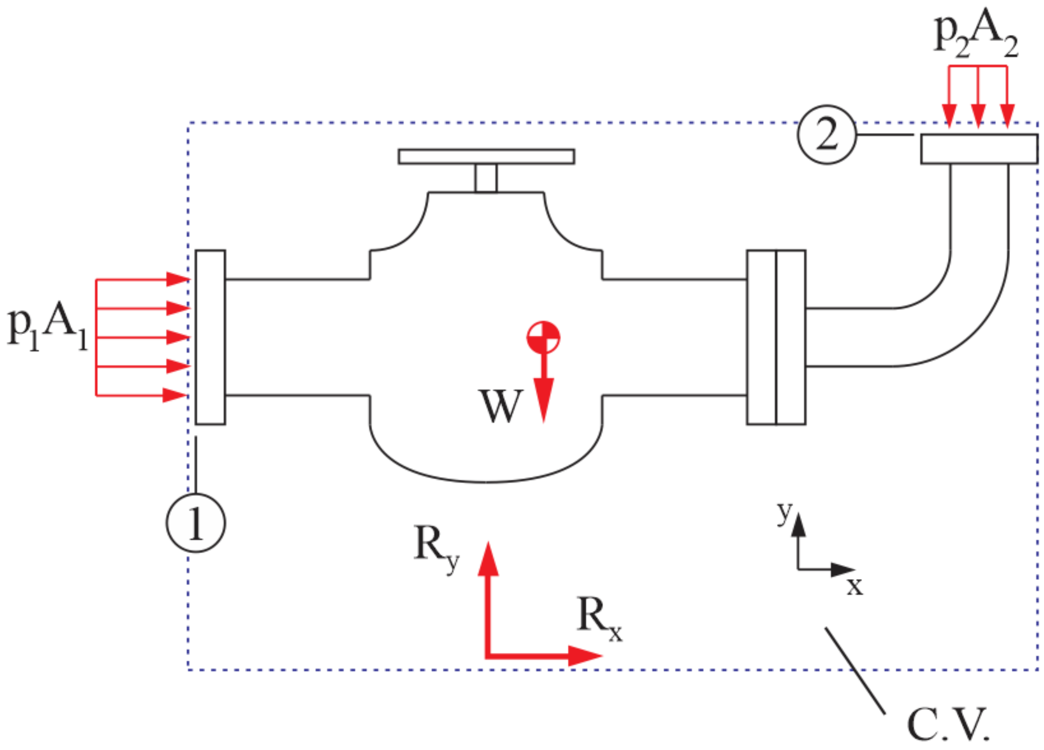
$$\sum \boldsymbol{F} = \dot{m}_2 \boldsymbol{V}_2 - \dot{m}_1 \boldsymbol{V}_1 = \dot{m}(\boldsymbol{V}_2 - \boldsymbol{V}_1)$$

All forces on C.V. — Rate of momentum: outflow – inflow

Example: Linear Momentum

- Vector equation: $\sum \boldsymbol{F} = \dot{m}(\boldsymbol{V}_2 - \boldsymbol{V}_1)$

$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$ $\quad$ $\boldsymbol{V} = u\,\boldsymbol{i} + v\,\boldsymbol{j}$



x-direction: $\overset{+}{\rightarrow}$

$$\sum F_x = \dot{m}(u_2 - u_1)$$

$u_2 = 0$ (no x-component at outlet)

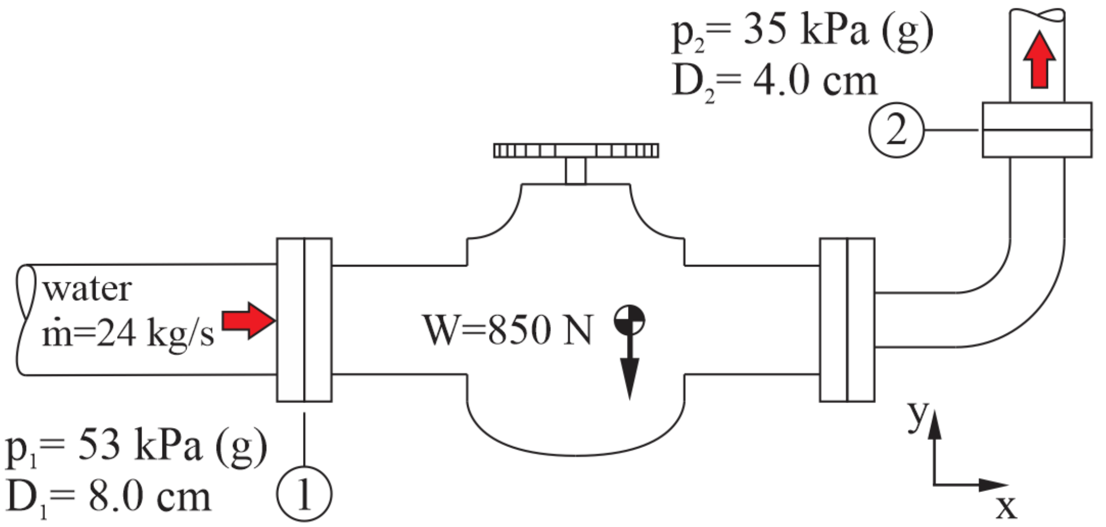
$$p_1 A_1 + R_x = \dot{m}(u_2 - u_1)$$

Solve for R_x : $\quad R_x = -p_1 A_1 - \dot{m} u_1$

- Force directions make sense?

Example: Linear Momentum

$$R_x = -p_1 A_1 - \dot{m} u_1$$



- Calculations:

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (0.08m)^2}{4} = 5.03x10^{-3} m^2$$

$$\dot{m} = \rho\, A_1\, u_1 \qquad u_1 = \frac{\dot{m}}{\rho A_1}$$

$$u_1 = \frac{24\ kg/s}{998\ kg/m^3 (5.03x10^{-3} m^2)} = 4.78\ m/s$$

$$R_x = -53x10^3 \frac{N}{m^2}(5.03x10^{-3} m^2) - 24\frac{kg}{s}\left(4.78\frac{m}{s}\right) = -266.4 - 114.8\ N = -381\ N$$

$$R_x = 381\ N \leftarrow$$ Ans.

Example: Linear Momentum

Vector equation: $\sum \boldsymbol{F} = \dot{m}(\boldsymbol{V}_2 - \boldsymbol{V}_1)$

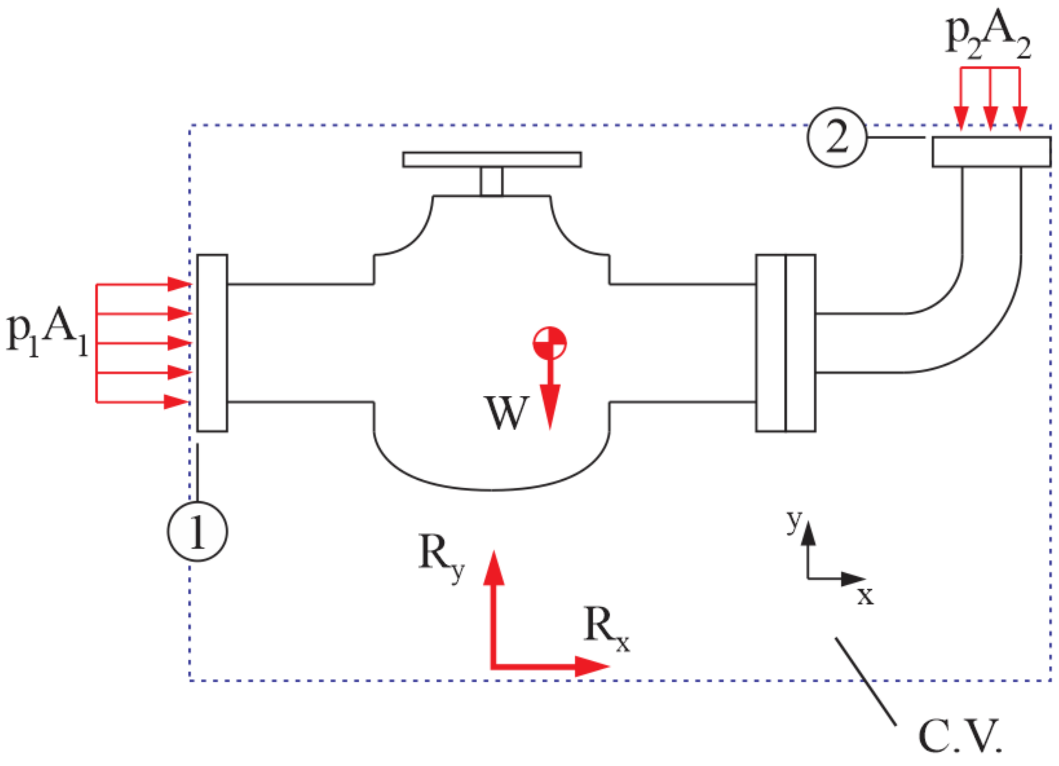
<u>y-direction:</u> + ↑

$$\sum F_y = \dot{m}(v_2 - v_1)$$

$v_1 = 0$ (no y-component at inlet)

$$-p_2 A_2 - W + R_y = \dot{m}(v_2 - v_1)$$

Solve for R_y: $R_y = p_2 A_2 + W + \dot{m} v_2$

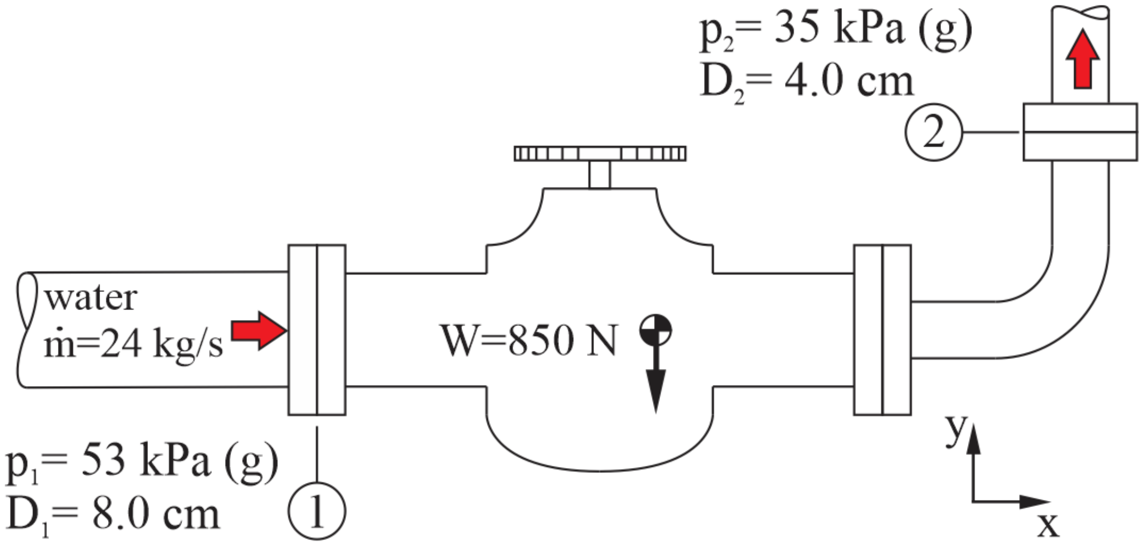


- Force directions make sense?

Example: Linear Momentum

$$R_y = p_2 A_2 + W + \dot{m} v_2$$

Calculations:



$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (0.04m)^2}{4} = 1.26x10^{-3} m^2$$

$$\dot{m} = \rho\, A_2\, v_2 \qquad v_2 = \frac{\dot{m}}{\rho A_2}$$

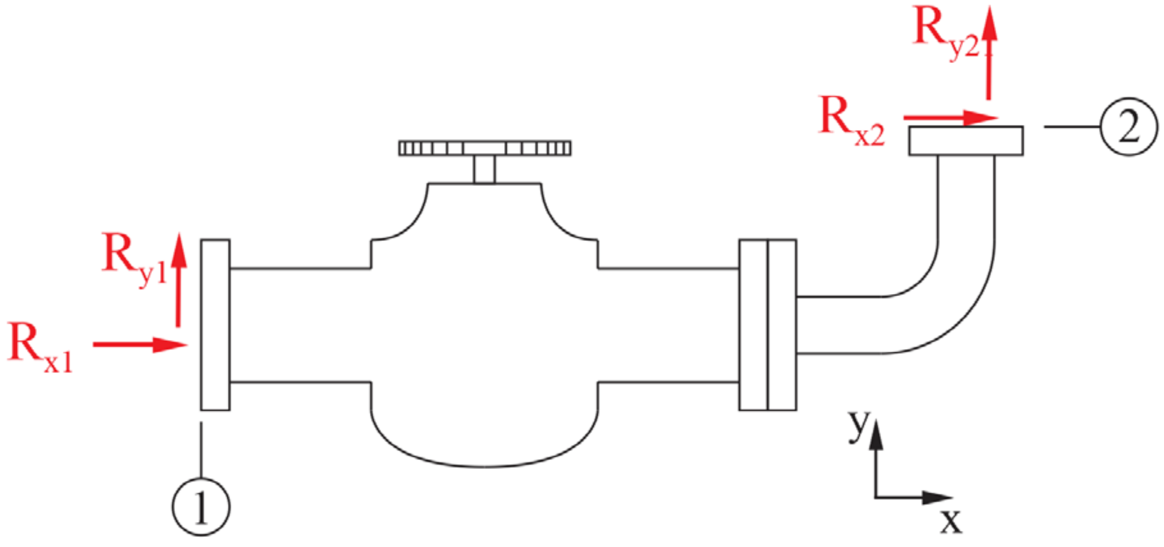
$$v_2 = \frac{24\ kg/s}{998\ kg/m^3 (1.26x10^{-3} m^2)} = 19.1\ m/s$$

$$R_y = 35x10^3 \frac{N}{m^2}(1.26x10^{-3} m^2) + 850\ N + 24 \frac{kg}{s}\left(19.1 \frac{m}{s}\right) = +1350\ N$$

$R_y = 1350\ N \uparrow$ Ans.

Example: Linear Momentum

- R_x and R_y act at the pipe flanges
- R_x and R_y are the net forces required to hold valve and pipe in place
- Linear momentum analysis can only determine *total* force in each direction:



$$R_x = R_{x1} + R_{x2} = -381\ N$$

$$R_y = R_{y1} + R_{y2} = 1350\ N$$

$R_x = 381\ N \leftarrow$

$R_y = 1350\ N \uparrow$

Streamlines/Streaklines over a Car
Source: http://6speedhaven.tumblr.com/post/25448453293

END NOTES

Presentation prepared and delivered by Dr. David Naylor.