MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis Part 6

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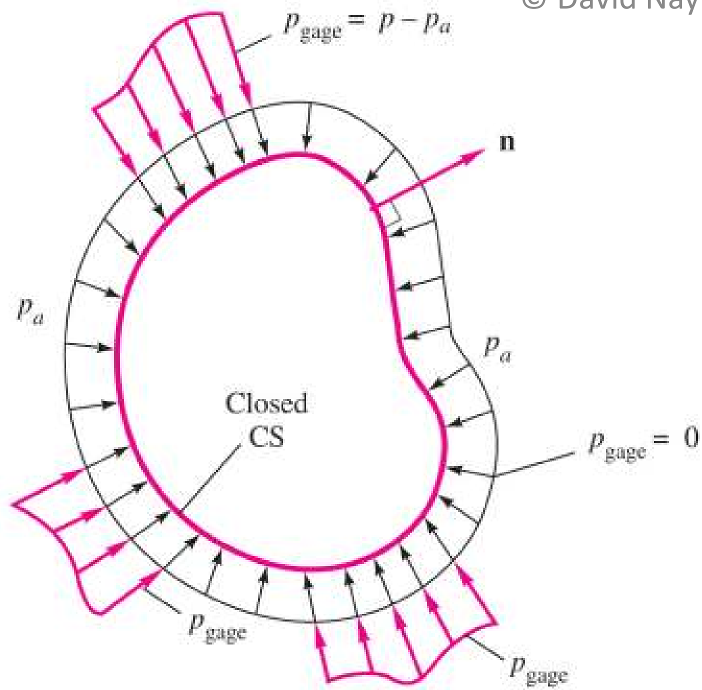
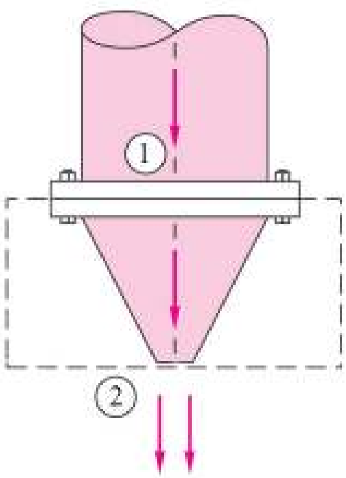
Overview

- Linear Momentum Equation continued…
- Net Pressure on a Closed Control Surface
    - use of gage pressure

- Numerical Example
    - Calculating bolt attachment forces in a nozzle.





Net Pressure Force on a Closed Control Surface

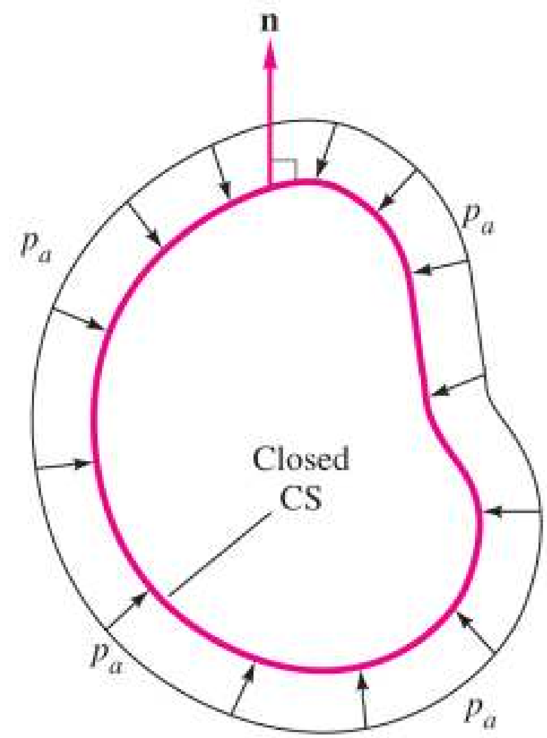
- In the application of linear momentum equation, we often have pressure forces on the control surface.
- Pressure acts normal to a control surface (c.s.) and inward.
- Since the unit normal vector **n** points outward, the net pressure force (acting inward) on a control surface is:

$$F_{press} = \int_{c.s.} p\,(-\boldsymbol{n})dA$$ where *p* is the absolute fluid pressure.

- This integral could be difficult to evaluate for complex c.v. shapes.
- Luckily, there is a “trick” to allow us to work in gage pressure. You will see that this makes the calculation the net pressure force much easier.

Net Pressure Force on a Closed Control Surface

- Consider an arbitrarily shaped closed c.s. in a with **uniform** pressure (p_a) on its surface.
- If the pressure is uniform, we know that:



$$F_{press} = \int_{c.s.} p_a \, (-\boldsymbol{n}) dA = 0$$

- For a closed surface, there can be no net pressure force, no matter how complex the shape.

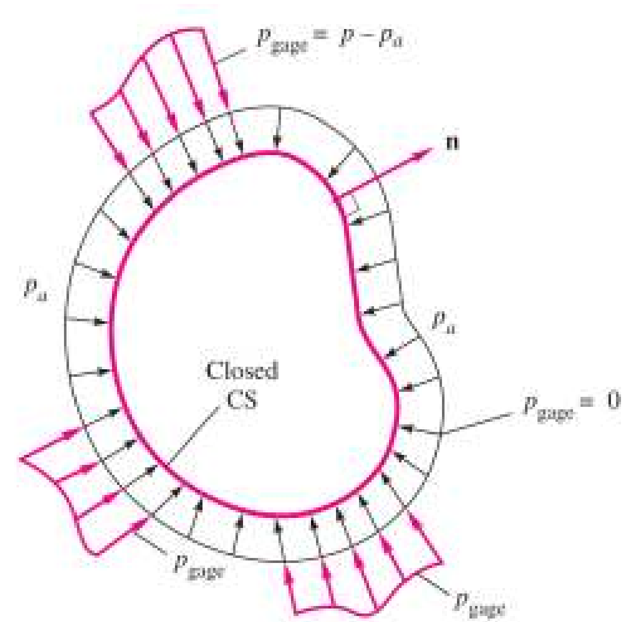
- How do we know this?

Nature takes care of the force balance for a closed surface.
If this were not true objects would move spontaneously!

Net Pressure Force on a Closed Control Surface

- Now consider a control volume with a non-uniform pressure distribution on the control surface.

$$F_{press} = \int_{c.s.} p\,(-\boldsymbol{n})dA$$



- This problem can be simplified by subtracting the uniform pressure, p_a.

$$F_{press} = \int_{c.s.} (p - p_a)\,(-\boldsymbol{n})dA = \int_{c.s.} p_{gage}\,(-\boldsymbol{n})dA$$

- This is equivalent because $\int_{c.s.} p_a\,(-\boldsymbol{n})dA = 0$

- We can integrate only the regions with gage pressure that remain. Simplifies the calculation greatly!

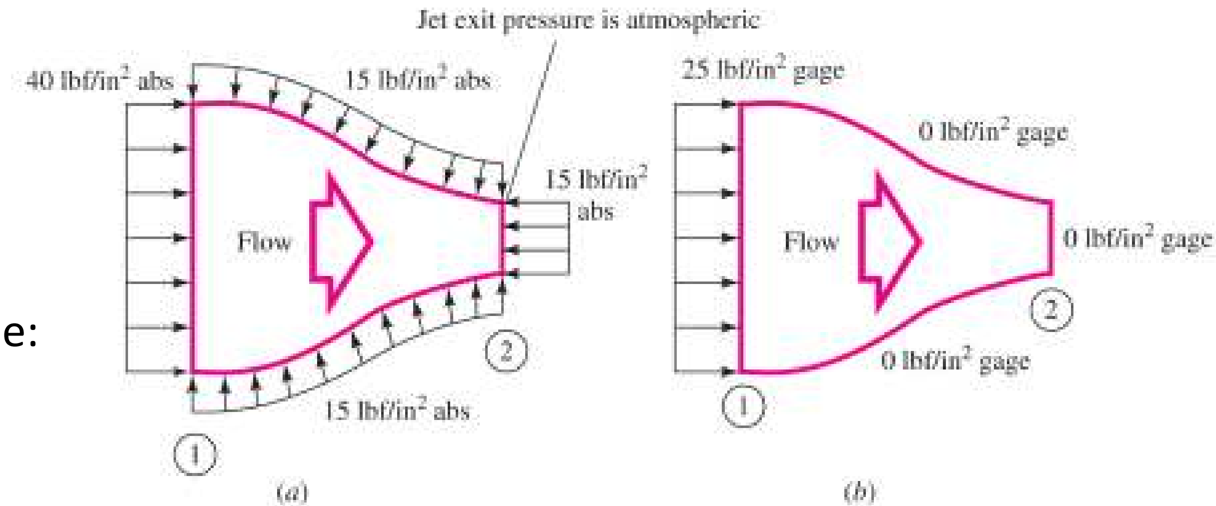
Example 3.6 (from textbook)

- The inlet of the nozzle has a fluid pressure of 40 psi (absolute). The nozzle discharges to atmosphere at p_a=15 psi.
- Press. distribution in Fig. (a) can be converted to Fig. (b), in terms of gage pressure.
- If the inlet diameter is D_1= 3 inches, the net pressure force is:

$$F_{press} = \int_{c.s.} p_{gage} (-\boldsymbol{n}) dA$$

Thus, net pressure on the control volume:

$$F_{press} = 25 \frac{lb}{in^2} \frac{\pi}{4} (3in)^2 = 177\ lb \rightarrow$$

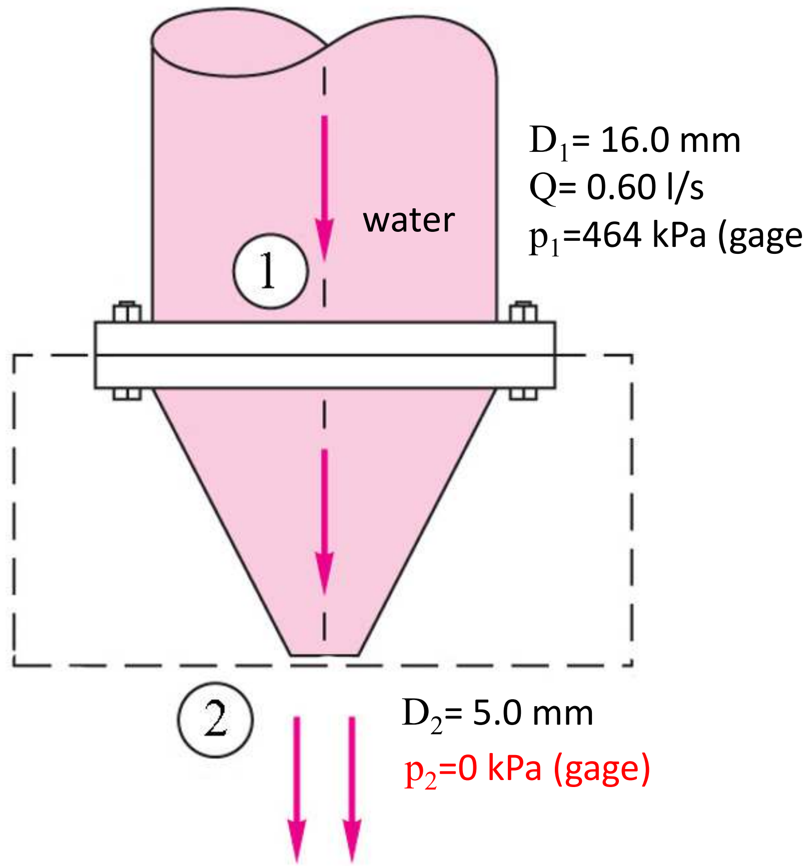


- Given this “trick”, we can now solve a linear momentum problems more easily!

Example

Water at 20°C flows through a vertical nozzle at a steady flow rate of 0.60 litres per second. The nozzle inlet and outlet diameters are D_1=16.0 mm and D_2=5.0 mm. The nozzle has a mass of 0.10 kg and the water contained in the nozzle has a mass of 3.0×10^{-3} kg. The gage pressure at section 1 is p_1=464 kPa. The nozzle discharges to atmospheric pressure.

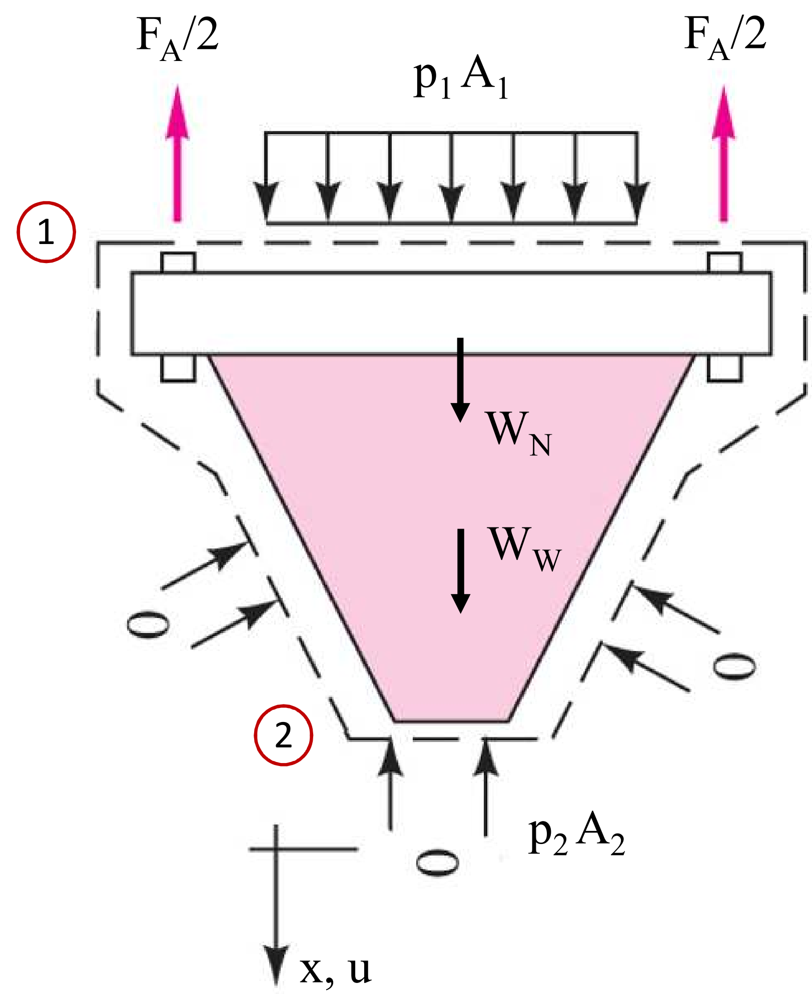
Calculate the force in the flange bolts required hold the nozzle in place.



Important!: You can set $p=p_{atm}$ where the flow discharges to atmosphere.

Example

- Free Body Diagram
- F_A is the bolt anchoring force
- W_N is the weight of the nozzle
- W_W is the weight of the water in the nozzle (between (1) and (2))
- p_1A_1 is the pressure force at (1)
- p_2A_2 is the pressure force at (2)

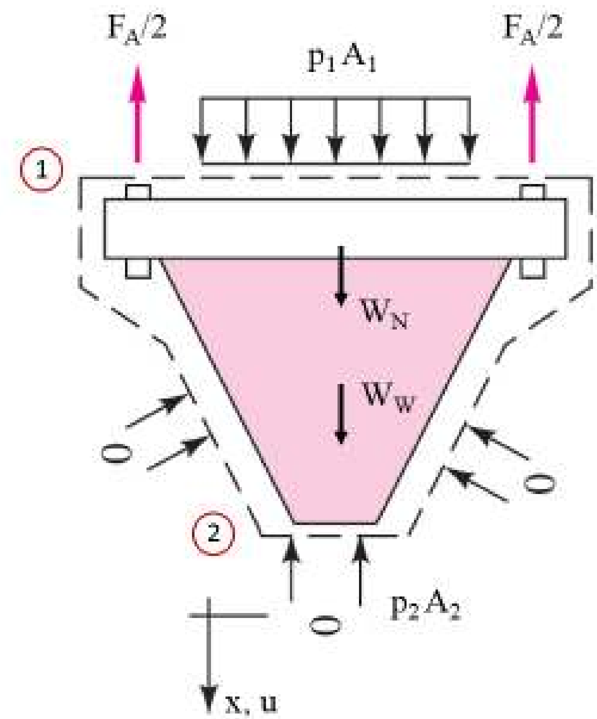


- All pressures are expressed in gage.
- So, the net pressure force can be calculated by integrating the gage pressure at (1) i.e., p_1A_1

Example

- For steady flow, the linear momentum equation with one inlet (1) and one outlet (2) is:

$$\sum \boldsymbol{F} = \dot{m}_2 \boldsymbol{V}_2 - \dot{m}_1 \boldsymbol{V}_1$$



- Note the x co-ordinate in the downward direction.
- Applying linear momentum:

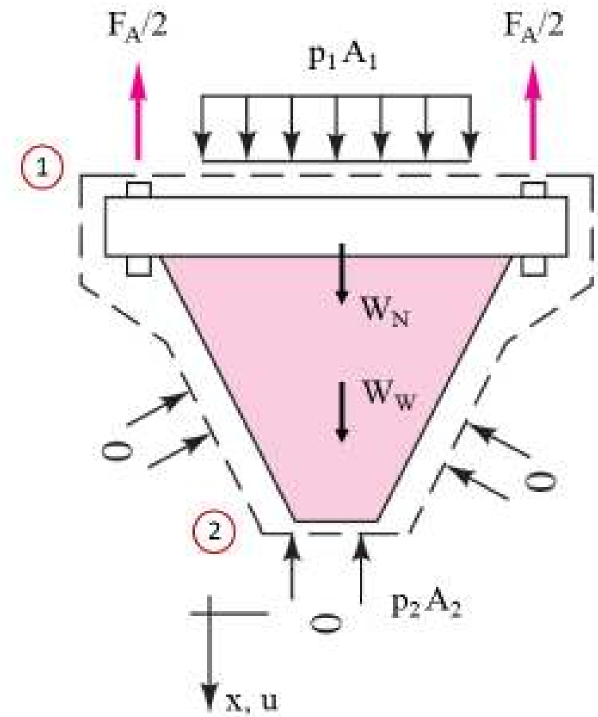
$$-F_A + W_N + W_w + p_1 A_1 - p_2 A_2 = \dot{m}_2\, u_2 - \dot{m}_1\, u_1$$

- Conservation of mass: $\dot{m}_1 = \dot{m}_2 = \dot{m} = \rho\, Q$ (Q= 0.6 litres/s)
- Solving for the bolt anchoring force, F_A

Example

$$F_A = +W_N + W_w + p_1A_1 - p_2A_2 - \dot{m}(u_2 - u_1)$$

- Stop here for a moment. Does this make sense?
  - weight of the nozzle and water increases F_A ☺
  - p_1 increases F_A, p_2 decreases F_A ☺
  - change in momentum of the water reduces F_A ☺
    ($u_2 > u_1$, like the action of a jet engine)



- Substituting numerical values:

$$\dot{m} = \rho A_1 u_1 = \rho\, Q = \left(998\ \frac{kg}{m^3}\right) 0.6 \frac{l}{s}\ \left(10^{-3}\ \frac{m^3}{l}\right) = 0.599 \frac{kg}{s}$$

Example

- Fluid velocity at (1):

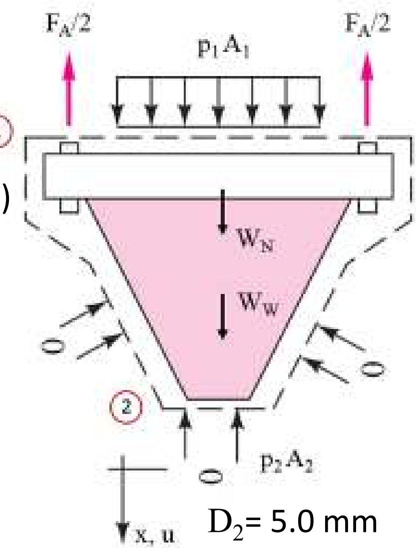
$$u_1 = \frac{Q}{A_1} = \frac{4\,Q}{\pi\, D_1^2} = \frac{4\,(0.6\times10^{-3}\frac{m^3}{s})}{\pi\ (0.016m)^2} = 2.98\,\frac{m}{s}$$

$$u_2 = \frac{Q}{A_2} = \frac{4\,Q}{\pi\, D_2^2} = \frac{4\,(0.6\times10^{-3}\frac{m^3}{s})}{\pi\ (0.005m)^2} = 30.6\,\frac{m}{s}$$

- Weight of nozzle: $W_N = m_N g = 0.1\ kg\ \left(9.81\ \frac{m}{s^2}\right) = 0.981\ N$
- Weight of water: $W_W = m_W g = 3\times10^{-3} kg \left(9.81\ \frac{m}{s^2}\right) = 2.94\times10^{-2} N$

$$F_A = +W_N + W_w + p_1A_1 - p_2A_2 - \dot{m}(u_2 - u_1)$$

D_1= 16.0 mm
Q= 0.60 l/s
p_1=464 Pa (gage)

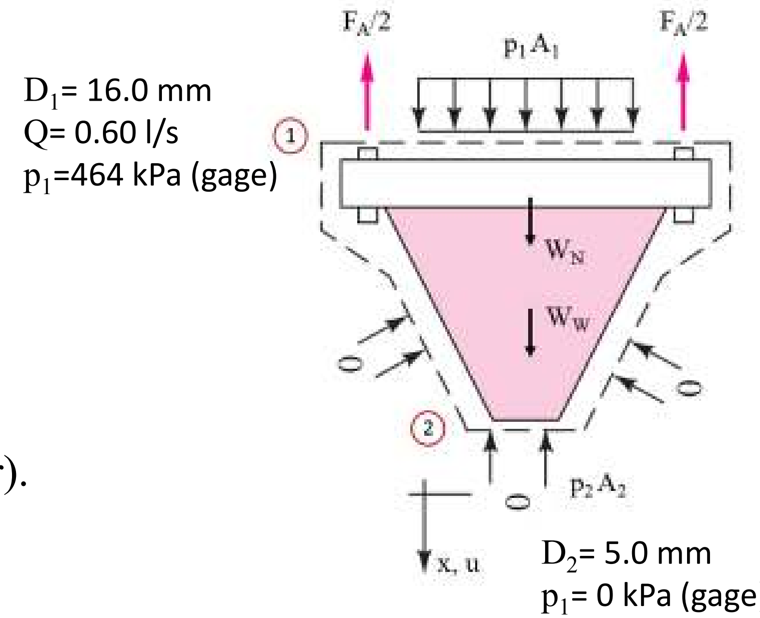


D_2= 5.0 mm
p_1= 0 Pa (gage)

Example

- So, the bolt anchoring force is:

$$F_A = +W_N + W_w + p_1A_1 - p_2A_2 - \dot{m}(u_2 - u_1)$$



$$F_A = 0.981N + 2.94\times10^{-2}N + \ 464\times10^{3}Pa\ \frac{\pi(0.016m)^2}{4}$$
$$-0\ - 0.599\frac{kg}{s}\left(30.6\frac{m}{s} - 2.98\frac{m}{s}\right)$$

$$F_A = 0.981N + 0.0294N + \ 93.3\ N\ \ - 16.54\ N = 77.8\ N \quad \text{(ans.)}$$

- The bolts will be in tension (even with the acceleration of the water).

Full Scale Visualization of Wing Tip Vortices
https://www.tumblr.com/tagged/fluid-dynamics

END NOTES

Presentation prepared and delivered by Dr. David Naylor.