MEC516/BME516: Fluid Mechanics I

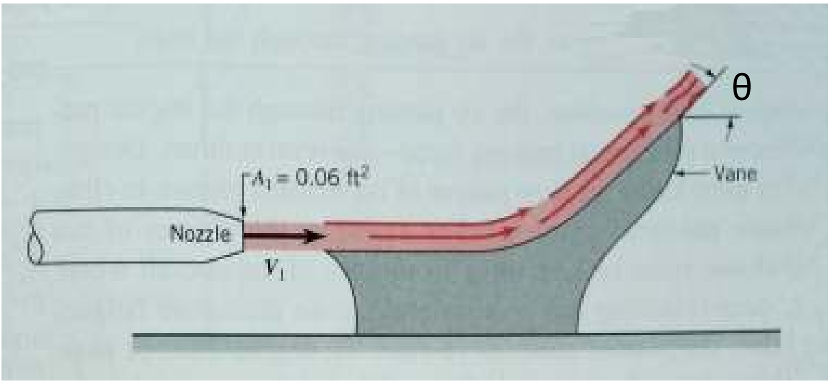
Chapter 3: Control Volume Analysis Part 5

Department of Mechanical
& Industrial Engineering

Overview

- Conservation of Momentum for a Control Volume
  - use RTT to derive the "Linear Momentum Equation"

- Numerical Example
  - Calculating fluid forces on a vane. Has applications to rotation machinery, e.g. Pelton Wheel Turbine



http://commons.wikimedia.org/wiki/File:Pelton_wheel_turbine_in_Barcelona.jpg

Derivation of the Linear Momentum Equation

- Newton's second law, $\Sigma \boldsymbol{F} = m\boldsymbol{a} = \frac{d}{dt}(m\boldsymbol{V})_{sys}$

Sum of external forces = Rate of change of linear momentum of **the system**

- Note: Typeset **Bold** variables are vectors (textbook notation),

- For a non-accelerating control volume, with 1-D flow normal to the control surface boundaries, RTT gives:

$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}\int_{CV} \beta \, dm + \sum_{outlets} \beta_o \dot{m}_o - \sum_{inlets} \beta_i \dot{m}_i$$

(relates the system to the c.v.)

- The quantity of interest is $\boldsymbol{B} = m\,\boldsymbol{V}$, linear momentum. So,

$$\beta = \frac{d\boldsymbol{B}}{dm} = \boldsymbol{V}$$

Derivation of the Linear Momentum Equation

So, we get: $\frac{d}{dt}(m\boldsymbol{V})_{sys} = \frac{d}{dt}\int_{CV} \boldsymbol{V}\, dm + \sum_{outlets} \boldsymbol{V}_o\, \dot{m}_o - \sum_{inlets} \boldsymbol{V}_i \dot{m}_i$

- Now, applying Newton's second law to the system:

$$\sum F = \frac{d}{dt}(m\boldsymbol{V})_{sys}$$

$$\sum F = \frac{d}{dt}\int_{CV} \boldsymbol{V}\, dm + \sum_{outlets} \boldsymbol{V}_o\, \dot{m}_o - \sum_{inlets} \boldsymbol{V}_i \dot{m}_i$$

In this course we will be dealing mostly with steady flows:

$$\boxed{\sum F = \sum_{outlets} \dot{m}_o \boldsymbol{V}_o - \sum_{inlets} \dot{m}_i \boldsymbol{V}_i}$$

Linear Momentum Equation

For steady 1-D flows (in a non-accelerating reference frame):

$$\sum F = \sum_{outlets} \dot{m}_o \boldsymbol{V}_o - \sum_{inlets} \dot{m}_i \boldsymbol{V}_i$$

Important Notes:

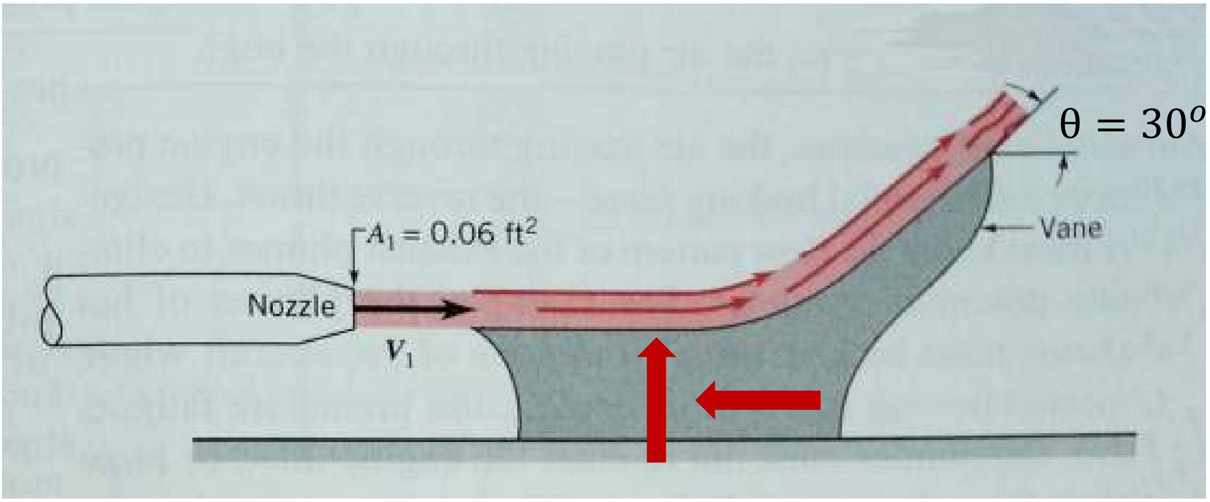
- $\boldsymbol{V}$ is a vector quantity.
- $\Sigma\ \boldsymbol{F}$ is the vector sum of all forces acting on the system, including
  - all forces at the control surfaces: pressure and viscous forces on the fluid & forces in solids cut by the c.s.
  - weight of fluid (or solid) contained in the system
- For a moving control volume, $\boldsymbol{V}$ is the relative velocity, i.e., $\boldsymbol{V} = \boldsymbol{V_r}$.

Example: Fluid forces on a stationary vane

A horizontal jet of water (at 50°F) exits a nozzle with a steady uniform velocity of 10 ft/s. The outlet area of the nozzle is 0.06 ft². A stationary vane redirects the jet upward, through an angle of θ=30°. Gravity and viscous forces are assumed to be negligible.

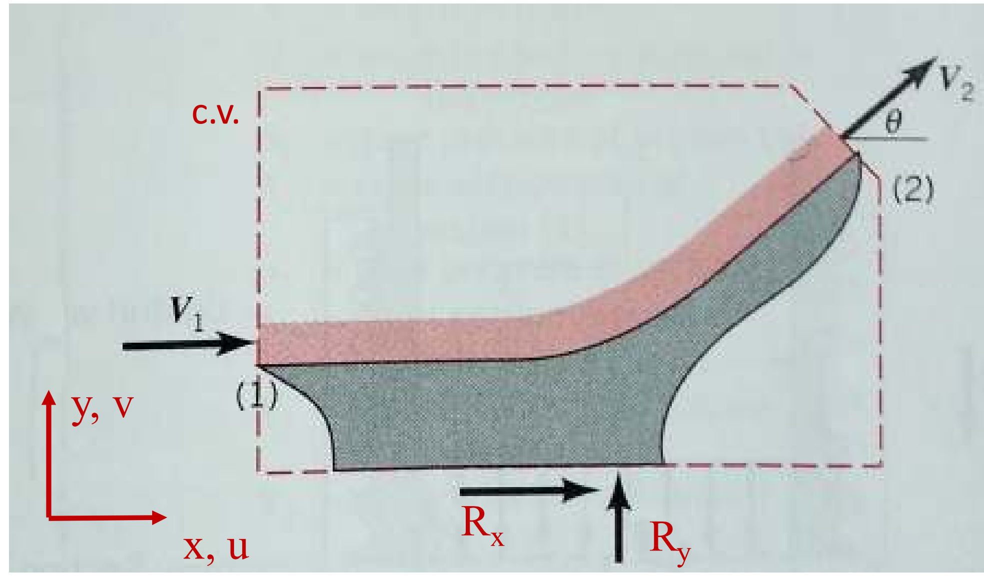
Calculate the force required to hold the vane stationary.

(What is your intuition?)



Example

- Control volume cuts through fluid & solid support.
- Free body diagram with the support reaction forces, R_x and R_y.
- The pressure at section (1) and (2) are the same, atmospheric pressure.
- With negligible gravity and viscous effect, and since $p_1=p_2$, the speed of the jet remains constant.



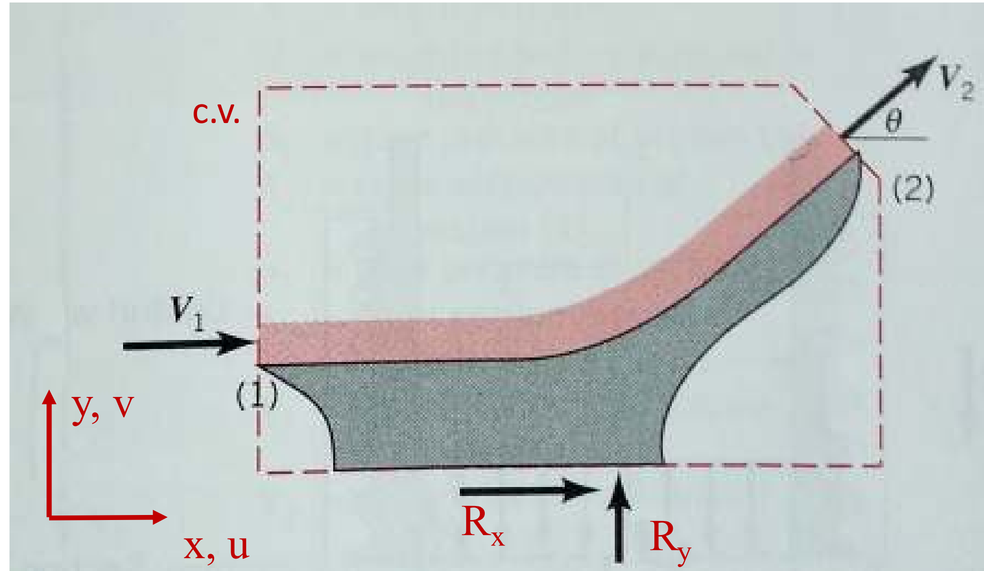
$$|\boldsymbol{V}_1| = |\boldsymbol{V}_2| = V = 10 \text{ ft/s}, \text{ also } \dot{m}_1 = \dot{m}_2 = \dot{m} = \rho A V$$

- For steady flow, the linear momentum equation with one inlet (1) and one outlet (2) is:

$$\sum \boldsymbol{F} = \dot{m}_2 \boldsymbol{V}_2 - \dot{m}_1 \boldsymbol{V}_1 = \dot{m}\, \boldsymbol{V}_2 - \dot{m}\, \boldsymbol{V}_1$$

Example

- Keeping in mind the vector nature of the forces and velocities, we get:



$$\boldsymbol{V}_1 = u_1\boldsymbol{i} + v_1\boldsymbol{j}\,;\; u_1 = V = 10\frac{ft}{s}, v_1 = 0$$

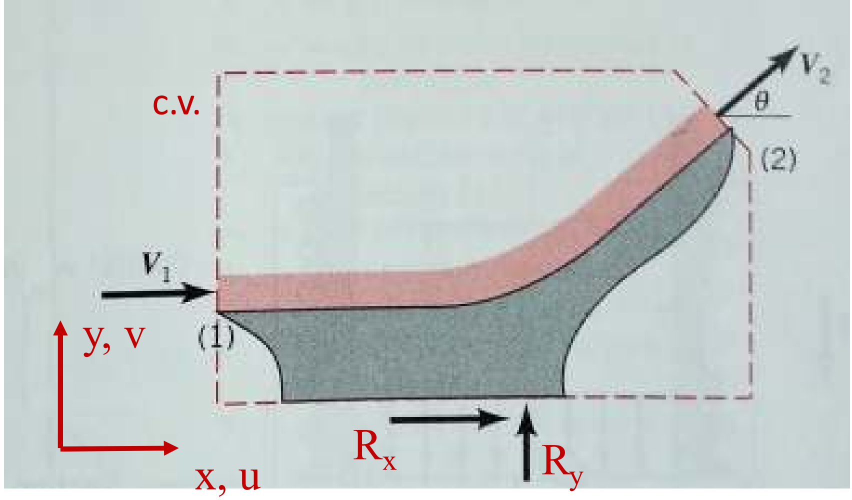
$$\boldsymbol{V}_2 = u_2\boldsymbol{i} + v_2\boldsymbol{j}\,;\;\; u_2 = V\cos\theta\,, v_2 = V\sin\theta$$

So,

$$\Sigma F_x = R_x = \dot{m}(u_2 - u_1) = \dot{m}(V\cos\theta - V) = \dot{m}V\,(\cos\theta - 1)$$

$$\Sigma F_y = R_y = \dot{m}(v_2 - v_1) = \dot{m}V\sin\theta$$

Example



Liquid water (at 50°F), $\rho = 1.94 \frac{slugs}{ft^3}$

$$\dot{m} = \rho\, A_1 V = 1.94 \frac{slugs}{ft^3}\,(0.06 ft^2)\left(10\frac{ft}{s}\right)$$

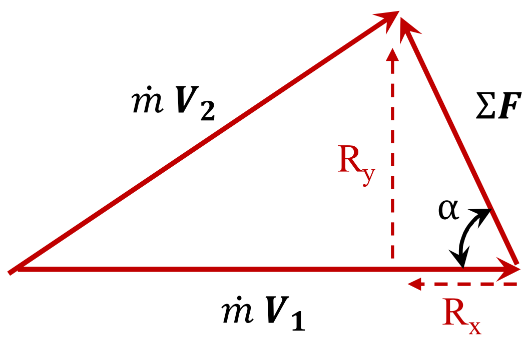
$$= 1.164 \text{ slugs/s}$$

$$R_x = \dot{m}V\,(\cos\theta - 1) = 1.164\,\frac{slugs}{s}\,(10\frac{ft}{s})\,\,(\cos 30^o - 1) = -\,1.559\,lb \qquad (R_x = +1.559 \text{ lb} \leftarrow)$$

$$R_y = \dot{m}V\sin\theta = 1.164\,\,\frac{slugs}{s}\,\left(10\,\frac{ft}{s}\right)\sin 30^0 = 5.820 \text{ lb } \uparrow$$

Magnitude of Total Reaction: $R = \sqrt{R_x^2 + R_y^2}$

Example

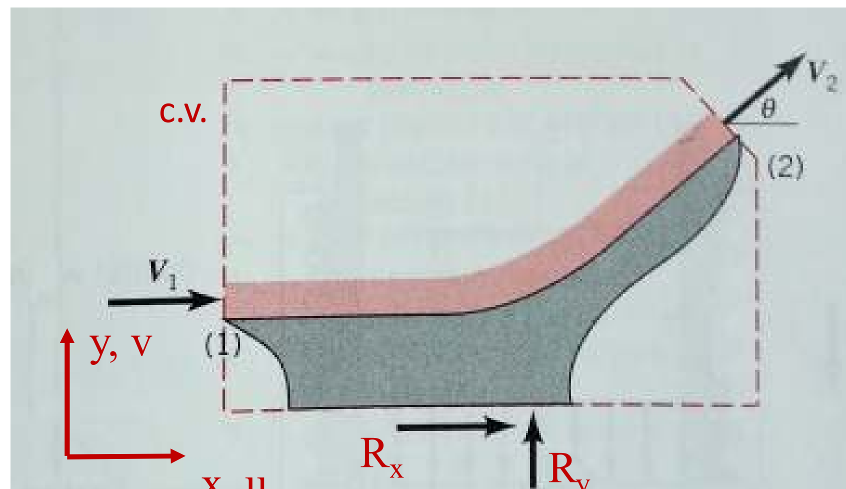


$$\Sigma \boldsymbol{F} = \boldsymbol{R} = \dot{m}\,\boldsymbol{V_2} - \dot{m}\,\boldsymbol{V_1}$$

Total Reaction $R = \sqrt{R_x^2 + R_y^2} = (1.56^2 + 5.82^2)^{1/2}$

$$R = 6.02\ lb \quad \alpha = \tan^{-1}\frac{R_y}{R_x} = 75.0^o$$

Ans.



How a Dog Drinks
Source: http://thesirens-oftitan.tumblr.com/post/80516755506/this-high-speed-footage-shows-how-a-dog-drinks

END NOTES

Presentation prepared and delivered by Dr. David Naylor.