MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis Part 4

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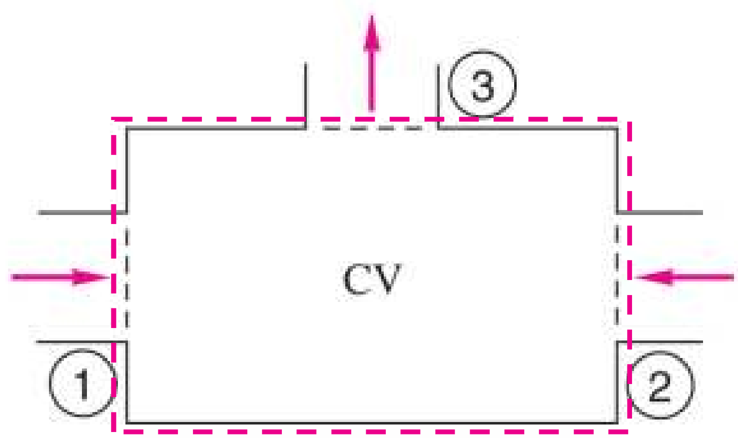
Overview

- Conservation of Mass for a Control Volume
  - using RTT to derive the “Continuity Equation”

- Numerical Example



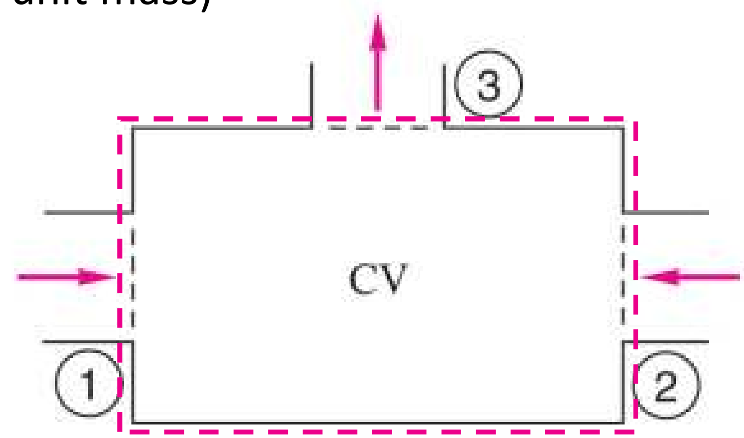
Conservation of Mass (Continuity Equation)

- For 1-D flow, Reynolds Transport Theorem (RTT) for arbitrary extensive property B is:

$$\cancelto{0}{\frac{d}{dt}(B_{SYS})} = \frac{d}{dt}\int_{CV} \cancelto{1}{\beta}\,\rho\, d\forall + \sum_{outlets} \cancelto{1}{\beta_o}\,\dot{m}_o - \sum_{inlets} \cancelto{1}{\beta_i}\,\dot{m}_i$$

$where\ \ \beta = \frac{dB}{dm}$ β is the intensive property (i.e., property per unit mass)

- Considering conservation of mass, we set $B = m$.
  Thus, $\beta = \frac{dm}{dm} = 1.$



- Principle of conservation of mass simply states:

$\frac{d}{dt}(B_{SYS}) = 0$ The mass of the **system** is constant!

Conservation of Mass (Continuity Equation)

- Conservation of mass for 1-D flow becomes:

$$0 = \frac{d}{dt}\int_{CV} \rho \, d\forall + \sum_{outlets} \dot{m}_o - \sum_{inlets} \dot{m}_i$$

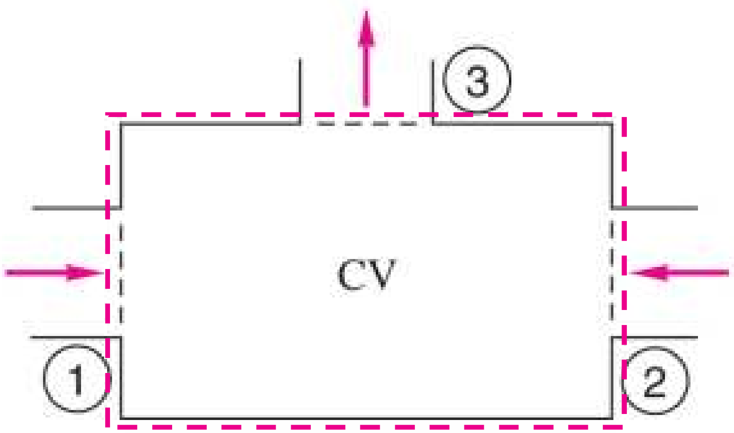
- Re-arranging, and noting that $\dot{m} = \rho \, A \, V$:

$$\frac{d}{dt}\int_{CV} \rho \, d\forall = \sum_{inlets} \rho_{in} \, A_{in} V_{in} - \sum_{out} \rho_{out} \, A_{out} V_{out}$$

Rate of accumulation of mass in the CV

Rate of mass flow **into** the CV

Rate of mass flow **out** of the CV



- This is very simple "mass" accounting. (Try replacing $\dot{m} \equiv \$$.)

Conservation of Mass (Continuity Equation)

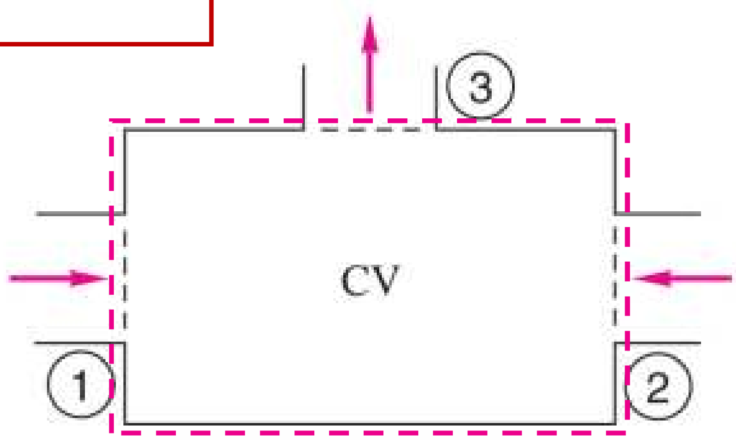
- For 1-D steady flow, $\frac{d}{dt}(m_{cv}) = 0$
- So, conservation of mass becomes:

$$\sum_{outlets} \dot{m}_o = \sum_{inlets} \dot{m}_i \qquad OR \qquad \boxed{\sum_{inlets} \rho_{in} A_{in} V_{in} = \sum_{out} \rho_{out} A_{out} V_{out}}$$

(Steady 1-D Flow)

- We will use this equation extensively.
- For the example, consider 1-D steady flow for the CV in the diagram:

$$\rho_1 A_1 V_1 + \rho_2 A_2 V_2 = \rho_3 A_3 V_3$$



If the flow is incompressible (ρ=constant):

$$A_1 V_1 + A_2 V_2 = A_3 V_3 \qquad OR \qquad Q_1 + Q_2 = Q_3$$ (volume flow rates)

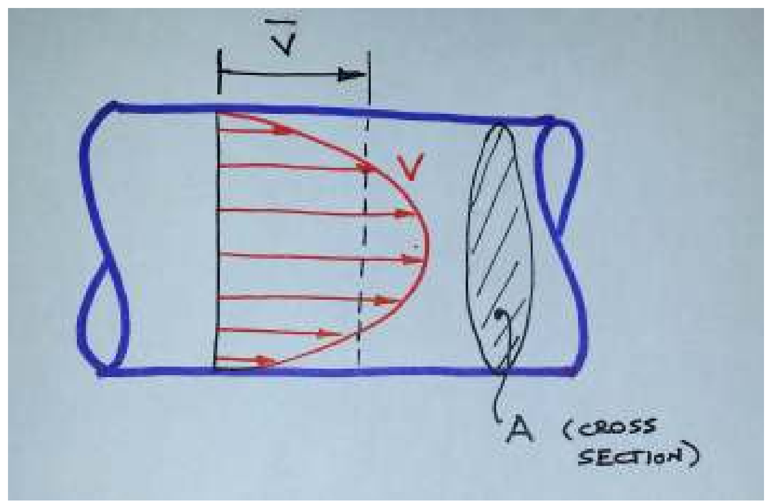
Calculating Average Velocities

- So far we have considered 1-D flows.
- Often the velocity across the sectional area A is not uniform.
- In this case, we use the average velocity $\bar{V}$ in the 1-D continuity equation.

- We calculate the average by equating the $\dot{m}$ for the 1-D flow to the $\dot{m}$ non-uniform flow:

$$\rho\, A\, \bar{V} = \int_A \rho\, V\, dA$$

$$\bar{V} = \frac{\int_A \rho\, V\, dA}{\rho\, A}$$



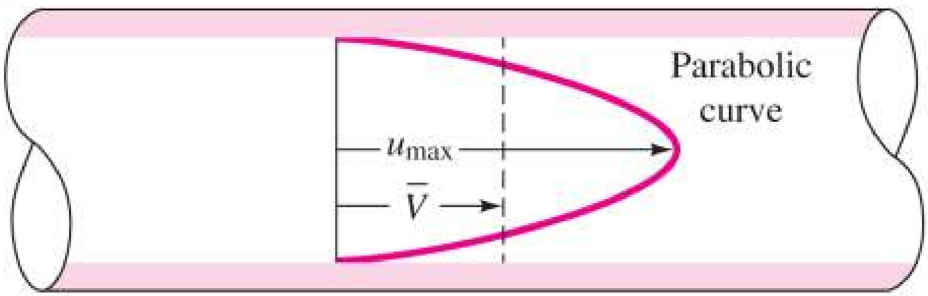
- See solved Example 3.4 in textbook.

An Aside: Laminar and Turbulent Velocity Profiles in a Pipe

- Jumping ahead… details in Chapter 6.
- Consider two fully developed flows with the same average velocity, $\bar{V}$

<u>Laminar flow (Re<2,300)</u>

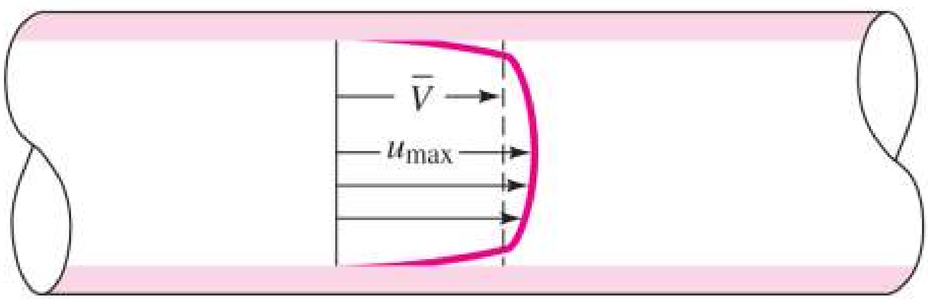
- smooth streamlines, no lateral mixing
- parabolic profile
- For round pipe: $\bar{V} = u_{max}/2$



(a)

<u>Fully Turbulent Flow (Re>10,000)</u>

- random eddies in the flow, mixing.
- much more uniform velocity profile (1-D flow is a better approx.)

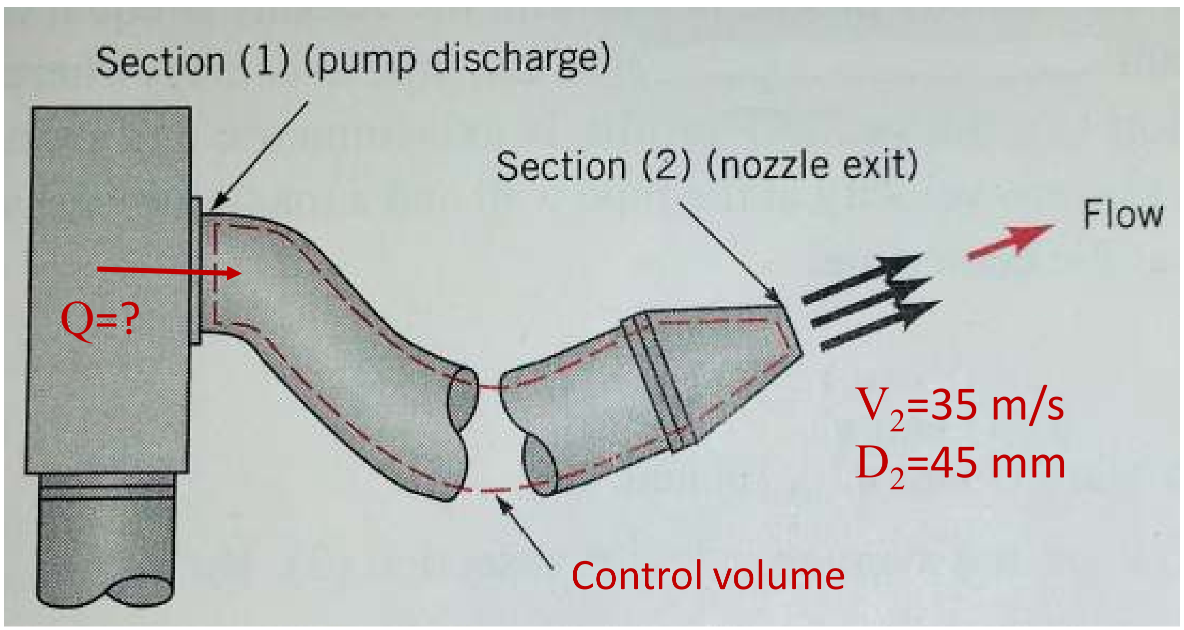


(b)

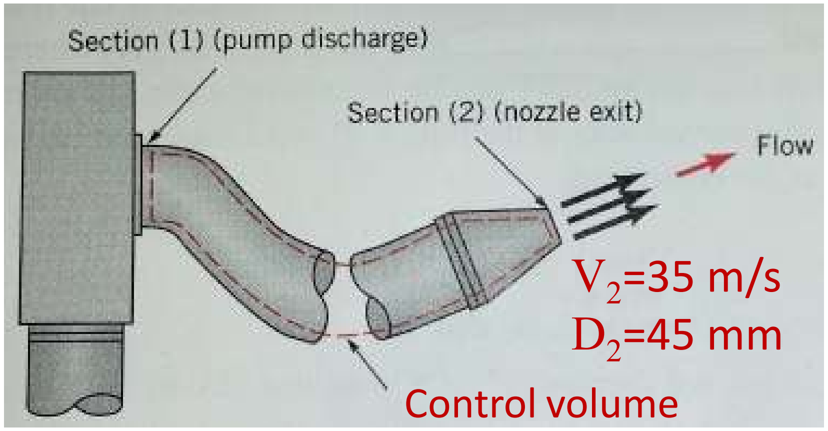
Example

Water at 20ºC is pumped steadily through a fire hose with a round nozzle. The mean jet velocity at the nozzle exit is 35 m/s. The exit of the nozzle has a diameter of 45 mm.

Calculate the flow rate Q generated by the pump in m^3/s.



Solution



Continuity equation:

$$\frac{d}{dt}\int_{CV} \cancel{\rho \, d\forall} = \sum_{inlets} \rho_{in} A_{in} V_{in} - \sum_{out} \rho_{out} A_{out} V_{out}$$

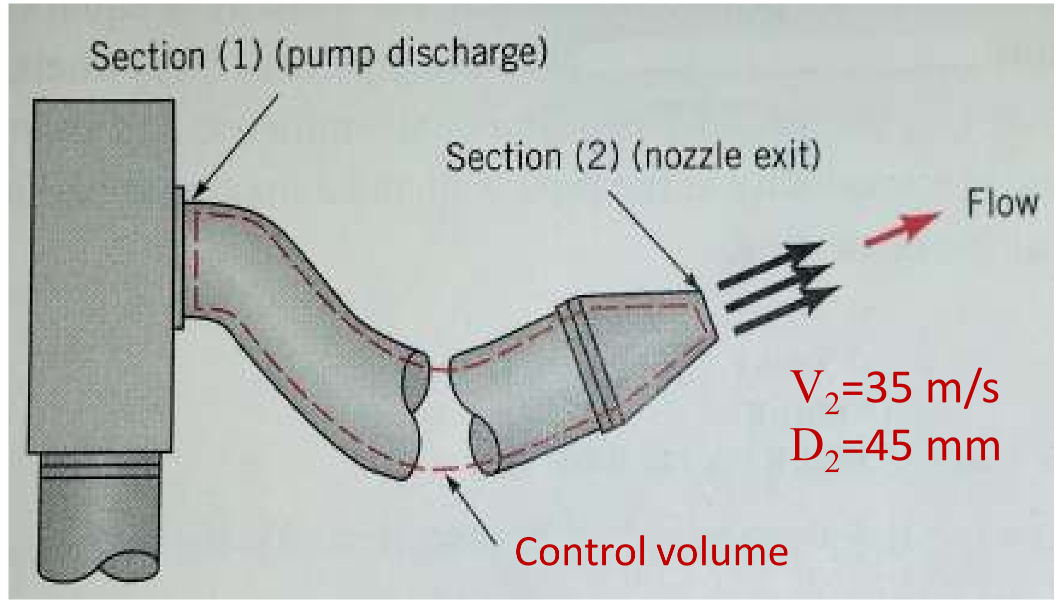
0 (steady flow)

No storage of mass in the control volume. Only one inlet at section (1) and one outlet at section (2). So,

$$\dot{m}_1 = \dot{m}_2 \qquad \rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad \rho_1 Q_1 = \rho_2 Q_2$$

Liquid water is incompressible, so: $\rho_1 = \rho_2$ Thus, $Q_1 = Q_2 = A_2 V_2$

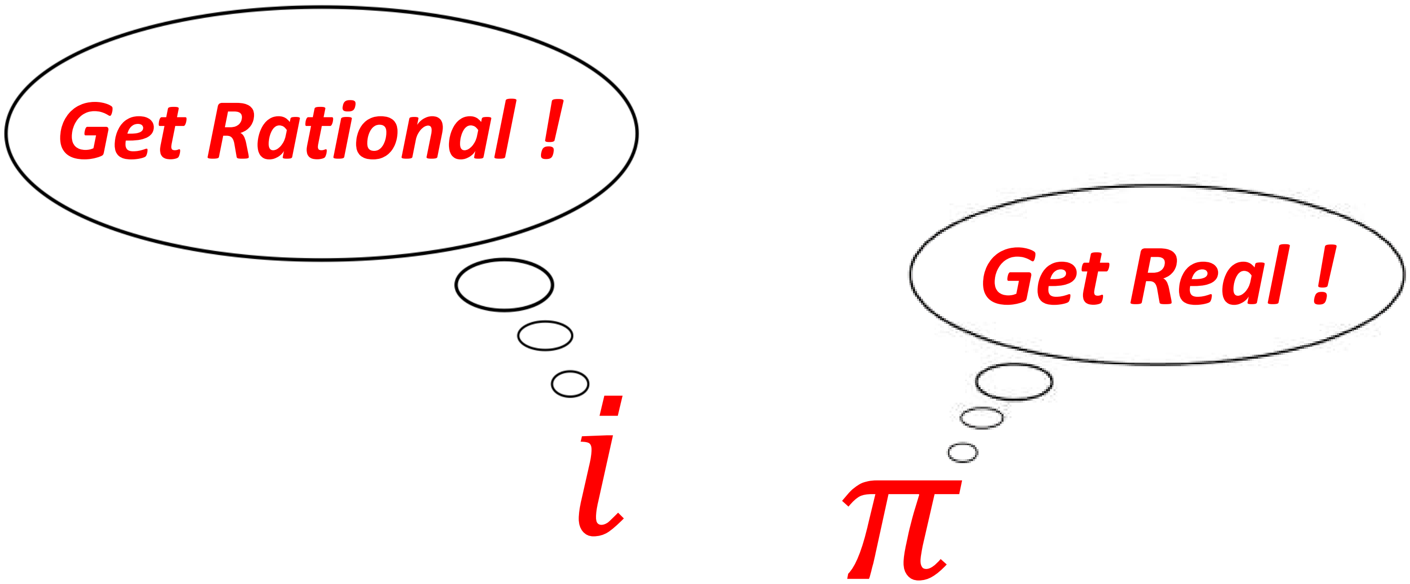
Solution



$$A_2 = \frac{\pi\, D_2^2}{4} = \frac{\pi\, (0.045m)^2}{4} = 1.59x10^{-3}\; m^2$$

$$Q = A_2\, V_2 = (1.59x10^{-3}\; m^2)\left(35\,\frac{m}{s}\right) = 5.57x10^{-2}\,\frac{m^3}{s} \qquad (55.7\,\frac{litres}{s})$$

(1 litre= 10^{-3} m^3)



END NOTES

Presentation prepared and delivered by Dr. David Naylor.