MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis Part 3

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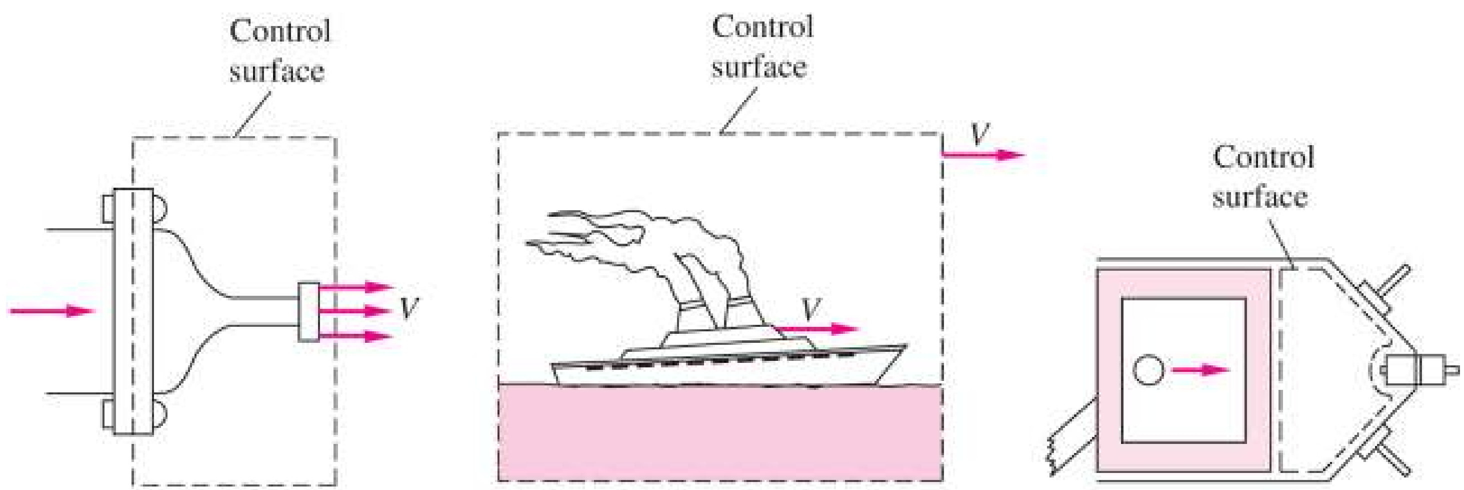
Overview

- Control Volume versus System approach
  - need for control volume analysis in fluid mechanics.
- Reynolds Transport Theorem (RTT)
  - rewriting basic laws for a defined volume in space (control volume)



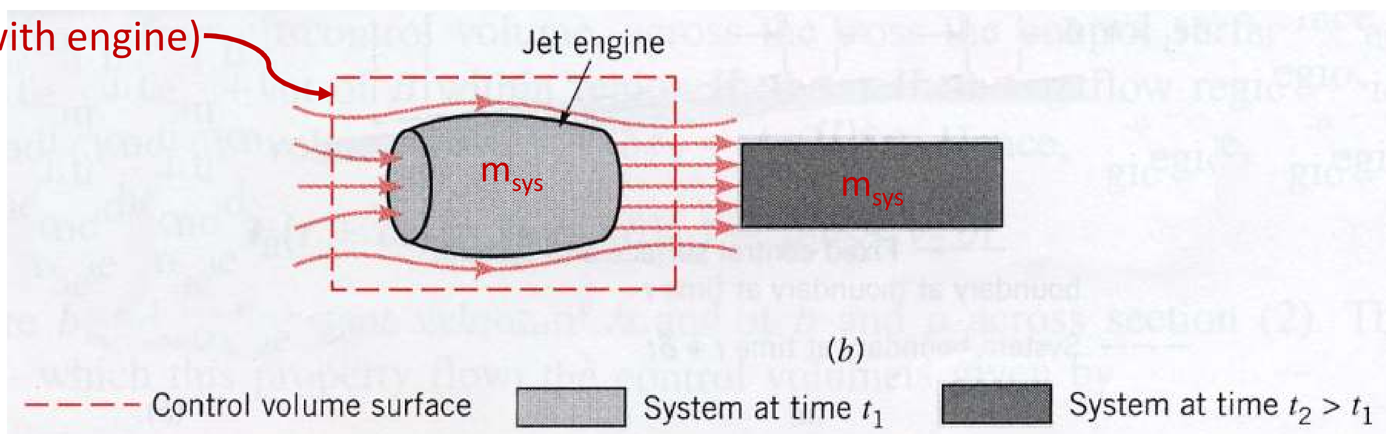
System versus Control Volume

From thermodynamics:

System: is a fixed quantity of mass, m_{sys}. It is a collection of matter of fixed identity, i.e., the same set of atoms.

Control Volume: is a volume in space through which a fluid may flow. It is a geometric identity.

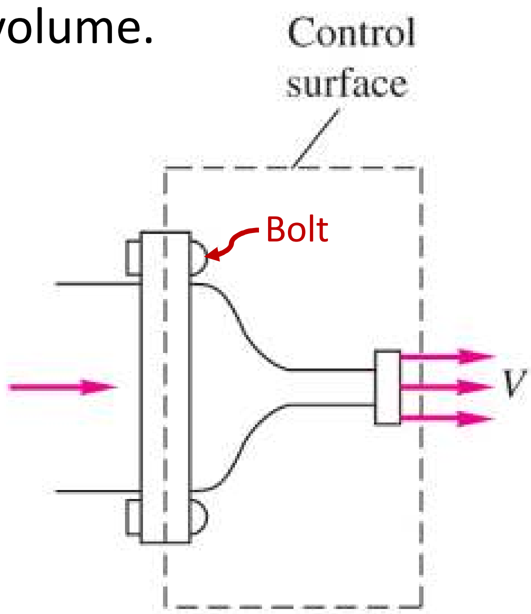
c.s. (moving with engine)



System versus Control Volume

- The laws governing motion are stated using a ***system*** approach i.e., for a fixed quantity of mass e.g., Newton's 2nd Law (F=ma), Conservation of Mass (m_{sys}=const.)

- In fluid mechanics we are often interested in analyzing a control volume.



For example, supposed we want to calculate the force needed to hold the nozzle in place (in order to size the bolts).

For this problem we need to analyze the change in momentum of the fluid that passes through the ***control volume***. So, we need to re-write $F = ma = \frac{d}{dt}(mV)$ for this example) for application to a control volume.

The general method for doing this is called ***Reynolds Transport Theorem***.

Derivation of Reynolds Transport Theorem (RTT)

- RTT is the analytical tool that relates a system to a control volume.
- But first, a little thermodynamics (review for Mech. and Ind.):

An ***Extensive Property*** depends on the mass of the system:
e.g., mass *m (kg)*, volume $\forall$ (m³), energy *E (J)*
An ***Intensive Property*** is independent of the mass of the system:
e.g., pressure, specific volume *v* (m³/kg), specific energy *e* (J/kg)

- Let B be any *extensive* property of the fluid,
  - e.g. mass (m), momentum (mV), total energy (E)
- Let β be the corresponding *intensive* property.

Thus: $$\beta = \frac{dB}{dm}$$

Derivation of Reynolds Transport Theorem (RTT)

$$\beta = \frac{dB}{dm}$$

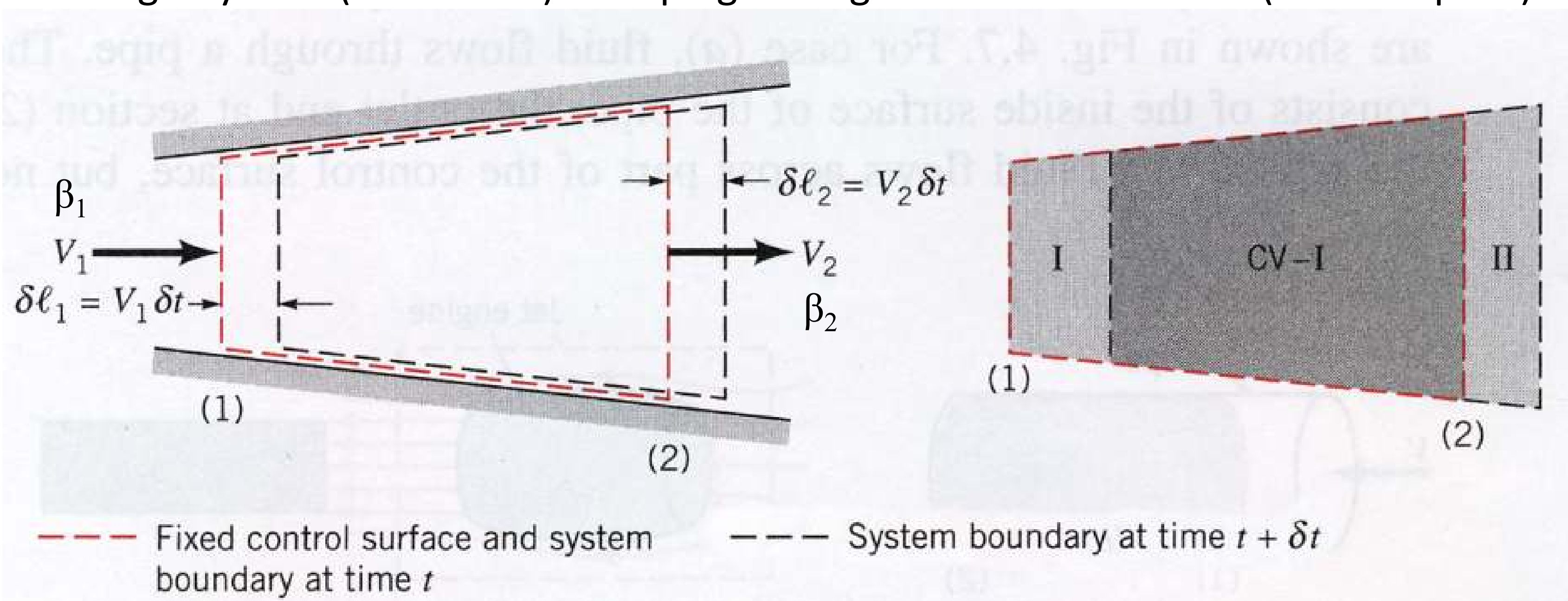
Examples:

Mass:	$B = m$ (kg)	then	β = 1 (dimensionless)
Momentum:	$B = m\boldsymbol{V}$ (kg m/s)	then	β = $\boldsymbol{V}$ (m/s)
Energy:	$B = E$ (J)	then	β = e (J/kg)

Note: These "examples" were chosen for a reason. We can set β=1 for considering conservation of mass, β=$\mathbf{V}$ for conservation of momentum, β=e for conservation of energy (for a control volume).

Derivation of Reynolds Transport Theorem (RTT)

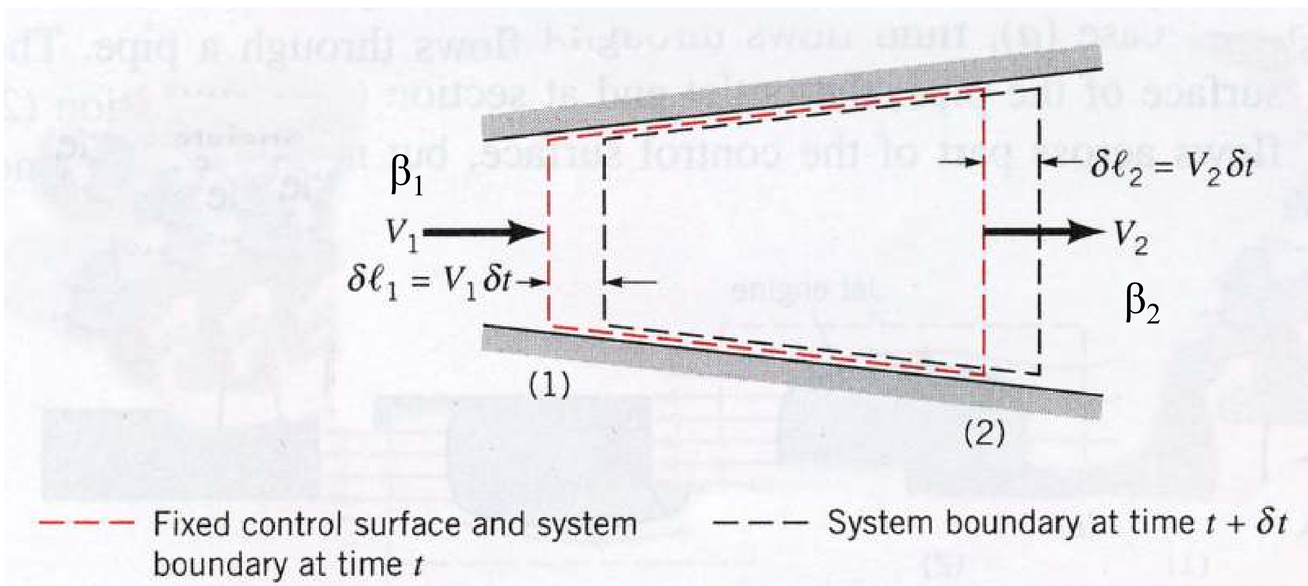
- General integral derivation of RTT in the textbook is difficult to follow.
- To simplify the derivation of RTT, we will start with a simplified case:
  - 1-D unsteady flow passing through a stationary control volume (a variable area pipe).
  - Inlet flow (1) and outlet flow (2) are normal to the control surfaces
- Considering a system (fixed mass) sweeping through the control volume (fixed in space).



Derivation of Reynolds Transport Theorem (RTT)

There are three sources of change in B (our arbitrary property) in the control volume:

- Time rate of change of B inside the control volume: $\frac{d}{dt}(B_{CV})$
- Time rate of outflow of B at control surface (2): $\beta_2 \dot{m}_2$
- Time rate of inflow of B at control surface (1): $\beta_1 \dot{m}_1$

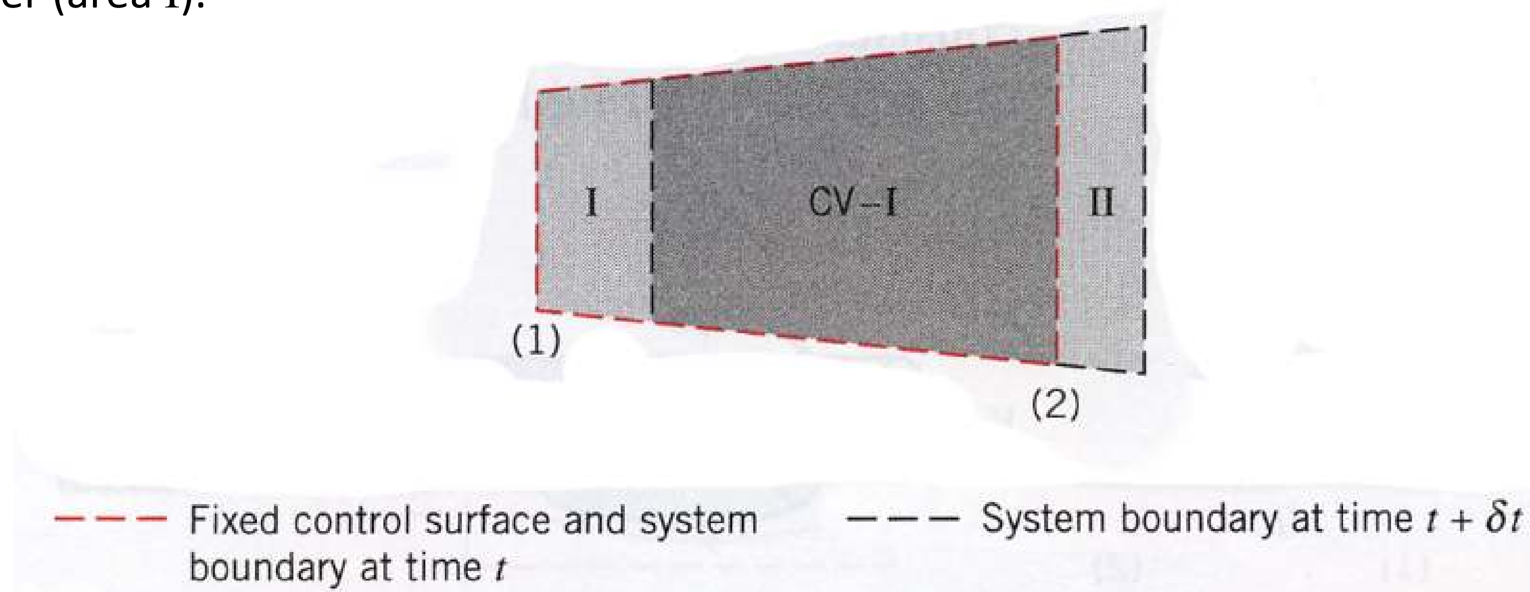


Derivation of Reynolds Transport Theorem (RTT)

At time t the system and the control volume are coincident (red dashed line): $\mathrm{SYS} \equiv \mathrm{CV}$

At time t+δt the system has moved to the right (black dashed line): $\mathrm{SYS} \equiv \mathrm{CV} - \mathrm{I} + \mathrm{II}$

Note that in the time interval δt, the system (SYS) has gained outflow sliver (area II) and has lost inflow sliver (area I).



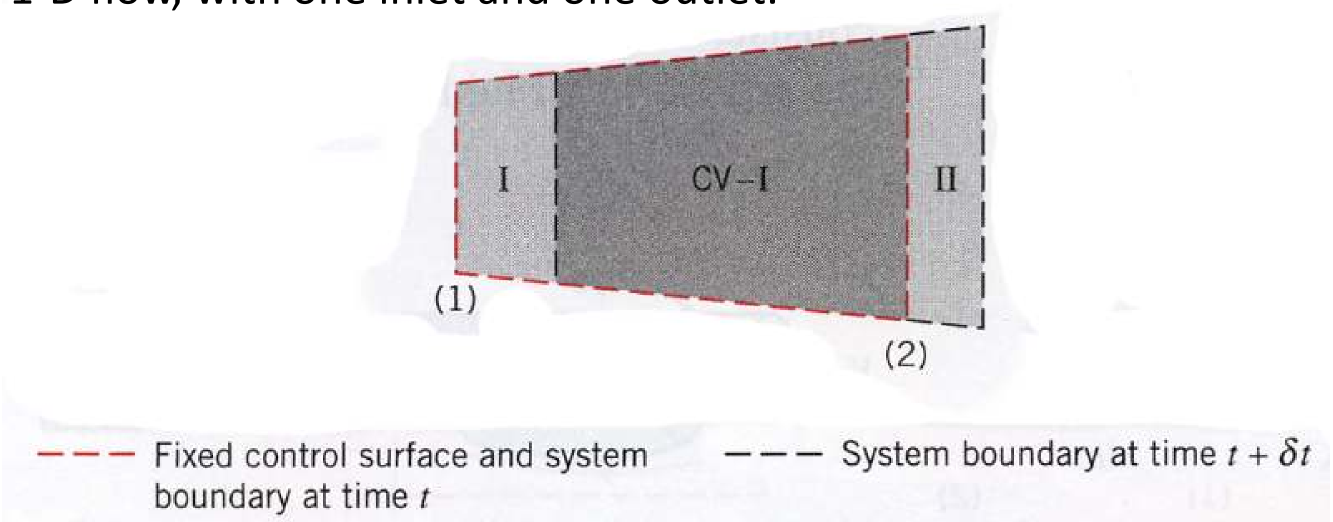
Derivation of Reynolds Transport Theorem (RTT)

Thus, in the limit as $\delta t \rightarrow 0$, the rate of change in the B of the system is:

$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}(B_{CV}) + \beta_2 \dot{m}_2 - \beta_1 \dot{m}_1$$

outflow inflow

This result is for 1-D flow, with one inlet and one outlet.



Derivation of Reynolds Transport Theorem (RTT)

- This result can be generalized for any number of inlet and outlet flows.

$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}\int_{CV} \beta \, dm + \sum_{outlets} \beta_o \dot{m}_o - \sum_{inlets} \beta_i \dot{m}_i$$

- This is Eq. (3.18) in the textbook.

- Eq. 3.18 is restricted to a fixed control volume, with 1-D flow normal to the control surface boundaries.

- In this course, the transient term will be normally be zero, i.e., steady flow.

Derivation of Reynolds Transport Theorem (RTT)

- For example, consider steady 1-D flow with 2 inflow and 3 outflows sections.

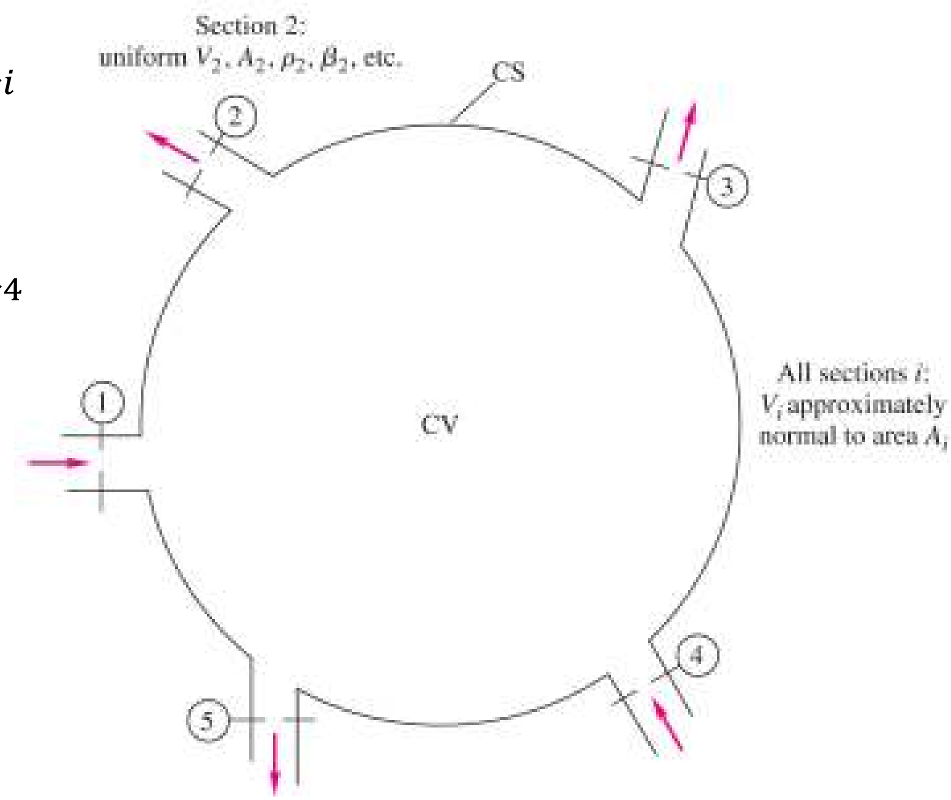
$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}\int_{CV} \beta \, dm + \sum_{outlets} \beta_o \dot{m}_o - \sum_{inlets} \beta_i \dot{m}_i$$

zero (steady flow)

$$\frac{d}{dt}(B_{SYS}) = \beta_2 \dot{m}_2 + \beta_3 \dot{m}_3 + \beta_5 \dot{m}_5 - \beta_1 \dot{m}_1 - \beta_4 \dot{m}_4$$

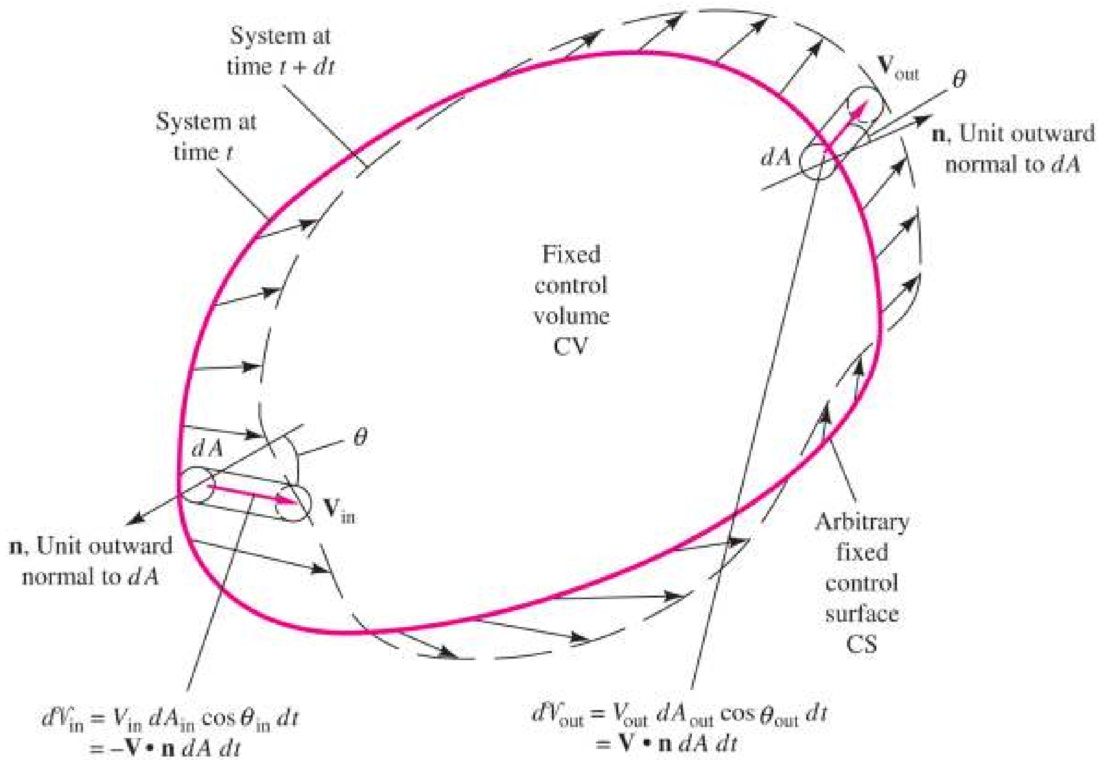
- Recall: $\dot{m} = \rho A V$
- So, for this control volume RTT gives:

$$\frac{d}{dt}(B_{SYS}) = \beta_2 (\rho A V)_2 + \beta_3 (\rho A V)_3 + \beta_5 (\rho A V)_5 - \beta_1 (\rho A V)_1 - \beta_4 (\rho A V)_4$$



Derivation of Reynolds Transport Theorem (RTT)

- So, far we have assume 1D flow, with the flow normal to the control surfaces.
- RTT can be generalized to remove these restrictions by repeating the analysis on the fixed control volume, shown below.



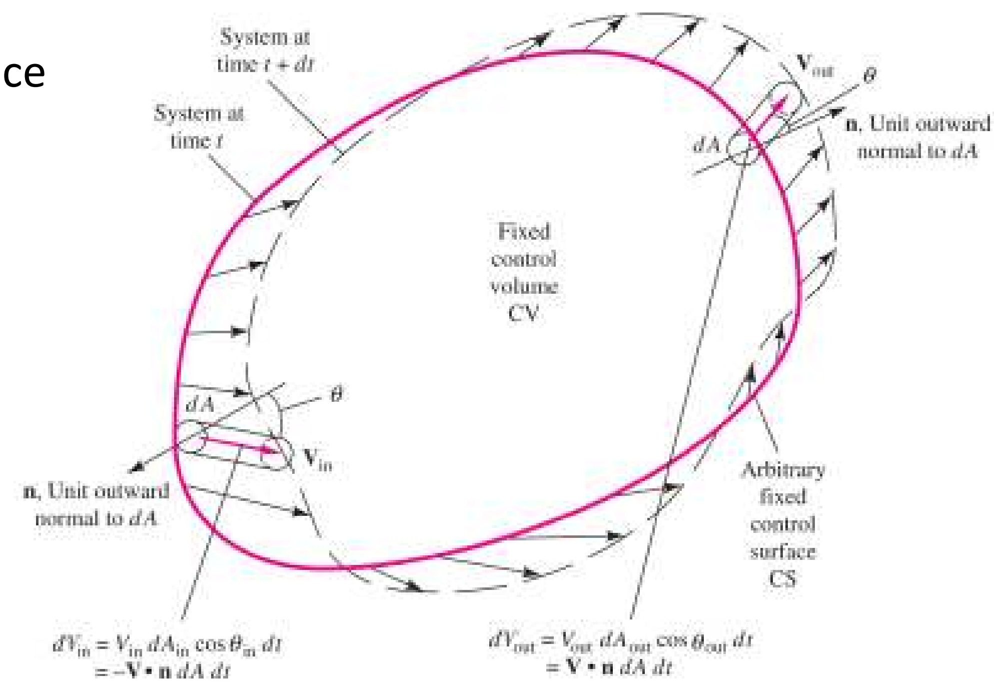
Derivation of Reynolds Transport Theorem (RTT)

- For completeness, the general expression of RTT for a fixed non-deforming control volume:

$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}\int_{CV} \beta\, \rho\, d\forall + \int_{CS} \beta\, \rho\, (\boldsymbol{V}\cdot\boldsymbol{n})dA \qquad \text{Eq. (3.12)}$$

- Flow has variable properties at control surface and $\mathbf{V}$ is not normal to the c.s.

We will use the simplified 1-D version in this course, Eq. (3.18). Also, normally, the unsteady term will be zero.

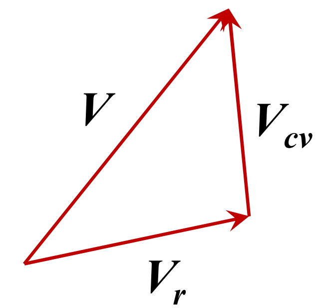


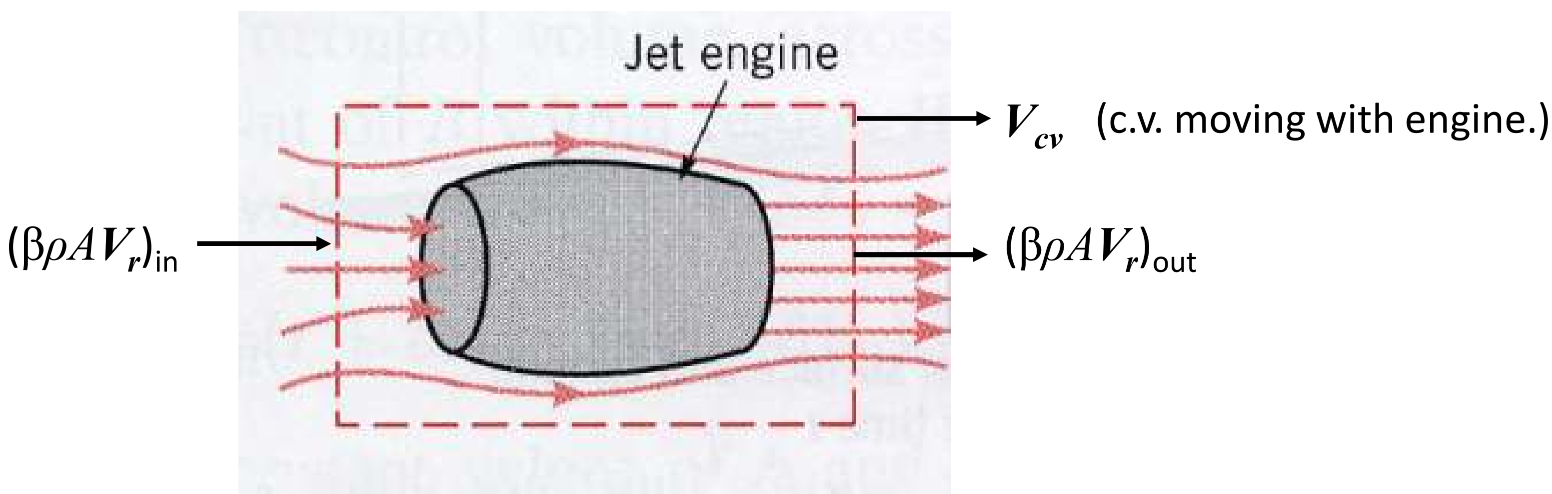
Reynolds Transport Theorem (RTT) for a Moving Control Volume

- Consider 1-D flow, but a control volume moving at constant velocity $\boldsymbol{V_{cv}}$.
- In this case, we write RTT using the relative velocity $(\boldsymbol{V_r})$ between the fluid and control volume.

$$\boldsymbol{V} = \boldsymbol{V_r} + \boldsymbol{V_{cv}}$$

- $\boldsymbol{V}$ is the absolute velocity vector.



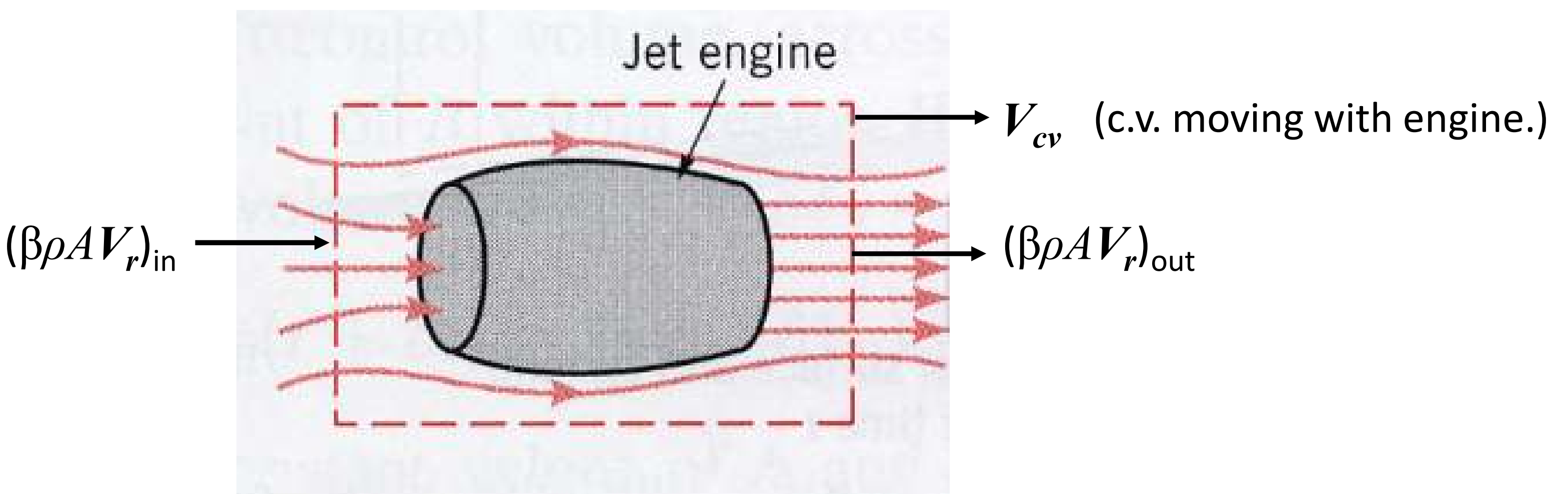


Reynolds Transport Theorem (RTT) for a Moving Control Volume

- RTT for the moving control volume with 1-D flow becomes:

$$\frac{d}{dt}(B_{SYS}) = \frac{d}{dt}(B_{CV}) + (\beta\rho A\boldsymbol{V_r})_{out} - (\beta\rho A\boldsymbol{V}_r)_{in}$$

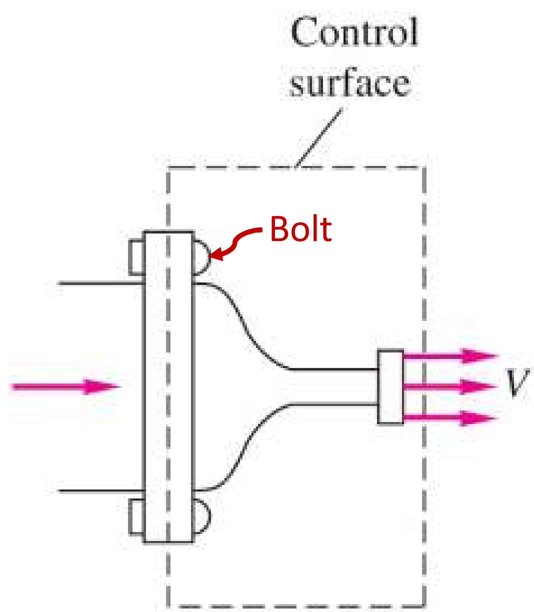
where $\boldsymbol{V}_r$ is the relative velocity between the fluid and engine: $\boldsymbol{V_r} = \boldsymbol{V} - \boldsymbol{V_{cv}}$



Derivation of Reynolds Transport Theorem (RTT)

- We will not cover the textbook sections on Moving Deformable control volumes and Non-inertial Reference Frames. See Chapter Summary on Blackboard.

- In the next videos we will apply RTT to:
  - conservation of mass
  - conservation of momentum (forces on control volumes)



END NOTES

Presentation prepared and delivered by Dr. David Naylor.

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