MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis Part 2

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Overview

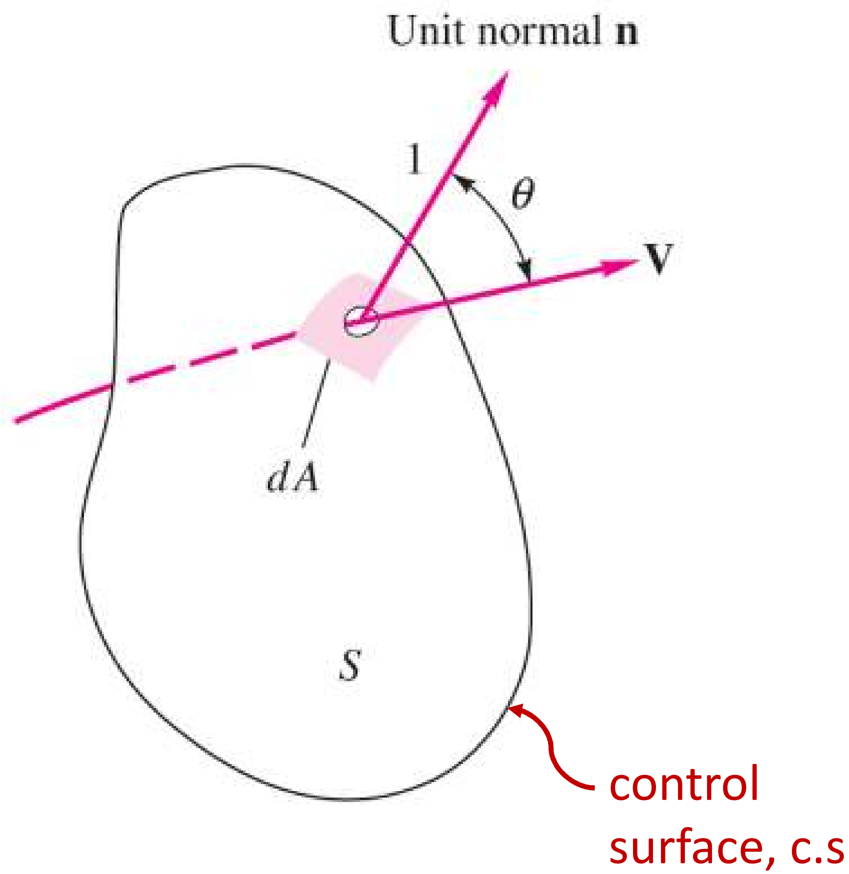
Some basic definitions:

- Volume Flow Rate, Q
- Mass Flow Rate, $\dot{m}$

  - For one-dimensional (1-D) flow

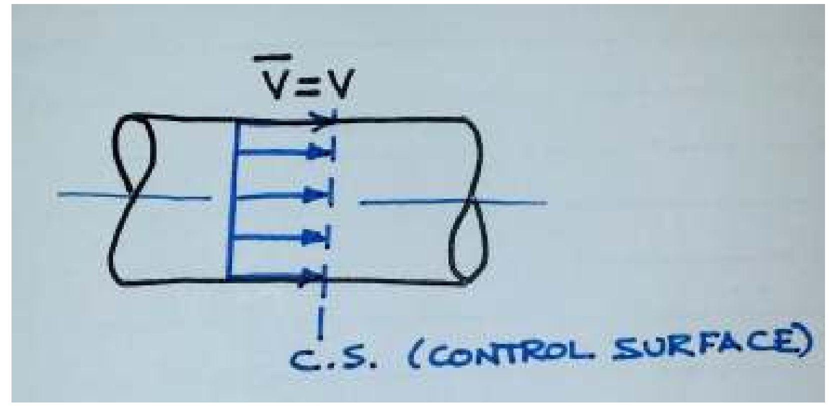
  - General integral representations for 3-D velocity vector field, $\boldsymbol{V}$

  (required for upcoming derivations)



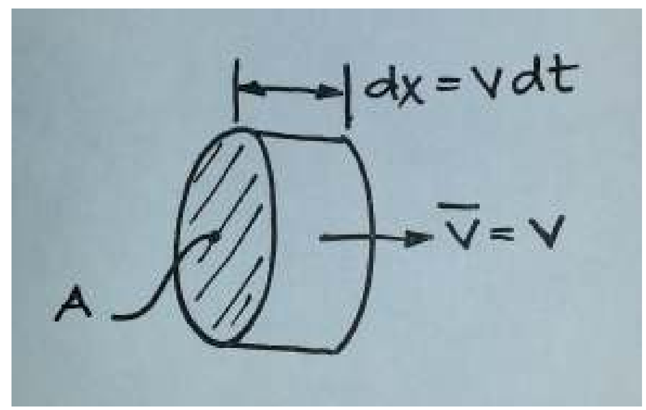
Volume Flow Rate (Q)

- Consider one-dimensional (1-D) flow in a pipe.



- In time interval dt, the incremental volume of fluid (d∀) to pass the c.s. is:

$$d\forall = A\,dx = A\,V dt$$



So, the rate of volume flow across the control surface is:

$$Q = \frac{d\forall}{dt} = \frac{AVdt}{dt} \qquad \boxed{Q = VA}$$

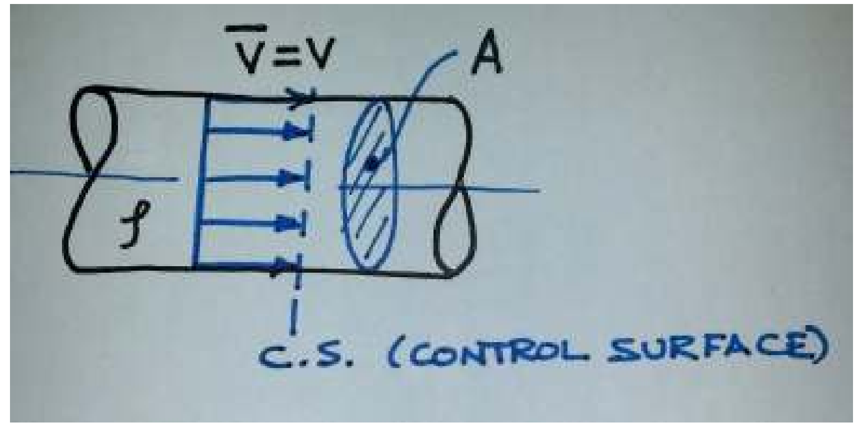
Units: m^3/s, ft^3/s

Mass Flow Rate ($\dot{m}$)

- Mass flow rate can be expressed as :

$$\dot{m} = \frac{mass}{volume} \times \frac{volume}{time} = \frac{mass}{time}$$

fluid density, ρ volume flow rate, Q



$$\boxed{\dot{m} = \rho\, Q = \rho\, V A}$$

Units: kg/s, slugs/s

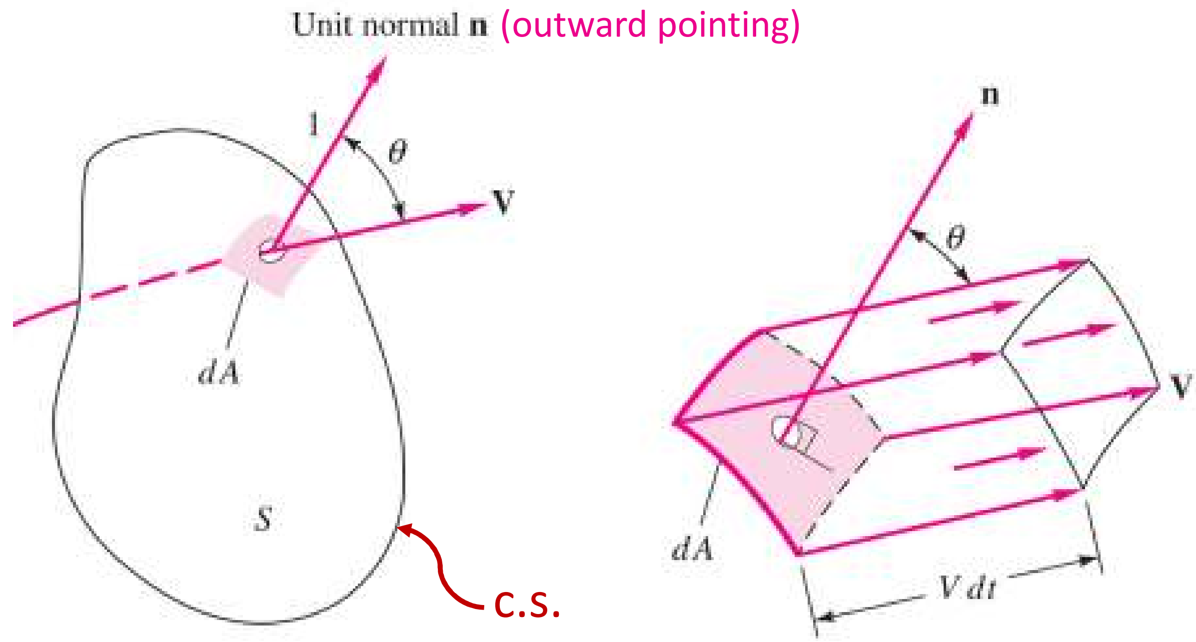
These equations apply for 1-D flow where the mean velocity is normal to the control surface.

General Integral Definitions for Q and $\dot{m}$

- Consider a general integral representation for a general 3-D velocity field, $\boldsymbol{V}$. The flow rate across the an arbitrary control surface is:

$$\boxed{\boldsymbol{Q} = \int_{c.s.} (\boldsymbol{V} \cdot \boldsymbol{n})\, \boldsymbol{dA}}$$

dot product



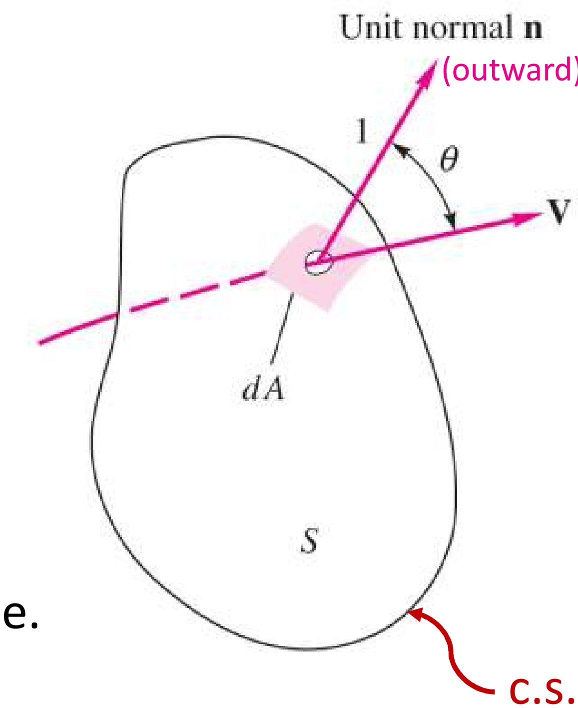
- dot (or scalar) product gives component of velocity vector normal to the c.s.

Recall: $\boldsymbol{V} \cdot \boldsymbol{n} = |\boldsymbol{V}||\cancelto{1}{\boldsymbol{n}}| \cos\theta = |\boldsymbol{V}| \cos\theta$

General Integral Definitions for Q and $\dot{m}$ (cont'd…)

- Similarly, for a general vector field $\boldsymbol{V}$, the mass flow rate can be expressed as:

$$\boxed{\dot{\boldsymbol{m}} = \int_{c.s.} \rho \, (\boldsymbol{V} \cdot \boldsymbol{n}) \, \boldsymbol{dA}}$$



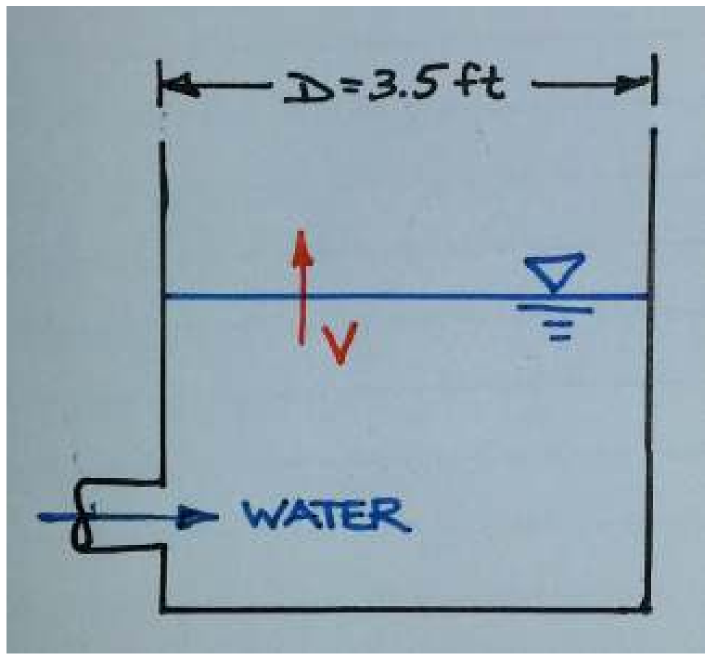
where ρ is the local fluid density, ρ=ρ(x,y,z). Note: $\boldsymbol{n}$ is the <u>outward</u> pointing normal. Thus, $\boldsymbol{V} \cdot \boldsymbol{n}$ is positive for outward flow and $\boldsymbol{V} \cdot \boldsymbol{n}$ is negative for inward flow.

- Thus, $\dot{m}$ is the net mass flow rate leaving the control volume.

- 1-D flow with $\boldsymbol{V}$ normal to the c.s. we get: $Q = VA$ and $\dot{m} = \rho \, Q = \rho \, VA$

Example

Liquid water at 68 ^{o}F flows into a circular tank with an inside diameter of 3.5 ft. If the water level in the tank is rising at V=10 in/s, calculate mass and volume flow rate of water flowing into the tank.



Example (cont'd)

- This flow can be well represented by the 1-D approximation.

$$Q = VA = \frac{10}{12}\frac{ft}{s} \cdot \frac{\pi(3.5)^2}{4} ft^2 = 8.02 \frac{ft^3}{s}$$

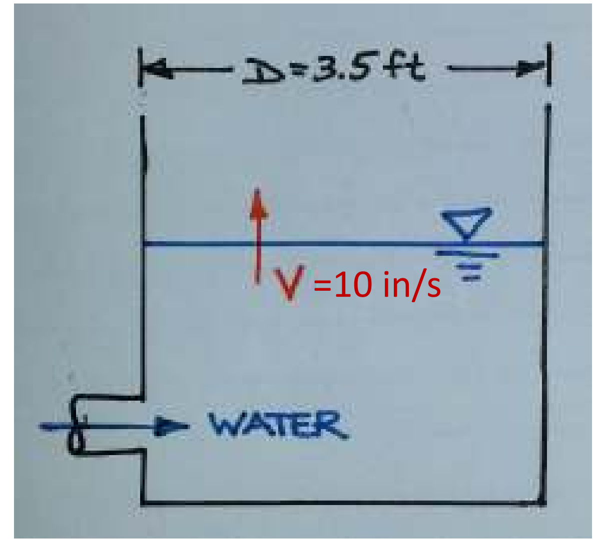
ANS

WATER AT 68°F

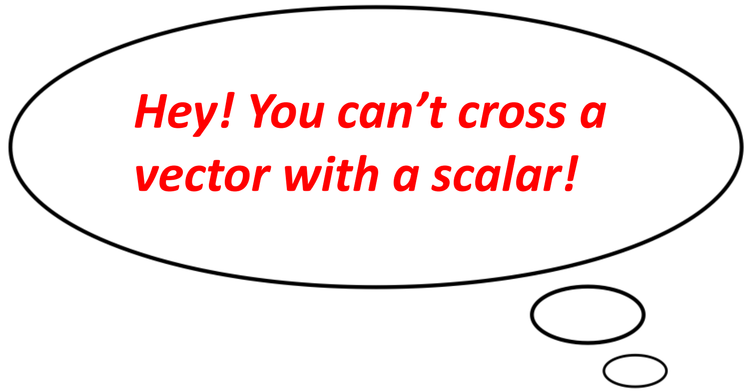
$$\rho = 998 \frac{kg}{m^3}\left(0.3048 \frac{m}{ft}\right)^3 \frac{1}{14.594}\frac{slug}{kg} = 1.936 \frac{slugs}{ft^3}$$

$$\dot{m} = \rho A V = \rho Q = 1.936 \frac{slugs}{ft^3}\left(8.02 \frac{ft^3}{s}\right) = 15.5 \frac{slugs}{s}$$

ANS



(Unit conversion factors from Appendix C of the textbook.)



END NOTES

Presentation prepared and delivered by Dr. David Naylor.