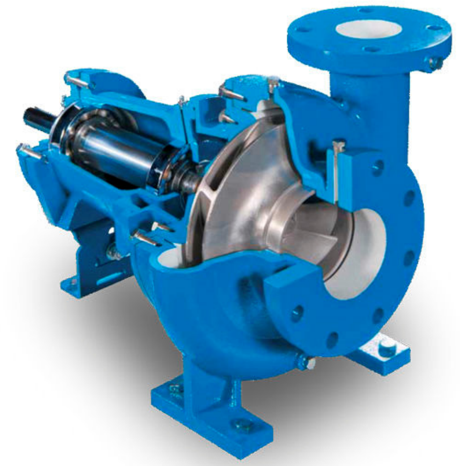
MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis
Part 10

RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

Overview

- **The Steady Flow Energy Equation**
  - Extending the Bernoulli equation for pumps, turbines and frictional losses (head losses).
  - Kinetic energy correction factor.
  - Hydraulic power and turbine/pump efficiency.



source: http://www.aurorapump.com/

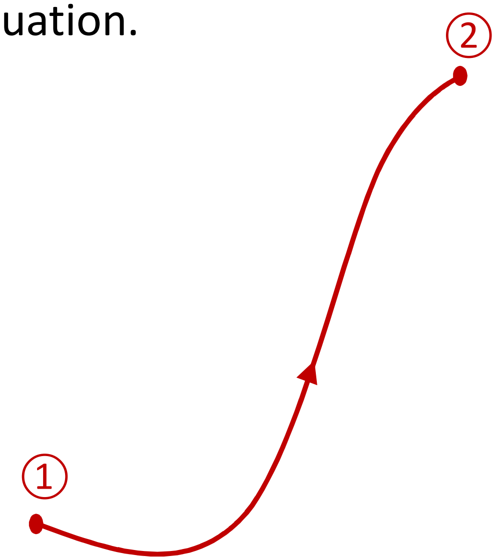
- **Numerical Example**
  - Analysis of a hydropower system using the energy equation.

- **Types of Water Turbines used for Power Generation**
  - Pelton, Francis and Kaplan Turbines.

The Steady Flow Energy Equation

- Taking a non-thermodynamic approach for the derivation of the Energy Equation. (BME students do not get MEC309 *Thermodynamics I*)

- Recall that there is no energy loss associated with the Bernoulli equation: (inviscid, frictionless flow)



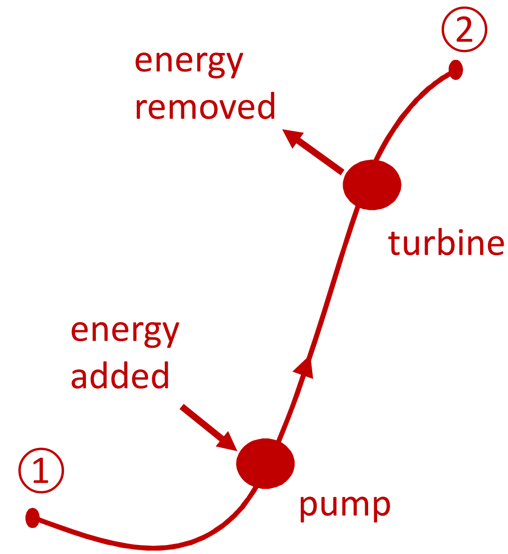
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$Total\ energy\ at\ (1) = Total\ Energy\ at\ (2)$$

- Total energy is constant between two points on a streamline.
- Each term represents the energy per unit weight of flowing fluid, a.k.a. “head”.

The Steady Flow Energy Equation

- Next, can will modify this equation to account for:
  - energy addition (by pump)
  - energy extraction (by a turbine)
  - energy "losses" (by pipe friction and turbulence)



- For a steady system, with these additional energy terms, we can say:

$$\left\{\begin{array}{l} Total\ energy\ at\ (1) - Energy\ extracted\ by\ turbine \\ + Energy\ added\ by\ pump - \ Energy\ losses\ by\ friction \end{array}\right\} = \left\{Total\ Energy\ at\ (2)\right\}$$

- This is simple energy "accounting".

The Steady Flow Energy Equation

- So, the steady energy equation becomes:

$$\underbrace{\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1}_{\text{Total fluid energy per unit weight at point (1)}} - h_{turbine} + h_{pump} - h_{friction} = \underbrace{\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2}_{\text{Total fluid energy per unit weight at point (2)}}$$

where $h_{turbine}$ is the energy *extracted* from (1) to (2) by the turbine per unit weight of fluid

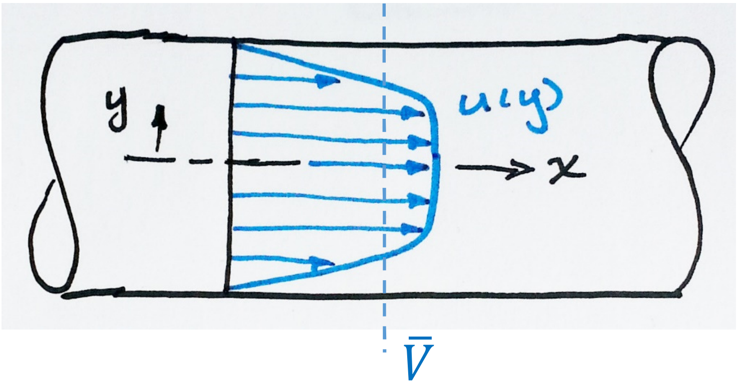
h_{pump} is the energy *added* from (1) to (2) by the pump per unit weight of fluid

$h_{friction}$ is the energy *"loss"* from (1) to (2) by fluid friction per unit weight of fluid

Terminology:

$h_{turbine}$ ≡ "turbine head" h_{pump} ≡ "pump head" $h_{friction}$ ≡ "head loss"

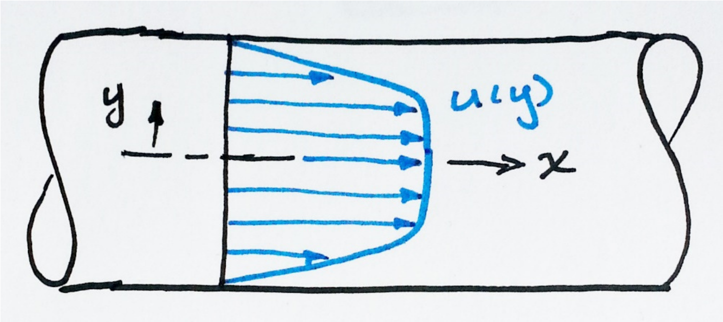
Kinetic Energy Correction Factor, α



- Real flows are not one-dimensional.
- For non-uniform flow, $\frac{1}{2}\,\bar{V}^2$ is not an accurate measure of the total kinetic energy per unit mass in the flow (particularly for laminar flows).
- This is a non-linear effect, since local fluid k.e. varies with $(u(y))^2$
- The correction factor α is defined such that:

$$\underbrace{\int_A \frac{1}{2}\, u^2\, \rho\, u\, dA}_{\text{Actual kinetic energy in non-uniform flow}} = \alpha\ \underbrace{\frac{1}{2}\,\bar{V}^2\,\overbrace{(\rho\,\bar{V}\,A)}^{\dot{m}}}_{\text{Kinetic energy based on uniform flow at } \bar{V}}$$

Kinetic Energy Correction Factor, α



$$\int_A \frac{1}{2}\, u^2\, \rho\, u\, dA = \alpha\, \frac{1}{2}\, \bar{V}^2\, (\rho\, \bar{V}\, A)$$

$$\int_A u^3\, dA = \alpha\, \bar{V}^3 A$$

$$\alpha = \frac{1}{A} \int \left(\frac{u}{\bar{V}}\right)^3 dA$$

- This correction factor, α, has been evaluated for various types of flows.

Kinetic Energy Correction Factor, α

- The correction factor (α) depends on the type of flow:

Uniform Flow

$\alpha = 1$

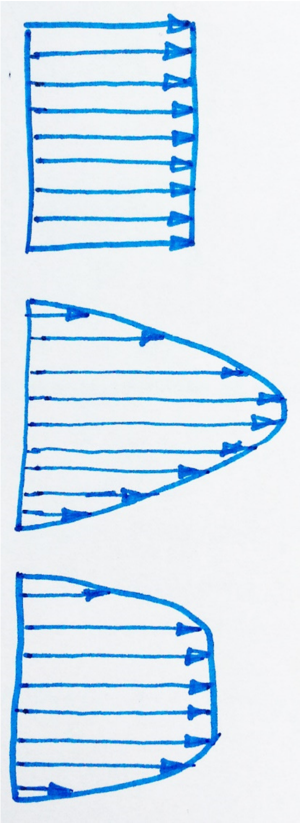
Laminar Flow
(Parabolic Profile)

$\alpha = 2$ (round pipe)

(A laminar flow has two times more kinetic energy than a uniform flow with the same $\bar{V}$)

Turbulent Flow

$\alpha = 1.04 - 1.11$



The Steady Flow Energy Equation

- With the kinetic energy correction factor (α):

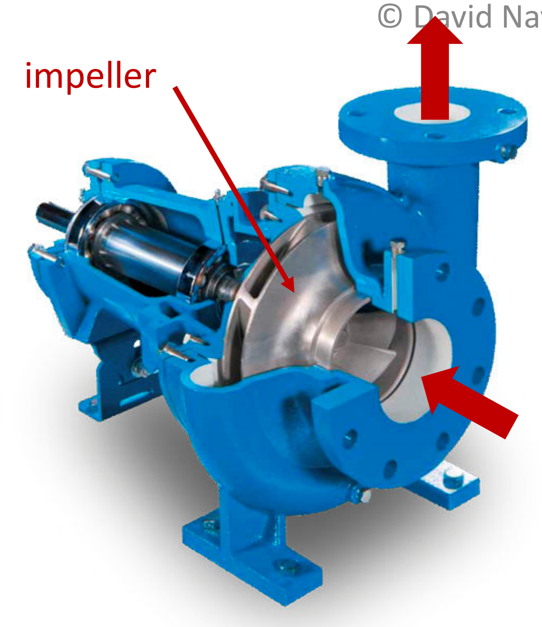
$$\boxed{\frac{p_1}{\gamma} + \frac{\alpha\, V_1^2}{2g} + z_1 - h_{turbine} + h_{pump} - h_{friction} = \frac{p_2}{\gamma} + \frac{\alpha\, V_2^2}{2g} + z_2}$$

- If α is not given in a problem, make the approximation that α=1. (Recall, most real world flows are turbulent, where α≈1.)

- Remember that the other restrictions on the Bernoulli equation apply:
  - steady flow
  - incompressible (ρ≈const.) For M<0.3, apply equation at flow average density.
  - flow along a streamline

Hydraulic Power, P

- A pump adds energy to the flow
  - increases pressure to drive a fluid along a pipe.
- As we saw, pump specifications give the "head" supplied by the pump, h_{pump}
- h_{pump} is the energy added per unit weight of fluid passing through the pump.



A Centrifugal Pump
source: http://www.aurorapump.com/

- The power input P_{pump} to the fluid by the pump is:

$$\left\{ \begin{array}{c} \text{Energy added to fluid} \\ \text{per unit time} \\ \frac{J}{s} = W \end{array} \right\} = \left\{ \begin{array}{c} \text{Weight of fluid pumped} \\ \text{per unit time} \\ \left(\frac{N}{m^3}\right)\frac{m^3}{s} \end{array} \right\} \left\{ \begin{array}{c} \text{Energy input per} \\ \text{unit weight of fluid} \\ \frac{J}{N} \end{array} \right\}$$

$$P_{pump} = \gamma\, Q\, h_{pump}$$

Hydraulic Power and Pump Efficiency

- Thus, the hydraulic power input to the fluid by a pump is:

$$\boxed{P_{pump} = \gamma\, Q\, h_{pump}}$$

- However, transmission of power from the shaft to the fluid has some energy losses.

- So, we define the overall *pump efficiency* (η_{pump}) as:

$$\boxed{\eta_{pump} = \frac{P_{pump}}{P_{input}}} \qquad (\,\eta_{pump} < 1.0\,)$$

where P_{input} is the required shaft power input from the motor (W).

(Recall: Shaft power is $P_{input} = Shaft\ Torque \times Angular\ Velocity$)

Turbine Efficiency

- Similarly, the hydraulic power extracted from the fluid by a turbine:

$$\boxed{P_{turbine} = \gamma\, Q\, h_{turbine}}$$

- Again, the extraction of power from fluid to the shaft has some energy losses.

- So, we define the overall *turbine efficiency* $(\eta_{turbine})$ as:

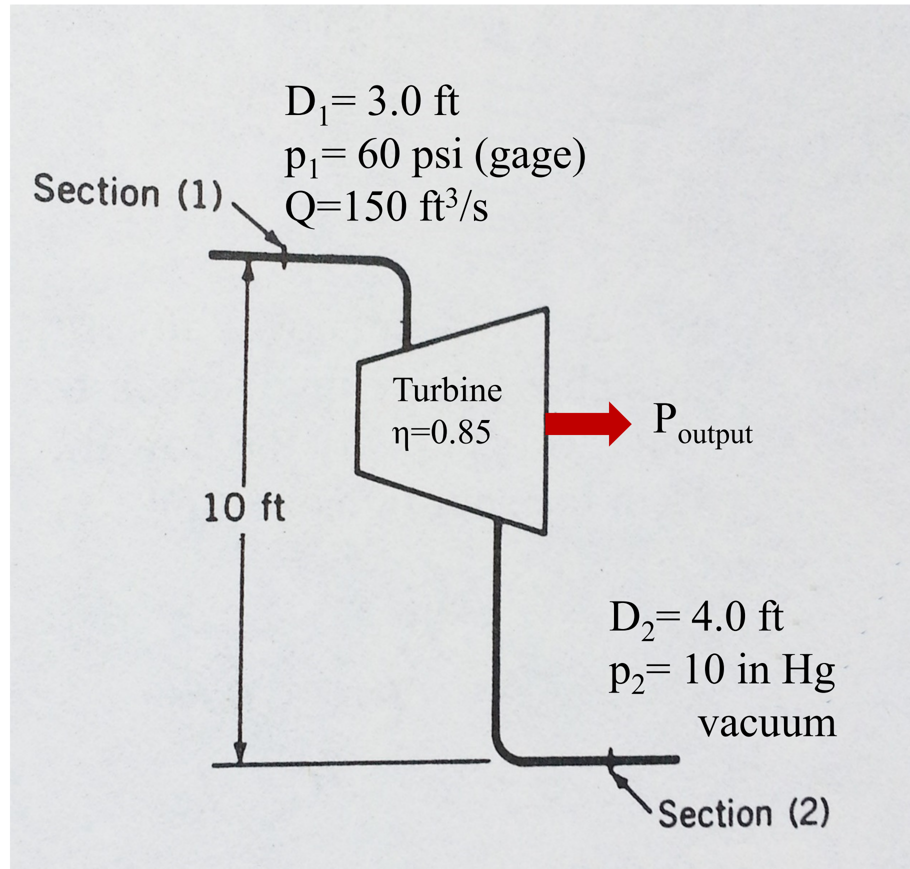
$$\boxed{\eta_{turbine} = \frac{P_{output}}{P_{turbine}}} \qquad (\eta_{turbine} < 1.0)$$

where P_{output} is the useful shaft power output to the generator (W).

Example

Water is supplied to a hydraulic turbine at a flow rate of 150 ft^3/s through a pipe with an inside diameter of 3.0 ft. The supply pressure at Section (1) is 60 psi(g). The discharge pipe has an inside diameter of 4.0 ft. The static pressure ten feet below the turbine inlet at Section (2) is 10 inches Hg vacuum. The frictional head loss in the pipes from (1) to (2) is 17.7 ft. The turbine has an overall efficiency of 85%.

Calculate the power output that is supplied to the electric generator.

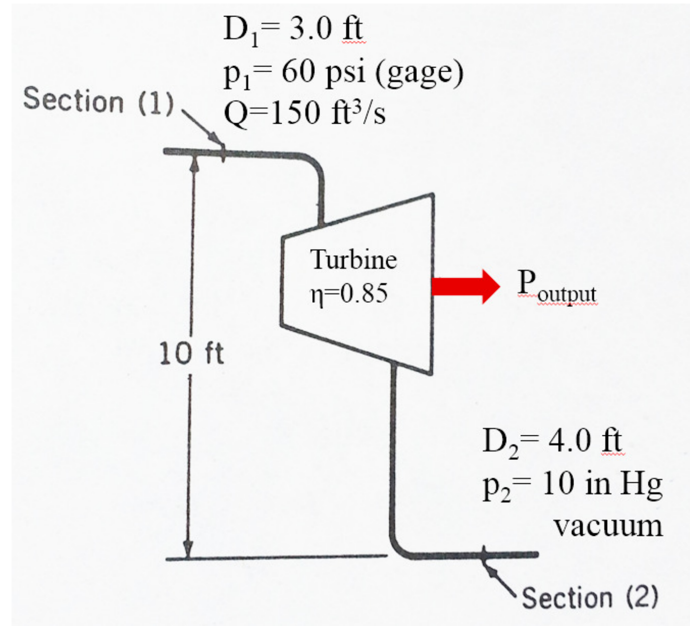


Example

The energy equation:

$$\frac{p_1}{\gamma} + \cancelto{1}{\alpha}\frac{V_1^2}{2g} + z_1 - h_{turbine} + \cancelto{0}{h_{pump}} - h_{friction} = \frac{p_2}{\gamma} + \cancelto{1}{\alpha}\frac{V_2^2}{2g} + z_2$$

- no pump in system, $h_{pump} = 0$
- Kinetic energy correction factor, $\alpha = 1$ (turbulent flow)

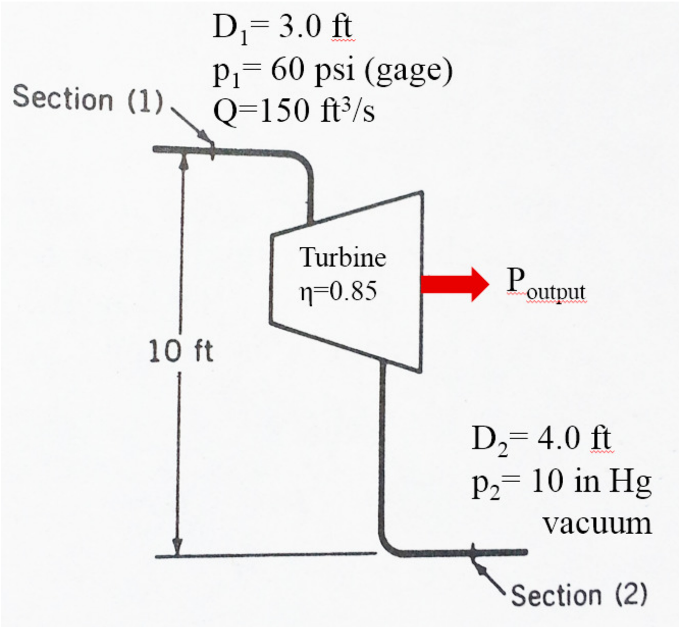


- Energy extracted from the fluid (per unit weight)

$$h_{turbine} = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) - h_{friction}$$

Example

$$h_{turbine} = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) - h_{friction}$$



Evaluating the terms:

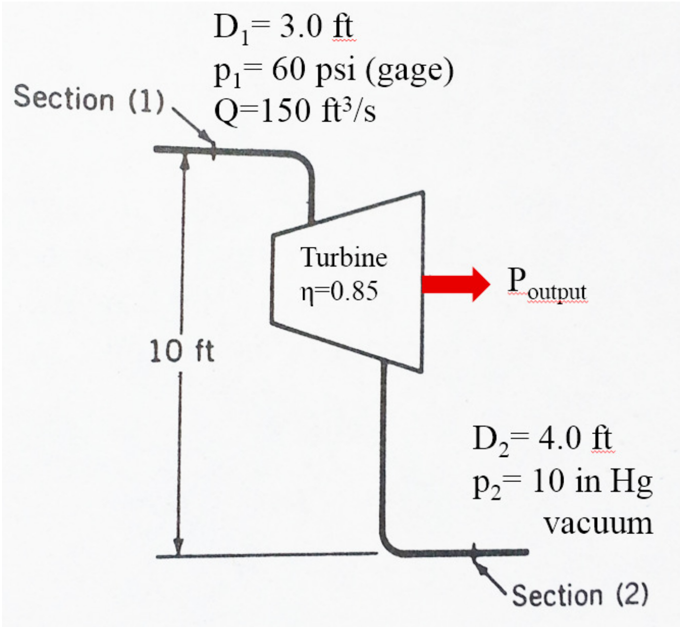
$$p_1 = 60\ \frac{lb}{in^2}\left(144 \frac{in^2}{ft^2}\right) = 8640\ \frac{lb}{ft^2}$$

$$p_2 = -\gamma_m h_m = -SG_m\ \gamma_w h_m$$ Specific gravity of mercury SG_m=13.6

$$p_2 = -13.6\ \left(62.4\ \frac{1b}{ft^3}\right)\frac{10}{12}ft = -707.2\ \frac{lb}{ft^2}$$ (gage, of course!)

Example

$$h_{turbine} = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) - h_{friction}$$



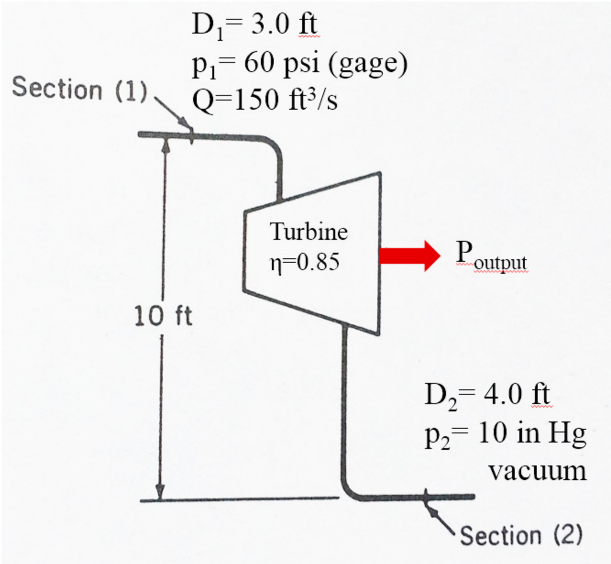
$$V_1 = \frac{Q}{A_1} = \frac{150\ \frac{ft^3}{s}}{\pi\ (3.0)^2\ ft^2/4} = 21.22\ \frac{ft}{s}$$

$$V_2 = \frac{Q}{A_2} = \frac{150\ \frac{ft^3}{s}}{\pi\ (4.0)^2\ ft^2/4} = 11.94\ \frac{ft}{s}$$

$$z_1 - z_2 = 10\ ft$$

$h_{friction} = 17.7\ ft$ i.e. $\frac{ft-lb}{lb}$ (problem statement)

Example



$$h_{turbine} = \frac{p_1 - p_2}{\gamma} + \frac{V_1^2 - V_2^2}{2g} + (z_1 - z_2) - h_{friction}$$

Substituting values:

$$h_{turbine} = \frac{(8640 - (-707))\,\frac{lb}{ft^2}}{62.4\,\frac{lb}{ft^3}} + \frac{(21.22^2 - 11.94^2)\,\frac{ft^2}{s^2}}{2(32.2)\,\frac{ft}{s^2}} + 10ft - 17.7\,ft$$

$$h_{turbine} = 149.8\,ft + 4.8\,ft + 10ft - 17.7\,ft = 146.9\,ft$$

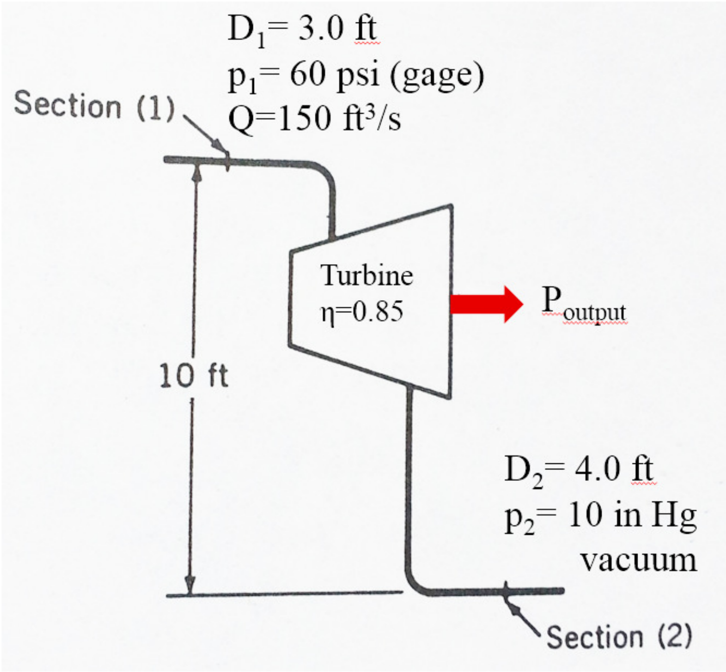
The power extracted from the water:

$$P_{turbine} = \gamma\, Q\, h_{turbine} = 62.4\frac{lb}{ft^3}\,(150\,\frac{ft^3}{s})\,146.9\,ft = 1.37 \times 10^6\,\frac{ft-lb}{s}$$

Example

The power extracted from the water:

$$P_{turbine} = \gamma\, Q\, h_{turbine} = 1.375 \times 10^6 \,\frac{ft-lb}{s}$$



This is the max. energy that could be extracted from the water.
Not all of this power becomes useful shaft power:

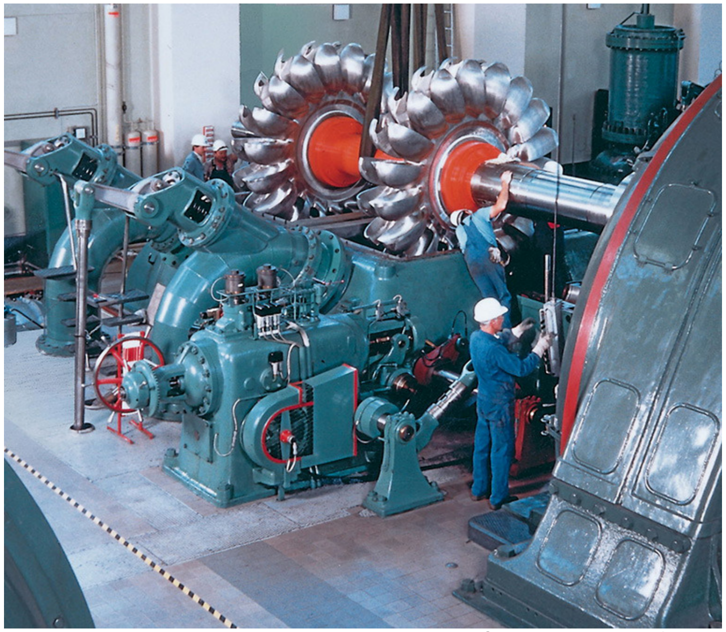
$$\eta_{turbine} = \frac{P_{output}}{P_{turbine}}$$

Thus, $P_{output} = \eta_{turbine} P_{turbine} = 0.85\left(1.375 \times 10^6\,\frac{ft-lb}{s}\right)\frac{1\;hp}{550\;ft-lb/s} = 2120\;hp\quad Ans.$

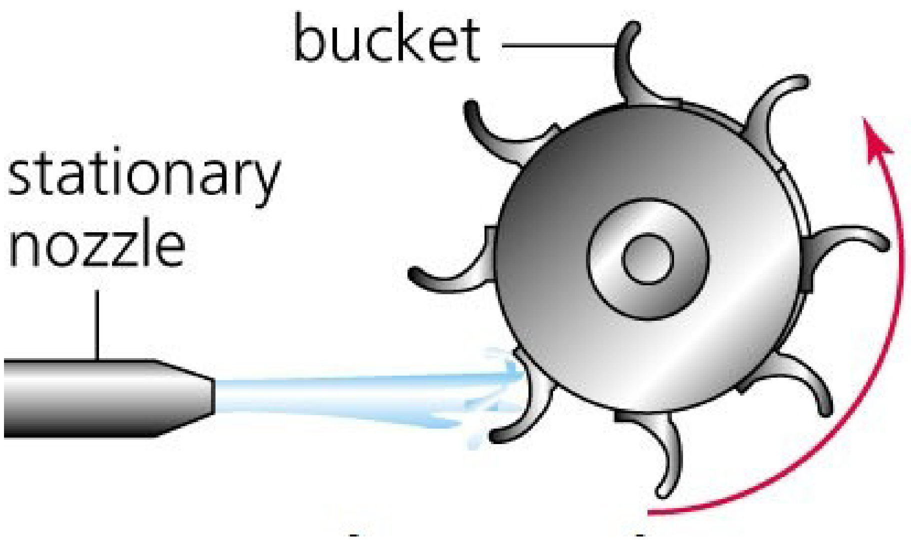
$$\left(550\,\frac{ft-lb}{s} = 1\;hp\right)$$

Types of Water Turbines

- Pelton Wheel (Impulse turbine)
  - used when high "head" is available.
  - high velocity jet(s) of water impinge on the buckets.
  - we calculated the forces on vanes using linear momentum.



(Source: wikipedia.org)



Types of Water Turbines

- Francis Turbine (reaction turbine)
  - one of the most common turbines
  - used for medium head installations
  - unlike Pelton wheel, mainly pressure energy extracted.

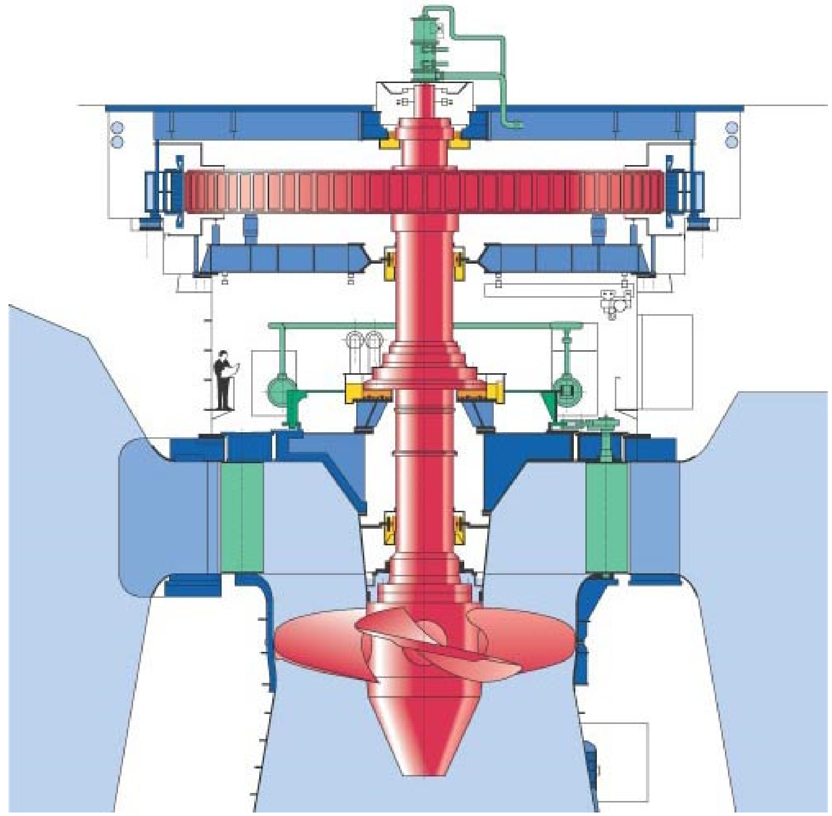
Sir Adam Beck Hydroelectric Power Station (Niagara Falls)

Runner of a Francis Turbine

(Source: wikipedia.org)

Types of Water Turbines

- Kaplan Turbine
  - used for relatively low head
  - usually has adjustable pitch blades



(Source: wikipedia.org)

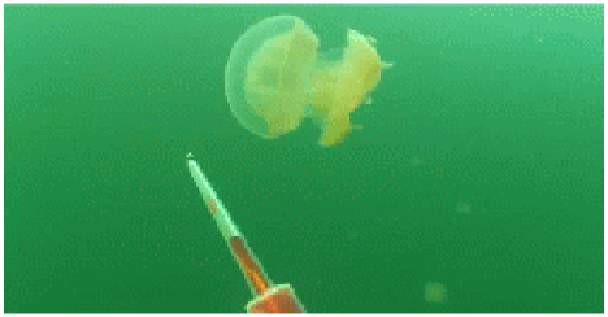


Photo: D. Naylor

Flow Visualization of How a Jellyfish Swims
Source:http://thecraftychemist.tumblr.com/post/70942784430/tracking-a-jellyfishs-movements-with-green-dye

END NOTES

Presentation prepared and delivered by Dr. David Naylor.