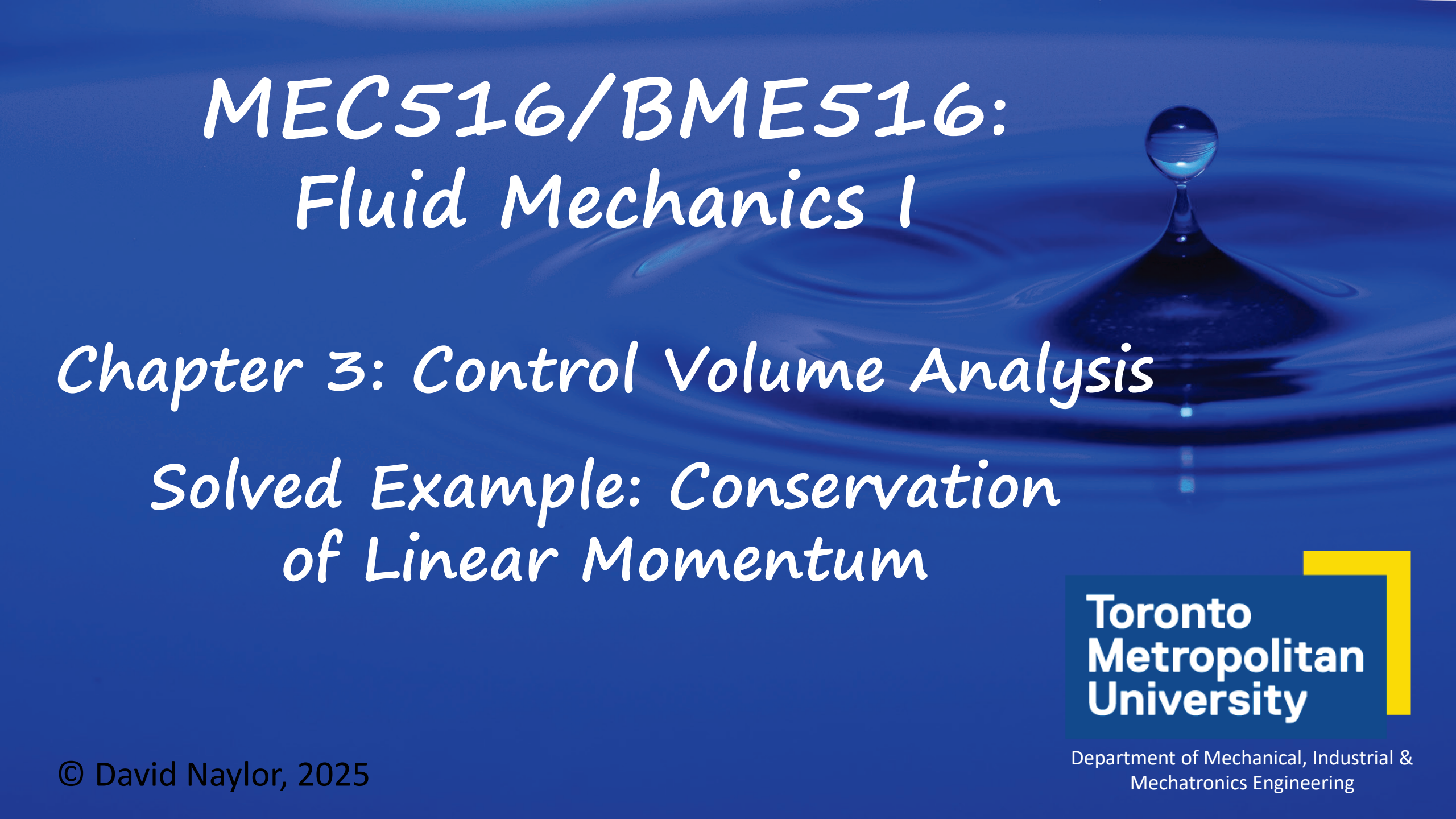
MEC516/BME516: Fluid Mechanics I

Chapter 3: Control Volume Analysis

Solved Example: Conservation of Linear Momentum



Toronto Metropolitan University

Department of Mechanical, Industrial & Mechatronics Engineering

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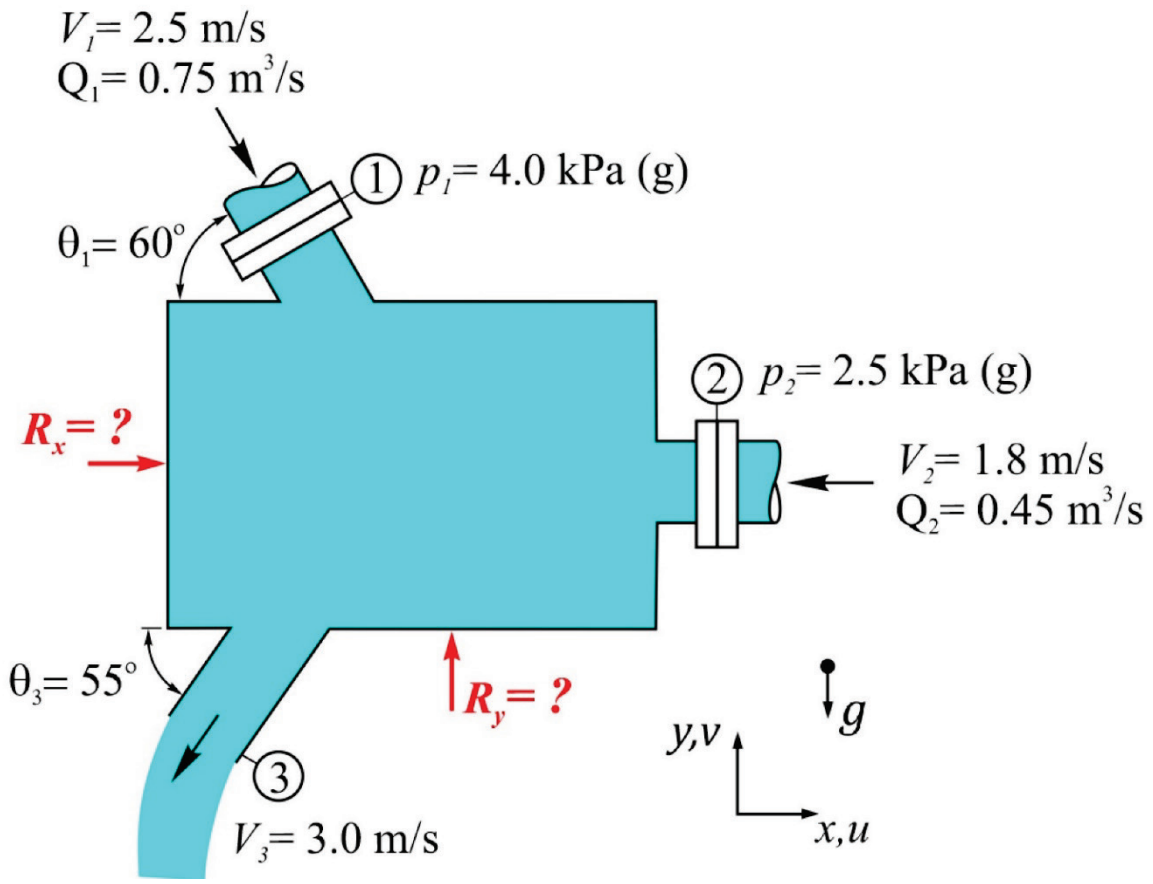
Fall 2024 Final Exam: Forces on a Fluid Mixing Box

Consider a steady flow of water (ρ=998 kg/m^3) in a fluid mixing box with two inlets and one outlet. The weight of the water in the box and the box is 4500 N. The gage pressures at the inlets are p_1=4.0 kPa and p_2=2.5 kPa. The outlet discharges to the atmosphere at 100 kPa.

(a) Calculate the volume flow rate at the outlet, Q_3

(b) Calculate the reaction forces R_x and R_y required to hold the box stationary



All course videos and pdfs available at:
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Link in the Description

Solution

(a) Volume flow rate at the outlet, Q_3

- Continuity: Mass flow rate into CV equals the mass flow rate out of CV

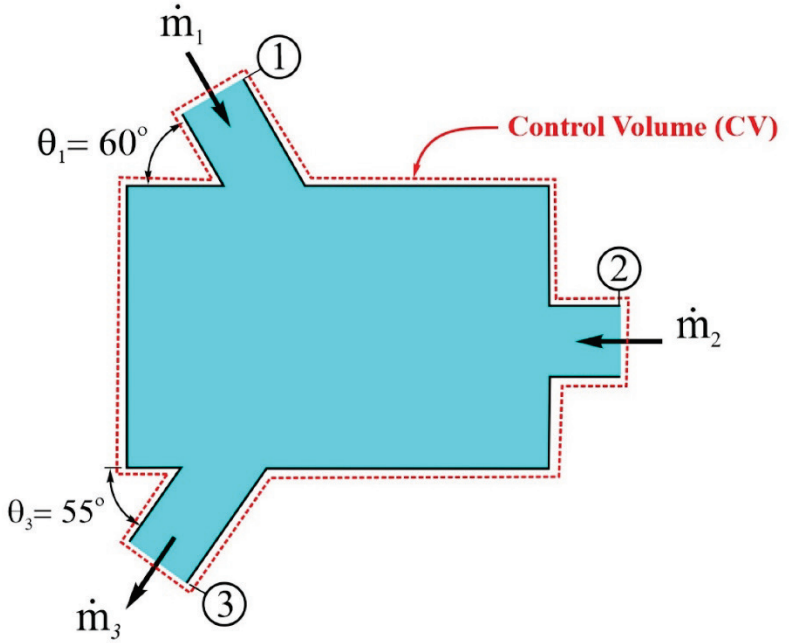
ρ= Constant (incompressible)

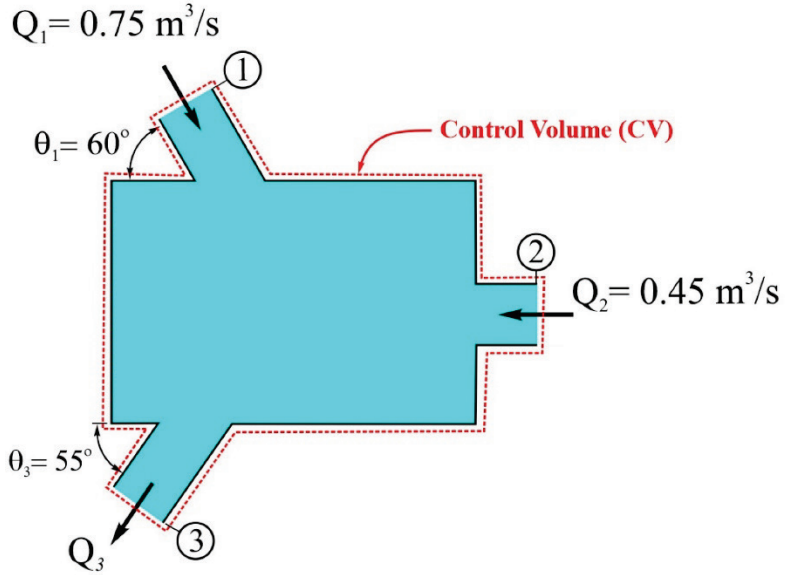
$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \qquad \cancel{\rho} Q_1 + \cancel{\rho} Q_2 = \cancel{\rho} Q_3$$

$$Q_3 = Q_1 + Q_2$$

- True for ***both*** steady and unsteady flows
(no mass storage in CV for incompressible fluid)

$$Q_3 = 0.75 \frac{m^3}{s} + 0.45 \frac{m^3}{s} = 1.2 \frac{m^3}{s}$$ **Ans.**

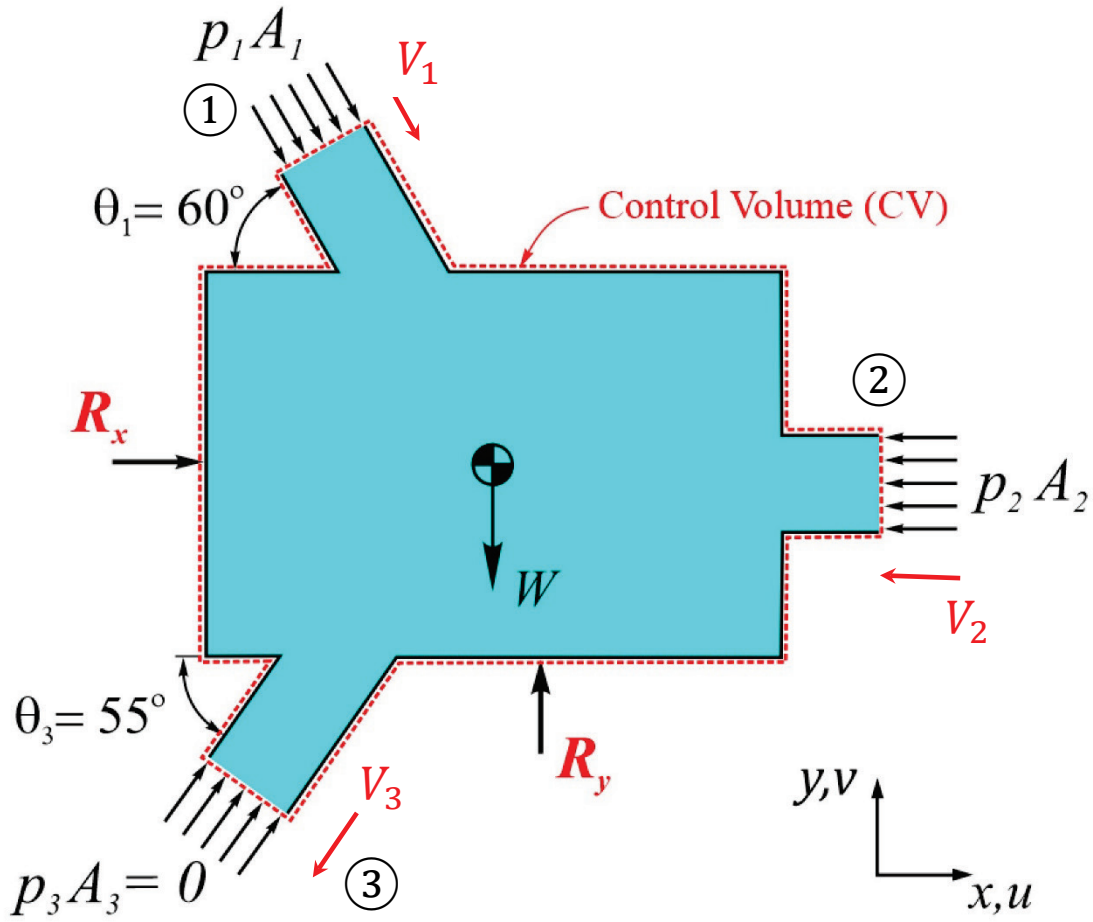




Solution

(b) Forces required to hold the box stationary

- **Free Body Diagram**
  - Unknown reaction forces (R_x, R_y) and weight
  - Pressure forces: Act inward & normal to the CV surface
  - **Must** use gage pressures! Why? See last slide



- Conservation of linear momentum:

$$\sum \boldsymbol{F} = \sum \dot{m}_i \boldsymbol{V}_i - \sum \dot{m}_i \boldsymbol{V}_i = \dot{m}_3 \boldsymbol{V}_3 - (\dot{m}_1 \boldsymbol{V}_1 + \dot{m}_2 \boldsymbol{V}_2)$$

All forces on CV

Outlets of CV $i = 3$

Inlets of CV $i = 1,2$

Rate of Momentum Out – Rate of Momentum In

Solution

- Vector equation: $\sum \boldsymbol{F} = \dot{m}_3 \boldsymbol{V}_3 - (\dot{m}_1 \boldsymbol{V}_1 + \dot{m}_2 \boldsymbol{V_2})$

$\boldsymbol{F} = F_x\, \boldsymbol{i} + F_y\, \boldsymbol{j}$ $\quad$ $\boldsymbol{V} = u\, \boldsymbol{i} + v\, \boldsymbol{j}$

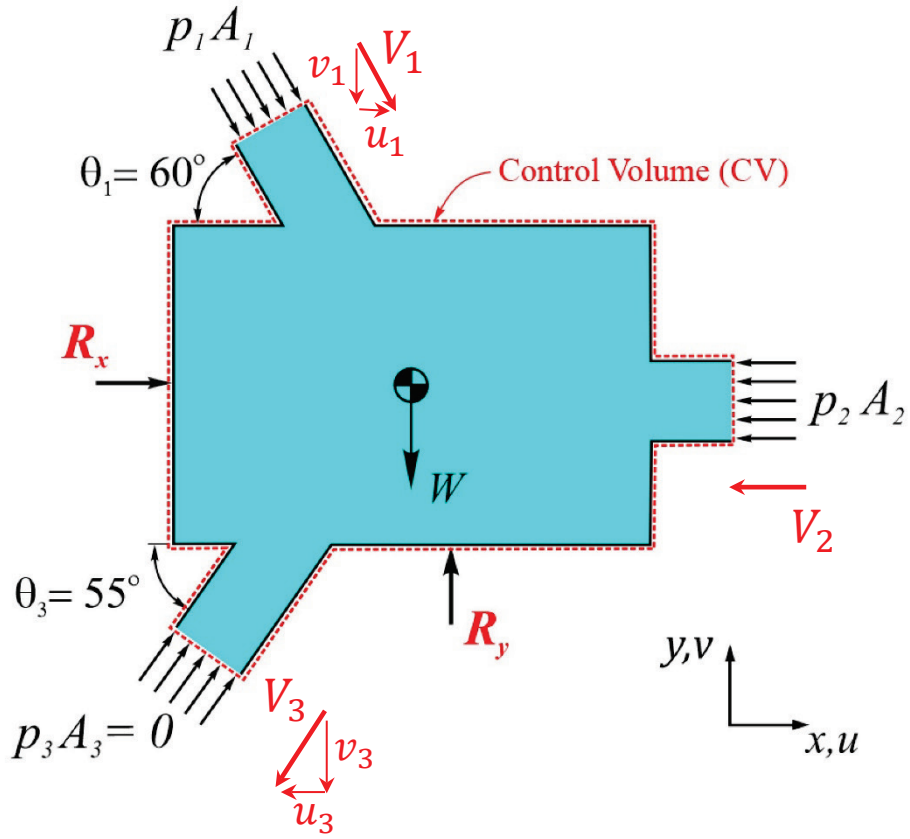
x-direction: + →

$u_3 = -V_3 \cos(\theta_3)$

$u_1 = V_1 \cos(\theta_1)$

$$\sum F_x = \dot{m}_3\, u_3 - [\dot{m}_1 u_1 + \dot{m}_2 u_2]$$

$u_2 = -V_2$



$$R_x + p_1 A_1 \cos(\theta_1) - p_2 A_2 = \dot{m}_3(-V_3 \cos(\theta_3)) - [\dot{m}_1 V_1 \cos(\theta_1) + \dot{m}_2(-V_2)]$$

Solving for R_x:

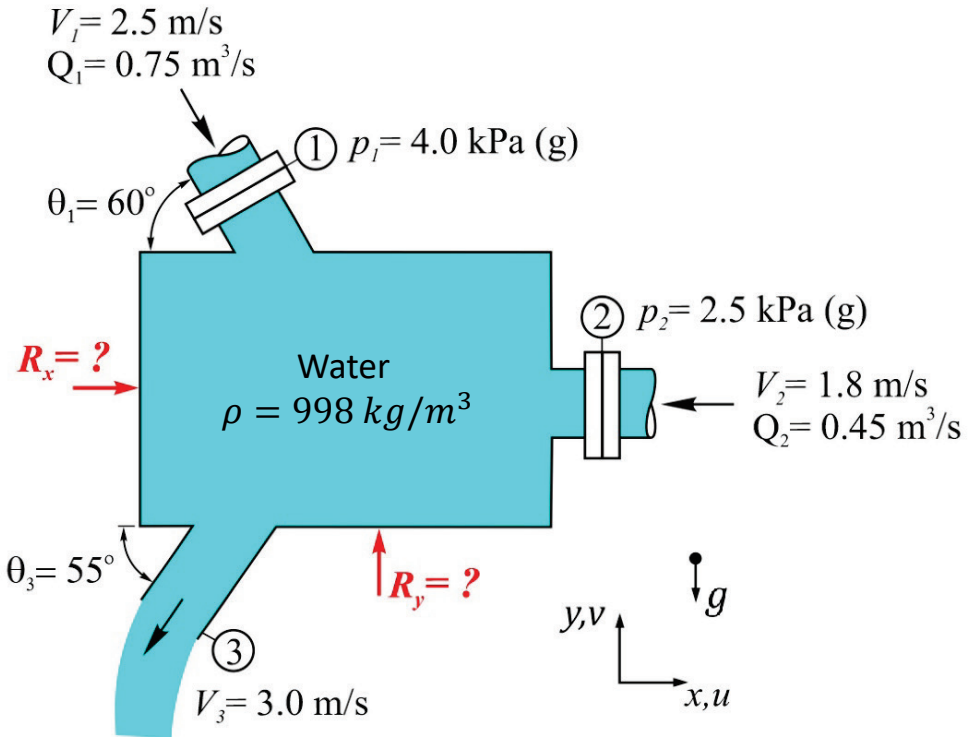
$$R_x = -p_1 A_1 \cos(\theta_1) + p_2 A_2 - \dot{m}_3 V_3 \cos(\theta_3) - \dot{m}_1 V_1 \cos(\theta_1) + \dot{m}_2 V_2$$

- Force directions make sense? ☑

Solution

$$R_x = -p_1 A_1 \cos(\theta_1) + p_2 A_2 - \dot{m}_3 V_3 \cos(\theta_3) - \dot{m}_1 V_1 \cos(\theta_1) + \dot{m}_2 V_2$$

Eq.(1)



- Work in symbolic form as long as you can!
- Calculations:

$$A_1 = \frac{Q_1}{V_1} = \frac{0.75\frac{m^3}{s}}{2.5\frac{m}{s}} = 0.30\ m^2 \qquad A_2 = \frac{Q_2}{V_2} = \frac{0.45\frac{m^3}{s}}{1.8\frac{m}{s}} = 0.25\ m^2$$

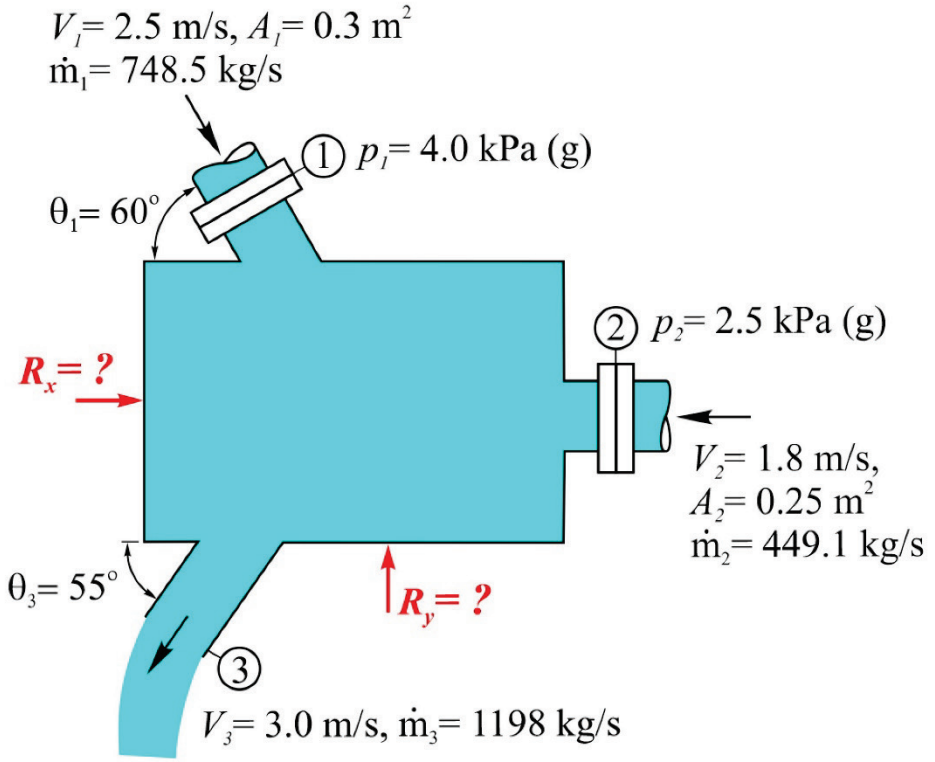
$$\dot{m}_1 = \rho Q_1 = 998\frac{kg}{m^3}\left(0.75\frac{m^3}{s}\right) = 748.5\frac{kg}{s}$$

$$\dot{m}_2 = \rho Q_2 = 998\frac{kg}{m^3}\left(0.45\frac{m^3}{s}\right) = 449.1\frac{kg}{s} \qquad \dot{m}_3 = \rho Q_3 = 998\frac{kg}{m^3}\left(1.2\frac{m^3}{s}\right) = 1198\frac{kg}{s}$$

Solution

$$R_x = -p_1 A_1 \cos(\theta_1) + p_2 A_2 - \dot{m}_3 V_3 \cos(\theta_3) - \dot{m}_1 V_1 \cos(\theta_1) + \dot{m}_2 V_2$$

Eq.(1)



- Making the substitutions into Eq (1):

$$R_x = -4000\frac{N}{m^2}\left(0.3m^2\right)\cos(60^o) + 2500\frac{N}{m^2}\left(0.25\,m^2\right) - 1198\frac{kg}{s}\left(3\frac{m}{s}\right)\cos(55^o)$$
$$-748.5\frac{kg}{s}\left(2.5\frac{m}{s}\right)\cos(60^o) + 449.1\frac{kg}{s}\left(1.8\frac{m}{s}\right) = -2160\ N$$

$$\therefore\ R_x = 2160\ N \leftarrow$$ **Ans.**

Solution

- Vector equation: $\sum \boldsymbol{F} = \dot{m}_3 \boldsymbol{V}_3 - (\dot{m}_1 \boldsymbol{V}_1 + \dot{m}_2 \boldsymbol{V}_2)$

$\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$ $\qquad$ $\boldsymbol{V} = u\,\boldsymbol{i} + v\,\boldsymbol{j}$

y-direction: + ↑

$v_3 = -V_3\,sin(\theta_3)$

$v_1 = -V_1\,sin(\theta_1)$

$$\sum F_y = \dot{m}_3\, v_3 - (\dot{m}_1 v_1 + \dot{m}_2 v_2)$$

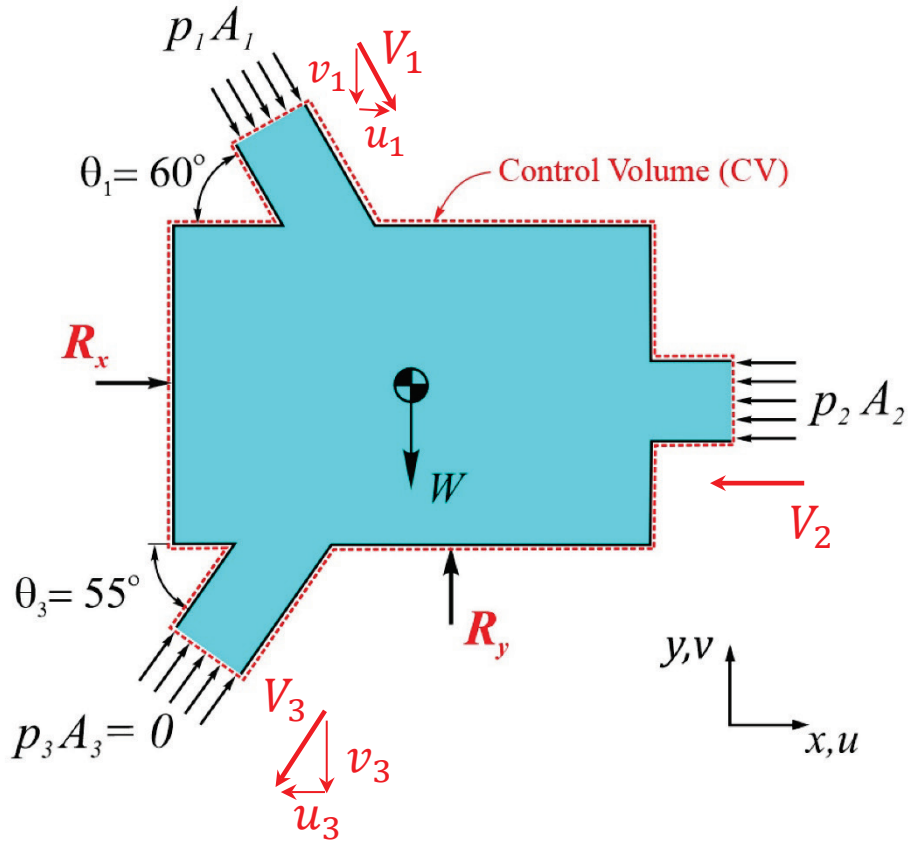
$v_2 = 0$

$$R_y - W - p_1 A_1 sin(\theta_1) = \dot{m}_3(-V_3\,sin(\theta_3)) - \dot{m}_1(-V_1\,sin(\theta_1))$$

Solving for R_y:

$$R_y = W + p_1 A_1 sin(\theta_1) - \dot{m}_3 V_3\,sin(\theta_3) + \dot{m}_1 V_1\,sin(\theta_1)$$

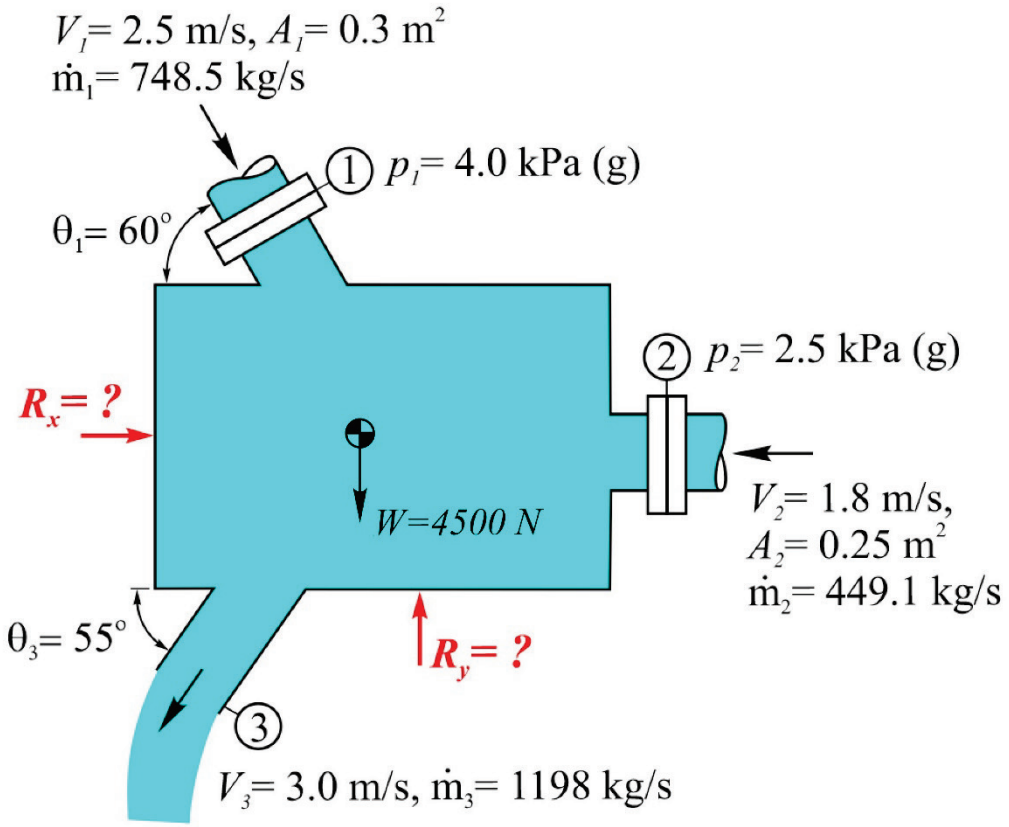
- Force directions make sense? ☑



Solution

$$R_y = W + p_1 A_1 sin(\theta_1) - \dot{m}_3 V_3 \, sin(\theta_3) + \dot{m}_1 V_1 \, sin(\theta_1)$$

Eq.(2)

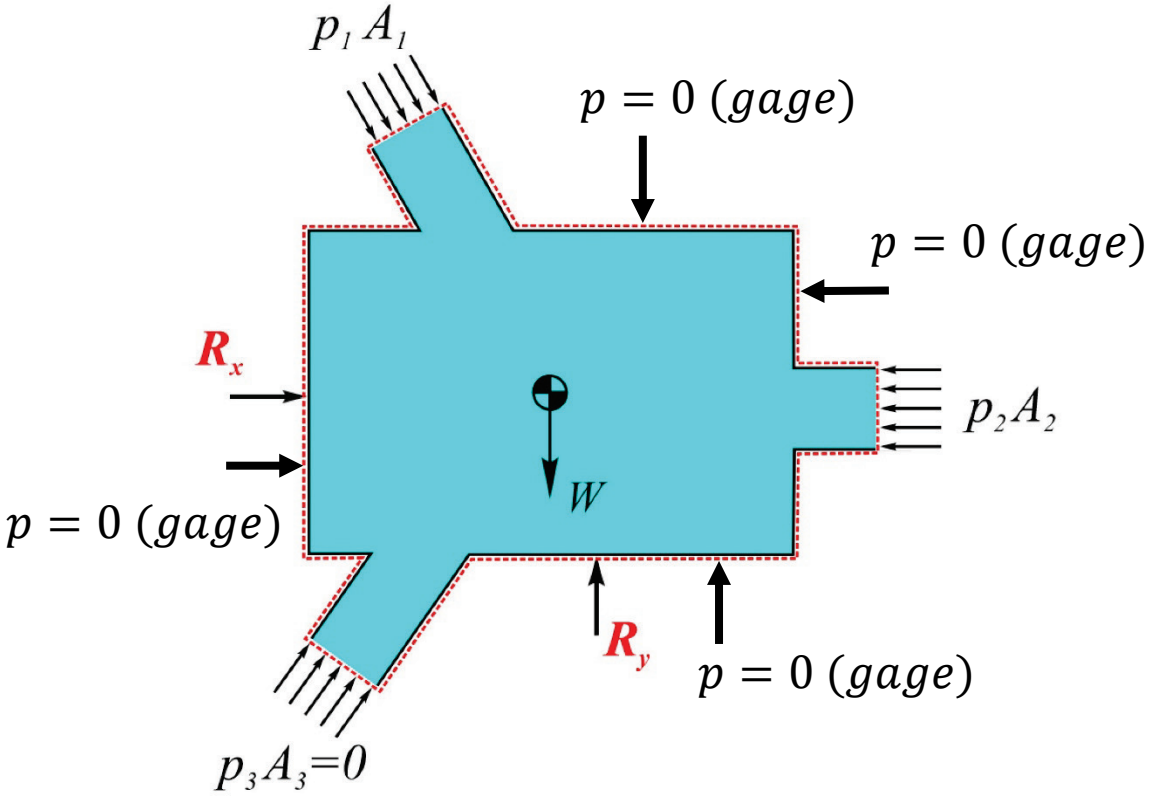


- Making the substitutions into Eq (2):

$$R_y = 4500\ N + 4000\frac{N}{m^2}\left(0.3\ m^2\right)\sin(60^o) - 1198\frac{kg}{s}\left(3.0\frac{m}{s}\right)\sin(55^o) + 748.5\frac{kg}{s}\left(2.5\frac{m}{s}\right) sin(60^o)$$

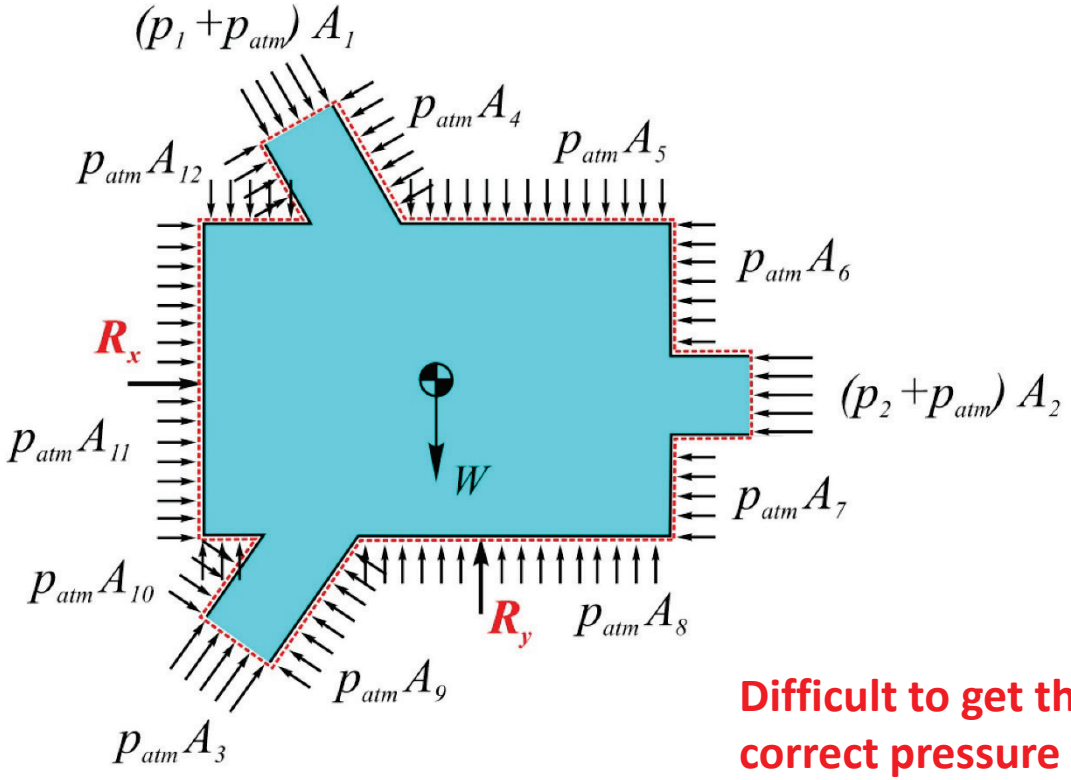
$$R_y = 4220\ N\ \uparrow \quad \textbf{Ans.}$$

Why use gage pressures?



Free Body Diagram
(Gage Pressures)

- Imagine using absolute pressures



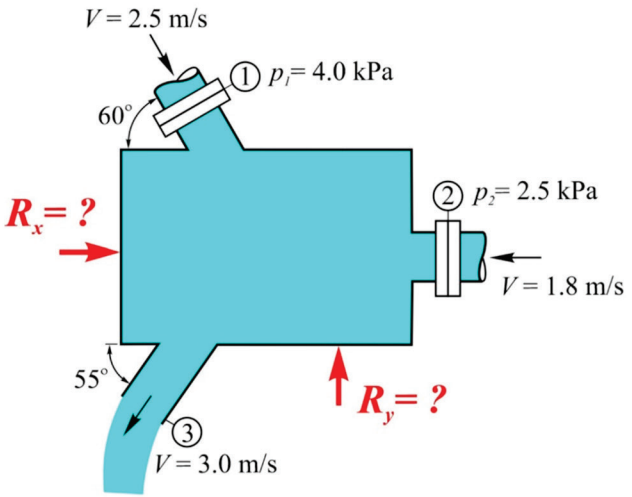
Free Body Diagram
(Absolute Pressures)

Difficult to get the correct pressure forces!

$p_{atm} = 100\ kPa$

Solved Exam Problem:

Forces on a Fluid Mixing Box



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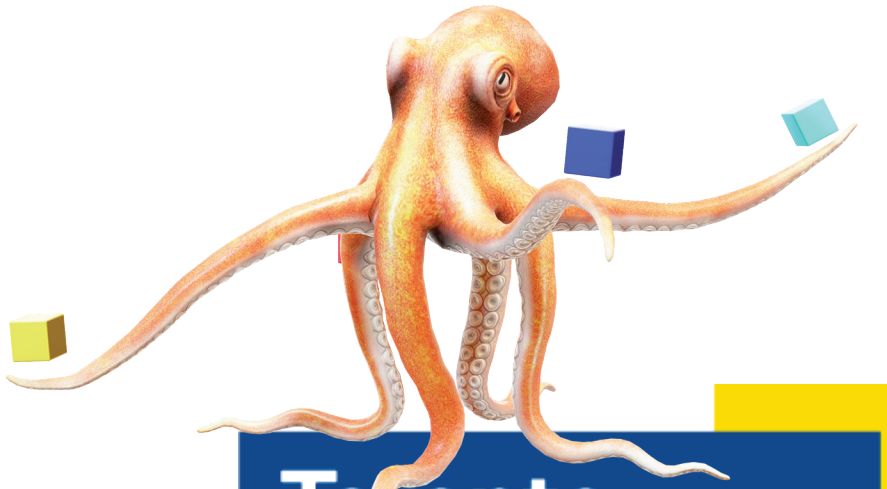

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END NOTES

Presentation prepared and delivered by Professor David Naylor

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