Last Updated: Sept. 2022

MEC516/BME516
Fluid Mechanics I

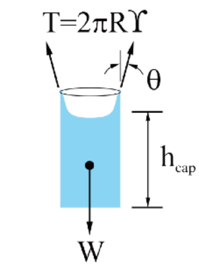
**Chapter 2**
Recommended Problem Solutions

**Caution:** Reading solutions can be deceiving. Solutions that look obvious at a glance can be difficult to reproduce later. Consult these solutions only after making an honest independent attempt at each problem.

**1.** Water at 20ºC, Table A.5: ϒ=0.728 N/m, $\gamma = \rho g = 9790 \frac{N}{m^3}$

See the free body diagram. The surface tension balances the weight of the column of water produced by the capillary effect:

$$T \cos\theta = W \qquad 2\pi R \Upsilon \cos\theta = \pi R^2 h_{cap} \gamma$$



Free Body Diagram

So, the height that the water raises up the tube due to capillary effects is:

$$h_{cap} = \frac{2\,\Upsilon \cos\theta}{\gamma R} = \frac{2\,(0.0728 \frac{N}{m})\ \cos(0)}{\left(9790 \frac{N}{m^3}\right) 0.0005m} = 0.0297\ m$$

This rise due to capillary effects must be subtracted from the total height to get the column height caused by the pressure at point A: $h_{press} = 0.250\ m - 0.0279\ m = 0.220\ m$. The gauge pressure at point A is then:

$$p_A = \gamma h_{press} = 9790 \frac{N}{m^3}(0.220m) = \ 2160 \frac{N}{m^2} = 2160\ Pa\ (g)$$

In this case the correction for the capillary effect is large because of the small tube diameter. Thus, it is important not to use small diameter tubes for a manometer or piezometer applications. It can lead to a significant error in the pressure measurement if capillary effects are neglected. In the remainder of this problem set (and course) we will assume that the manometer/piezometer tubes are sufficiently large that capillary effects are negligible.

**2.** The pressure at the top of all three liquid surfaces is the local atmospheric pressure, i.e. *p*=0 gauge. The gauge pressure in the lower horizontal section can be expressed in terms of the liquid column heights:

$$\gamma_w(0.27m) + \gamma_m h_1 = \gamma_m(0.08m) = \gamma_o h_2 + \gamma_m(0.05m)$$

Specific weights at 20ºC (Tables A.3 and A.1):

$$\gamma_m = \rho_m g = 13{,}550 \frac{kg}{m^3}\left(9.81 \frac{m}{s^2}\right) = 132{,}900 \frac{N}{m^3}$$

$$\gamma_w = \rho_w g = 998 \frac{kg}{m^3}\left(9.81 \frac{m}{s^2}\right) = 9790 \frac{N}{m^3}$$

$$\gamma_o = SG \rho_w g = 0.780\ (1000 \frac{kg}{m^3}) 9.81 \frac{m}{s^2} = 7650 \frac{N}{m^3}$$

Solving for the unknown heights:

$$9790 \frac{N}{m^3}\ (0.27m) + 132{,}900 \frac{N}{m^3} h_1 = 132{,}900 \frac{N}{m^3} (0.08m) \qquad h_1 = 0.060\ m = 6.00\ cm$$

$$132{,}900 \frac{N}{m^3} (0.08m) = 7650 \frac{N}{m^3} h_2 + 132{,}900 \frac{N}{m^3} (0.05m) \qquad h_2 = 0.521\ m = 52.1\ cm$$

**3.** Specific weights at 20ºC (Tables A.3 and A.1), as in a previous problem:

$$\gamma_w = 9790 \frac{N}{m^3}, \qquad \gamma_B = 8640 \frac{N}{m^3}, \qquad \gamma_K = 7890 \frac{N}{m^3}, \qquad \gamma_m = 132{,}900 \frac{N}{m^3}$$

Working from point A to point B:

$$p_A + \gamma_B\ (0.20\ m) - \gamma_m (0.08\ m) - \gamma_K (0.32\ m) + \gamma_w (0.40\ m) - \gamma_w (0.14\ m) = p_B$$

$$p_A - p_B = -\gamma_B\ (0.20\ m) + \gamma_m (0.08\ m) + \gamma_K (0.32\ m) - \gamma_w (0.40\ m) + \gamma_w (0.14\ m)$$

$$p_A - p_B = -\ 8640 \frac{N}{m^3} (0.20\ m) + 132{,}900 \frac{N}{m^3} (0.08\ m) + 7890 \frac{N}{m^3} (0.32\ m) - 9790 \frac{N}{m^3} (0.40\ m) + 9790 \frac{N}{m^3} (0.14\ m) = 8880\ Pa$$

**4.** (a) Specific weights at 20ºC (Tables A.3 and A.1), as in a previous problem:

$$\gamma_w = 9790 \frac{N}{m^3}, \qquad \gamma_m = 132{,}900 \frac{N}{m^3}$$

Working from the open end on the right side:

$$p_{atm} + \gamma_w\ L \cos 35^o - \gamma_w h_3 + \gamma_m h_2 - \gamma_m h_1 = p_A$$

Recall that the gauge pressure is $p_A - p_{atm}$, where $p_A$ is the absolute pressure at point A. Thus, the *gauge pressure* at point A is:

$$p_A - p_{atm} = \gamma_w\ L \cos 35^o - \gamma_w h_3 + \gamma_m h_2 - \gamma_m h_1$$

$$p_A - p_{atm} = 9790 \frac{N}{m^3}\ (1.20m) \cos 35^o - 9790 \frac{N}{m^3} (0.18\ m) + 132{,}900 \frac{N}{m^3} (0.32m - 0.15)$$

$$p_A - p_{atm} = 30{,}500 \frac{N}{m^2} = 30.5\ kPa$$

(b) The *absolute pressure* at point A is the gauge pressure plus the local atmospheric pressure:

$$p_A = 30{,}500 \frac{N}{m^2} + p_{atm} = 30.5\ kPa + 99.5\ kPa = 130.0\ kPa$$

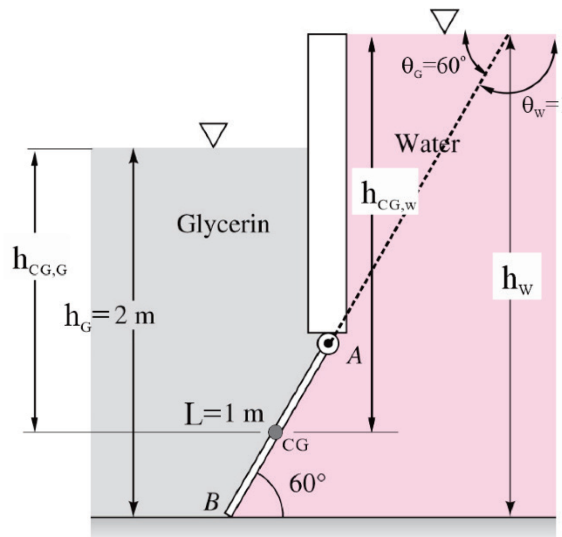
(c) The density of air must be calculated using the **absolute pressure**. From the ideal gas equation of state:

$$\rho = \frac{p_A}{RT} = \frac{130x10^3 \frac{N}{m^2}}{287 \frac{Nm}{kgK}(20+273)K} = 1.55 \frac{kg}{m^3}$$

**5.** Specific weights at 20ºC (Tables A.3 and A.1): $\gamma_W = 9790 \frac{N}{m^3}$, $\gamma_G = 1{,}260 \frac{kg}{m^3}\left(9.81 \frac{m}{s^2}\right) = 12{,}360 \frac{N}{m^3}$

On the glycerin side, the depth of the centroid of the gate is:

$$h_{CG,G} = h_G - \frac{L}{2}\sin 60^o = 2.0m - 0.5m\ (0.8660) = 1.567m$$




The hydrostatic force on the gate due to the glycerin, and its location relative to the centroid are:

$$F_G = \gamma_G h_{CG,G} A = 12{,}360 \frac{N}{m^3}(1.567m)1.2m^2 = 23{,}242\ N$$

$$y_{CP,G} = -\frac{I_{xx}\ sin\theta}{h_{CG,G}\ A} = -\frac{\frac{wL^3}{12} sin\theta_G}{h_{CG,G}\ A} = -\frac{\frac{1.2(1)^3}{12} m^4 (\sin 60^o)}{1.567m\ (1.2m^2)} = -0.04605m$$

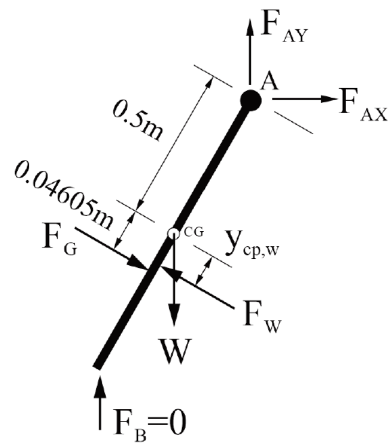
To find the depth of the water $h_W$, express the forces on the water side in terms of the depth of the centroid of the gate on the water side, $h_{CG,w}$:

$$F_W = \gamma_W h_{CG,w} A = 9790 \frac{N}{m^3} h_{CG,w} 1.2m^2 = 11{,}748\ h_{CG,w}$$

$$y_{CP,w} = -\frac{\frac{wL^3}{12} sin\theta_w}{h_{CG,w}\ A} = -\frac{\frac{1.2(1)^3}{12} m^4\ (\sin 120^o)}{h_{CG,w}\ (1.2m^2)} = -\frac{0.07217}{h_{CG,w}}$$

Consider the free body diagram for the gate:

The weight of the gate is: $W = mg = 180\ kg\left(9.81\frac{m}{s^2}\right) = 1766\ N$




The force at point B will be zero just as the gate starts to open. Taking moments about point A (to avoid calculating the forces at the hinge), for static equilibrium:

$$\sum M_A = 0$$

$$F_G(0.5461\ m) + W(0.5m\cos 60^o) - F_W\left(-y_{CP,w} + 0.5\ m\right) = 0$$

$$23{,}242\ N(0.5461m) + 1766\ N(0.25m) = 11{,}748\ h_{CG,w}\left(\frac{0.07217}{h_{CG,w}} + 0.5m\right) = 0$$

Solving for $h_{CG,w}$:

$$h_{CG,w} = 2.09m$$

Noting that:

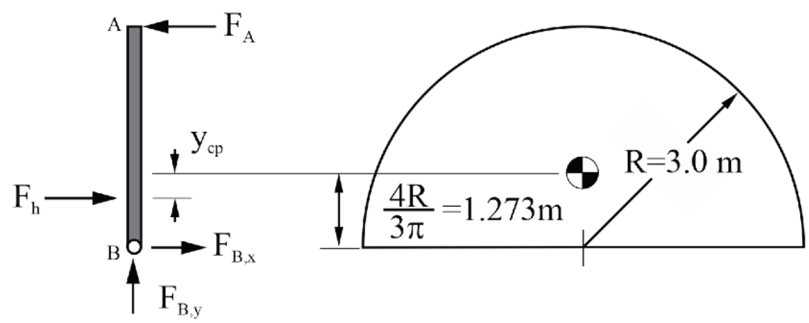
$$h_{CG,w} = h_w - \frac{L}{2}\sin 60^o\,, \qquad 2.09m = h_w - 0.5m\ (0.8660)$$

$$h_w = 2.52\ m$$

**6.** Referring to the centroid figure in the textbook, the centroid of the semi-circle is located at $\frac{4R}{3\pi} = 1.273m$ from the hinge at B. Also, the second moment of area is $I_{xx} = 0.10976R^4$. The hydrostatic force of the water on the gate and its location are:

$$F_h = \gamma_w h_{CG} A = \gamma_w\left(h + \left(R - \frac{4R}{3\pi}\right)\right)\frac{\pi R^2}{2} = 9790\frac{N}{m^3}(5m + 1.727m)14.14m^2 = 931{,}000\ N \rightarrow$$

$$y_{cp} = -\frac{I_{xx}\ sin\theta}{h_{CG}\ A} = -\frac{0.10976\ R^4\ sin\theta}{h_{CG,G}\ A} = -\frac{8.891m^4\sin 90^o}{6.727m\ (14.14m^2)} = -0.0935\ m$$




Free Body Diagram

Taking moments about point B (to avoid calculating the forces at the hinge), for static equilibrium:

$$\sum M_B = 0$$

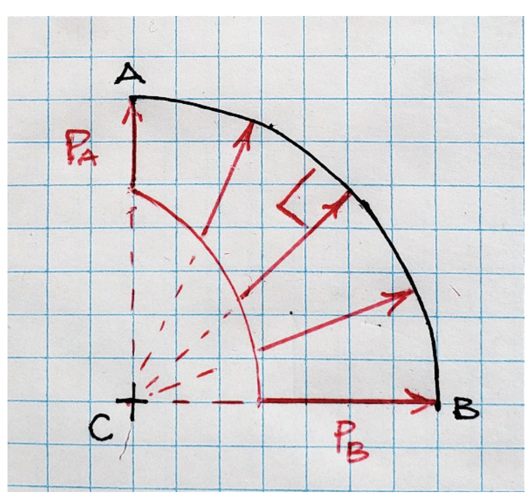
$$F_h(1.273m - 0.0935m) - F_A(3.0\ m) = 0$$

$$931{,}000\ N(1.179m) = F_A(3.0\ m)$$

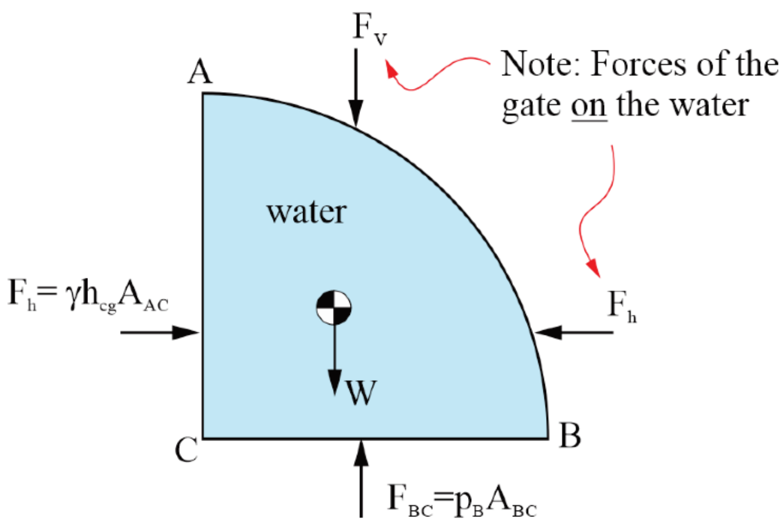
$$F_A = 366{,}000\ N = 366\ kN\ \leftarrow$$

7. (a) These are the key features your sketch should have to show a full understanding: The hydrostatic pressure acts normal to the surface – hence, all the pressure vectors must pass through centre of curvature, point C. The pressure at point A is upward. Pressure increases with depth. So, the pressure at point B is horizontal and higher than at point A.

   An aside: This sketch demonstrates that the force of the water on the curved gate AB is upward ↑ and to the right →.  This insight is helpful for part (b).



(b) There is more than one way to solve this type of problem. My preferred method is to perform a force balance using a free body diagram of the water adjacent to the gate. A free body diagram of the water segment ABC is shown below. (Other choices will also work. But this is the simplest.)




Note that the horizontal hydrostatic force acts on a plane rectangular surface AC. So, the horizontal force $F_h$ is calculated in the same way as for plane gates:

$$F_h = \gamma_w h_{CG} A_{AC} = \gamma_w \left(h + \frac{R}{2}\right)(Rw)$$

$$F_h = 9790\frac{N}{m^3}(1.5m + 0.375m)0.90m^2 = 16{,}500\ N \rightarrow$$

The force of the <u>water *on* the gate</u> is to the right, as was also shown in part (a).

To get the vertical force, use static equilibrium in the vertical (z) direction:

$$\sum F_z = 0 \quad F_{BC} - F_V - W = 0$$

The weight of the fluid segment ABC is:

$$W = \gamma_w \forall_{ABC} = \gamma_w \left(\frac{\pi R^2}{4}\right) w = 9790\frac{N}{m^3}\left(\frac{\pi(0.75m)^2}{4}(1.2m)\right) = 5190\ N$$

Note that the vertical force $F_{BC}$ acts on a plane rectangular surface BC. Surface BC is at a constant depth of (h+R). So, the pressure is constant on this surface:

$$F_{BC} = \gamma_w (h + R) A_{BC} = \gamma_w (h + R)(Rw)$$

$$F_h = 9790\frac{N}{m^3}(1.5m + 0.75m)0.90m^2 = 19{,}825\ N$$

Making the substitutions into the vertical force balance equation:

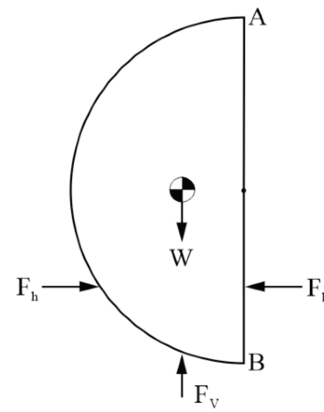
$$19{,}825N - F_V - 5190\ N = 0 \qquad F_V = 14{,}600\ N\ \uparrow$$

The vertical force of the water *on* the gate is upward, as was also seen in part (a). (The force of the gate on the water in the free body diagram has the opposite direction.)

**8.** Again, the preferred method is to perform a force balance on a free body diagram of the water adjacent to the gate, shown below.

The free body diagram shows that the horizontal force on the gate equal to the force on plane surface AB:

$$F_h = \gamma_w h_{CG} A_{AB} = \rho g \left(h + \frac{d}{2}\right) d\ w\ \leftarrow$$



Free Body Diagram

The horizontal force of the water on the gate is to the left.

The free body diagram shows that the vertical force on the gate is equal to the weight of the water volume contained by the semi-circular gate:

$$F_v = W = \gamma_w \forall_{AB} = \rho g \left(\frac{\pi R^2}{2}\right) w\ \downarrow$$

The vertical force of the water on the gate is the weight, which acts downward. (The force of the gate on the water in the free body diagram has the opposite direction.)

**9.** Using the density in Table A.3, the specific weight of SAE 30W oil is $\gamma_{oil} = 8740 \frac{N}{m^3}$

Note that the horizontal cross section is square. The force on surface AB is the pressure at the depth of the centroid of surface AB ($p_{cg,AB}$) times the area of AB:

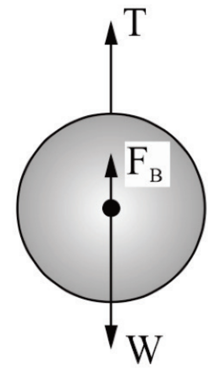
$$F_{AB} = p_{cg,AB} A_{AB} = (p_{air} + \gamma_o h_2 + \gamma_w \frac{h_1}{2}) A_{AB}$$

$$48{,}000\ N = [p_{air} + 8740 \frac{N}{m^3}(0.80m) + 9790 \frac{N}{m^3}(0.45m)](0.90m)1.6m$$

$$p_{air} = 21{,}900 \frac{N}{m^2} = 21.9\ kPa\ (gauge)$$

Note: This is the gauge pressure, which is what a Bourdon gauge reads.

**10.** Consider a free body diagram for each sphere shown in the sketch. The tension in the suspension wire is the weight of the sphere minus the upward buoyancy force:




$$T = W - F_B = \gamma_{solid}\, \forall_{sphere} - \gamma_{liquid}\, \forall_{sphere}$$

Equating the tension from the force balance on both spheres gives:

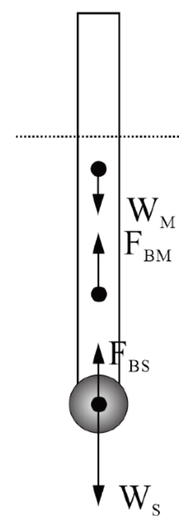
$$T = SG_A\,\gamma_w \left(\frac{\pi D_A^3}{6}\right) - \gamma_w \left(\frac{\pi D_A^3}{6}\right) = SG_B\,\gamma_w \left(\frac{\pi D_B^3}{6}\right) - \gamma_w \left(\frac{\pi D_B^3}{6}\right)$$

Simplifying:

$$D_A^3(SG_A - 1) = D_B^3(SG_B - 1)$$

$$D_B = D_A \sqrt[3]{\frac{(SG_A - 1)}{(SG_B - 1)}} = 7.0\ cm \sqrt[3]{\frac{(2.70 - 1)}{(8.50 - 1)}} = 4.27\ cm$$

**11.** See the free body diagram of the spar. For static equilibrium, the buoyancy cause by the submerged section of the maple wood and the steel balance the weights of the maple wood and steel:




$$\sum F_z = 0$$

$$W_M + W_S = F_{BM} + F_{BS}$$

$$SG_M\,\gamma_w \forall_M + SG_S\,\gamma_w \forall_S = SG_f\,\gamma_w \forall_{M,i} + SG_f \gamma_w \forall_S$$

The specific weight of water cancels:

$$SG_M\,\forall_M + SG_S\,\forall_S = SG_f\,\forall_{M,i} + SG_f \forall_S$$

The only unknown in this equation is the volume of steel (from which the weight can be found). The total volume of the maple wood is

$$\forall_M = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right) 12\ ft = 0.3333\ ft^3$$

The immersed volume of the spare is:

$$\forall_{M,i} = \left(\frac{2}{12}\right)\left(\frac{2}{12}\right) 10.5\ ft = 0.2917\ ft^3$$

Applying the force balance:

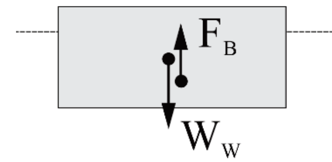
$$0.60(0.333\ ft^3) + 7.85\,\forall_S = 1.025(0.2917\ ft^3) + 1.025\forall_S$$

$$\forall_S = 0.01450 ft^3$$

Thus, the weight of this volume of steel is:

$$W_S = SG_S \gamma_w \forall_S = 7.85\left(62.4 \frac{lb}{ft^3}\right) 0.0145 ft^3 = 7.10\ lb$$

**12.** This solution involves two steps. First determine the specific weight of the unknown fluid $SG_f$ using a free body diagram of the wood block. For a floating block the weight equals the buoyancy force:




$$\sum F_z = 0$$

$$W_W = F_B$$

$$SG_W \gamma_W \forall_W = SG_f \gamma_W 0.75 \forall_W$$

$$SG_f = \frac{SG_W}{0.75} = \frac{0.60}{0.75} = 0.80$$

Knowing the specific gravity of the fluid, this is a simple manometry problem. Working from the open end of tube:

$$p_{atm} + \gamma_f(0.70m) - \gamma_f(0.70m + 0.40m) = p_{Air}$$

The gauge pressure of the air in the tank is:

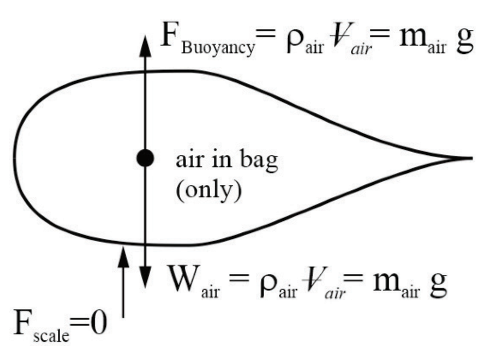
$$p_{Air} - p_{atm} = -\gamma_f(0.40m) = -0.80\left(9810\frac{N}{m^3}\right)(0.40\ m) = -3140\ Pa$$

The negative value means that the air in the tank is lower than local atmospheric pressure. There is a partial vacuum in the tank. This can also be expressed as $3140\ Pa\ vacuum$.

**13.** (a) Air at 20ºC and 1 atm has a density of 1.20 kg/m$^3$ or 1.20 g/litre (1 litre = 0.001 m$^3$). So, the mass of air in the bag is 1.2g. Weight is mass times acceleration due to gravity:

$$W_{air} = m_{air}g = 0.0012kg\left(9.806\frac{m}{s^2}\right) = 0.0118\ N$$

(b) The free body diagram of the air in the bag illustrates that weight of the air (which is not zero) is balanced by the buoyancy force due to displaced room air. So, the force on the scale is zero.




(c) Despite the fact that the air in the bag has weight, the scale will still read zero. Why? This is because a bag of air (at the same density at the room air) displaces its own weight in room air. According to Archimedes' principle, the bag experiences an upward buoyancy force equal to its weight. So, the downward force on the scale will still read zero after the bag is filled with air, as you might expect from everyday experience.

Of course, standard “weigh scales”, like the one shown in the figure, measure only the downward force of the object. They do not measure true weight nor mass. If you are measuring the weight low density materials (e.g. Styrofoam) the measurement error due to the buoyancy of the room air can become large. A correction is required.

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**Want more solved problems?** The midterm exams cover Chapters 1 and 2. There are several old midterm exams ***with full solutions*** posted on D2L.