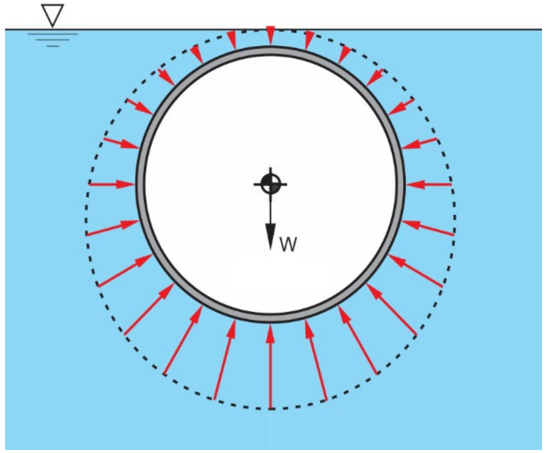
MEC516/BME516: Fluid Mechanics I

Chapter 2: Fluid Statics

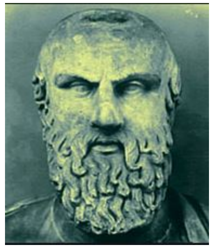
RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

The Buoyancy Force

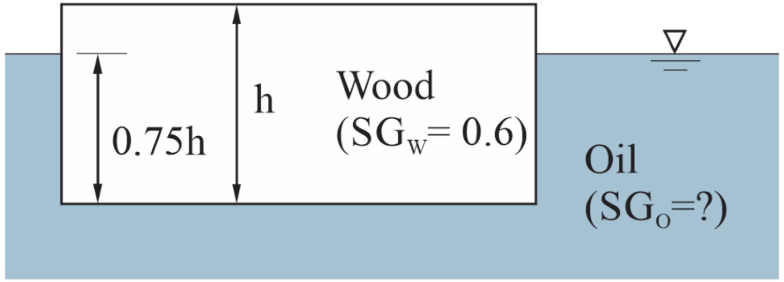
- Fundamental physics of buoyancy



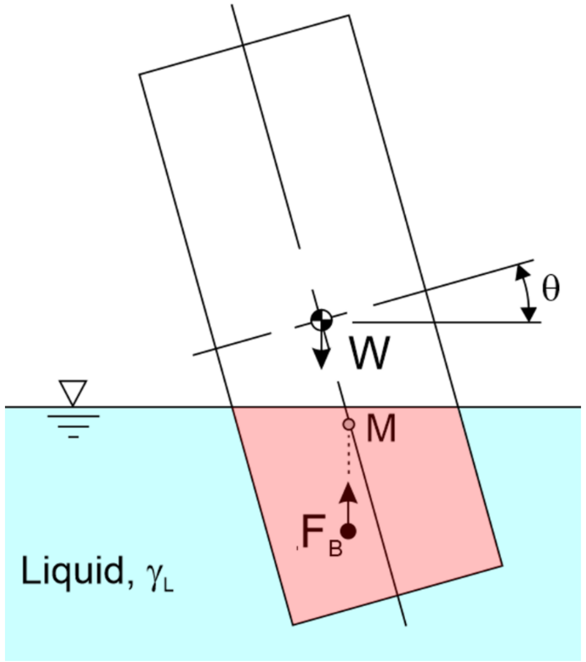
- Archimedes' principle



- Example problem

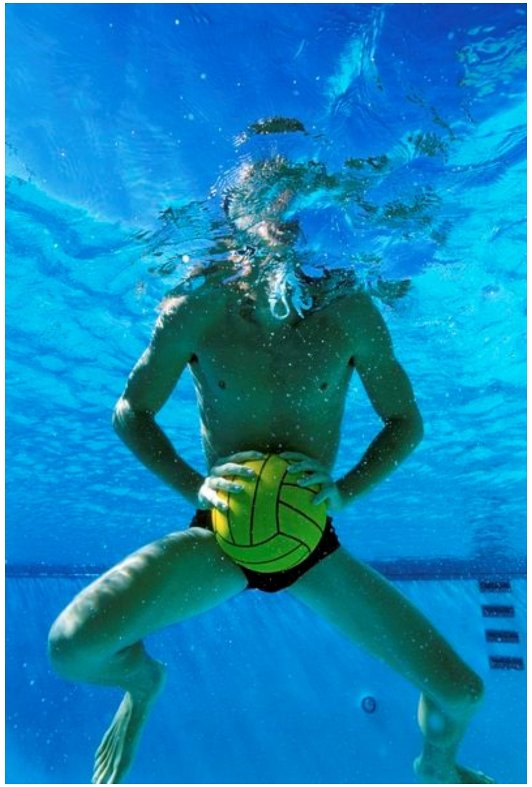


- Stability of floating objects



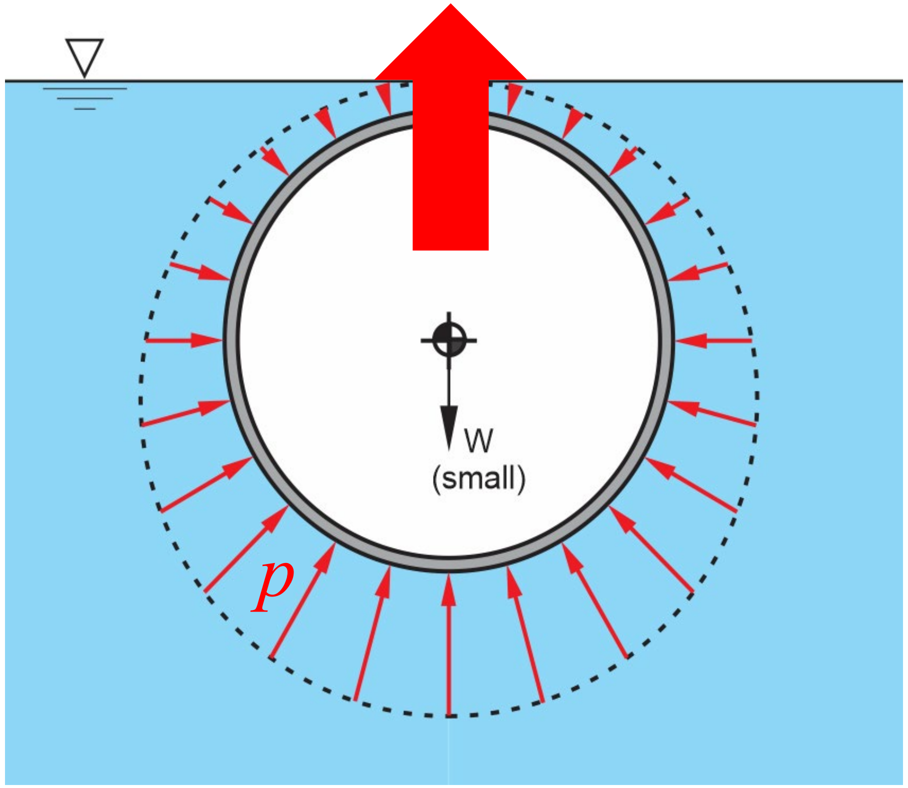
Buoyancy

- ***Fundamentally***, what is the cause of the upward buoyancy force on an submerged object?



- Key Concept: Buoyancy is the net result of the hydrostatic pressure distribution

Net upward pressure force, F_B

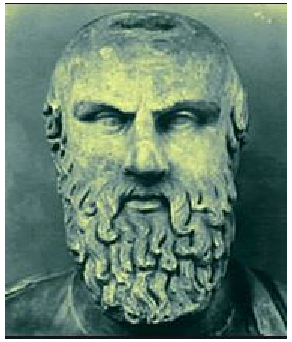


Hydrostatic pressure distribution on ball

Buoyancy Force: $F_B = \int_A p \, dA$

Luckily we don't have to solve this integral (difficult for complex shapes)

Buoyancy



- Archimedes' Principle (Greek mathematician/inventor, 3rd century BC):

"A body immersed in a fluid experiences a buoyancy force equal to the weight of displaced fluid"

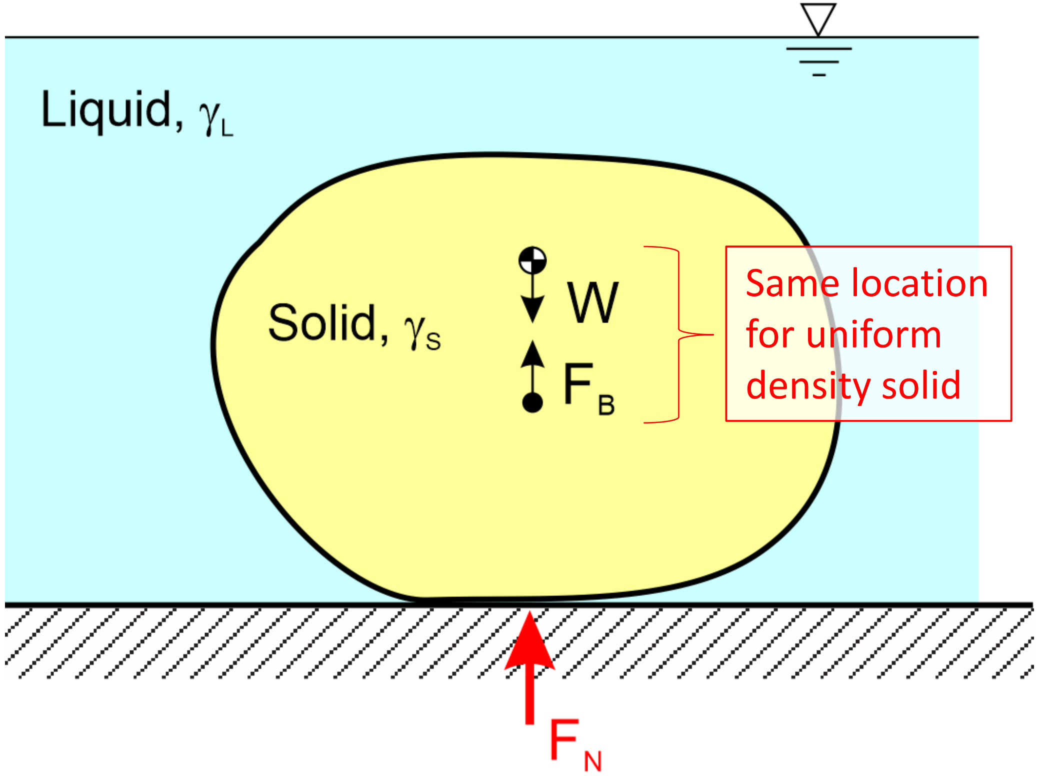
- W acts at the centre of mass of the solid
- F_B acts at the centroid of the displaced volume

So, $F_N = W - F_B = \gamma_S \forall_S - \gamma_L \forall_S$

F_B Weight of fluid displaced by solid

where $\forall_S$ is the volume of the solid

- Buoyancy force is small for gases (low density)



Buoyancy

- For a floating object: $W = F_B$

Weight of fluid displaced by solid

- It follows from Archimedes' Principle that:

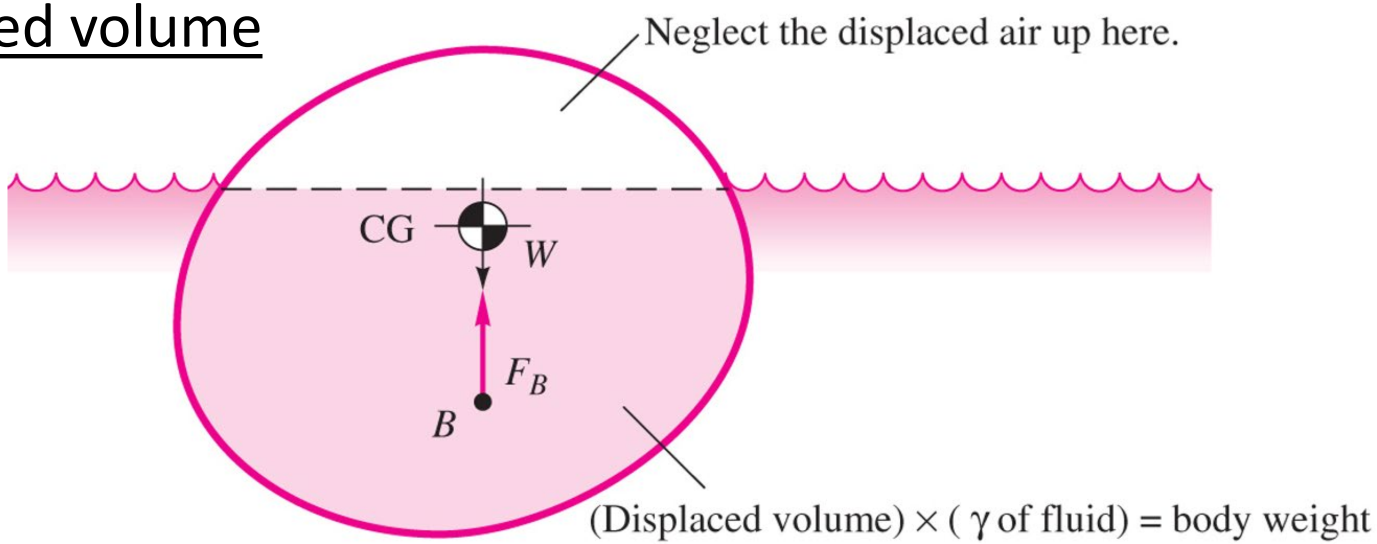
"A floating body displaces its own weight of the fluid in which it floats"

- W acts at the centre of mass of the solid
- F_B acts at the centroid of the displaced volume

$$W = F_B \qquad \gamma_S \forall_S = \gamma_L \forall_{displaced}$$

Solid Liquid

To float: $\gamma_S \leq \gamma_L$



Buoyancy

- We can show Archimedes' principle using simple hydrostatic analysis:

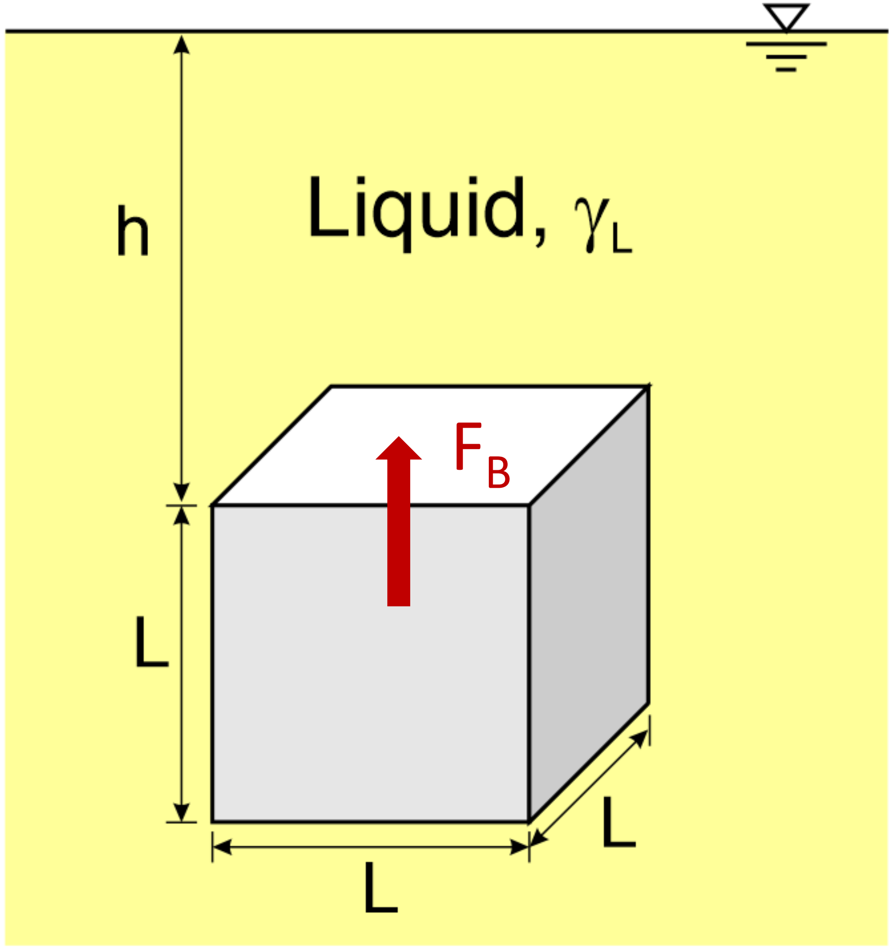
- Consider a cube with side length L, fully immersed a in liquid, γ_L

- Using Archimedes' principle, the buoyancy force is:

$$F_B = \gamma_L \forall_{displaced} = \gamma_L L^3 \quad \text{(weight of displaced fluid)}$$

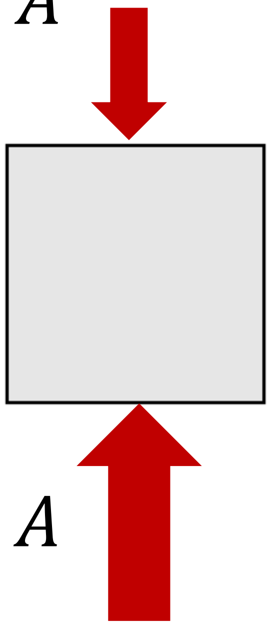
- Now calculate the buoyancy force using the hydrostatic forces on the top and bottom faces

  $(F_B = \int_A \, p \, dA)$

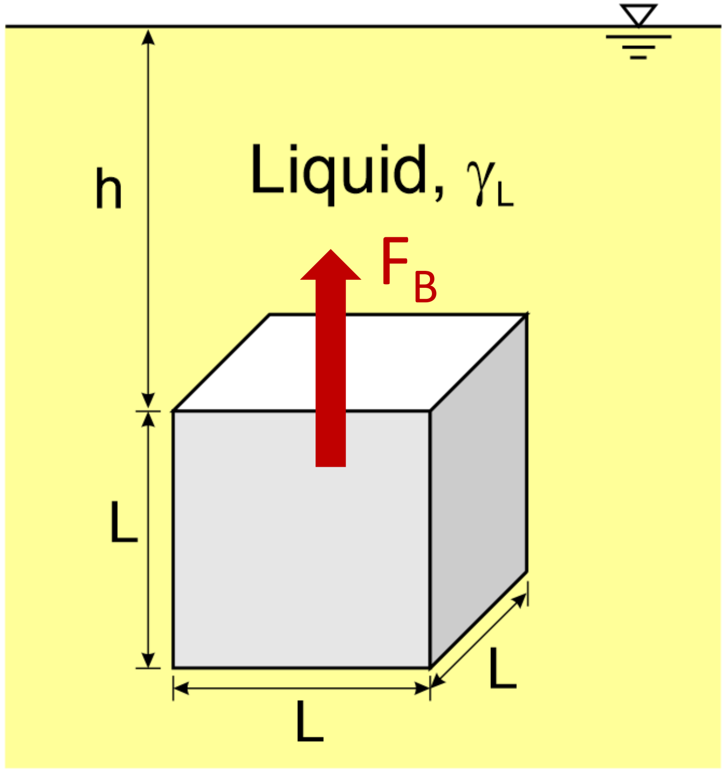


Buoyancy

$$F_{Top} = p_{Top}A = \gamma_L h\ A$$



$$F_{Bot} = p_{Bot}A = \gamma_L (h + L)\ A$$



Noting that $A=L^2$, the net upward pressure force on the cube is:

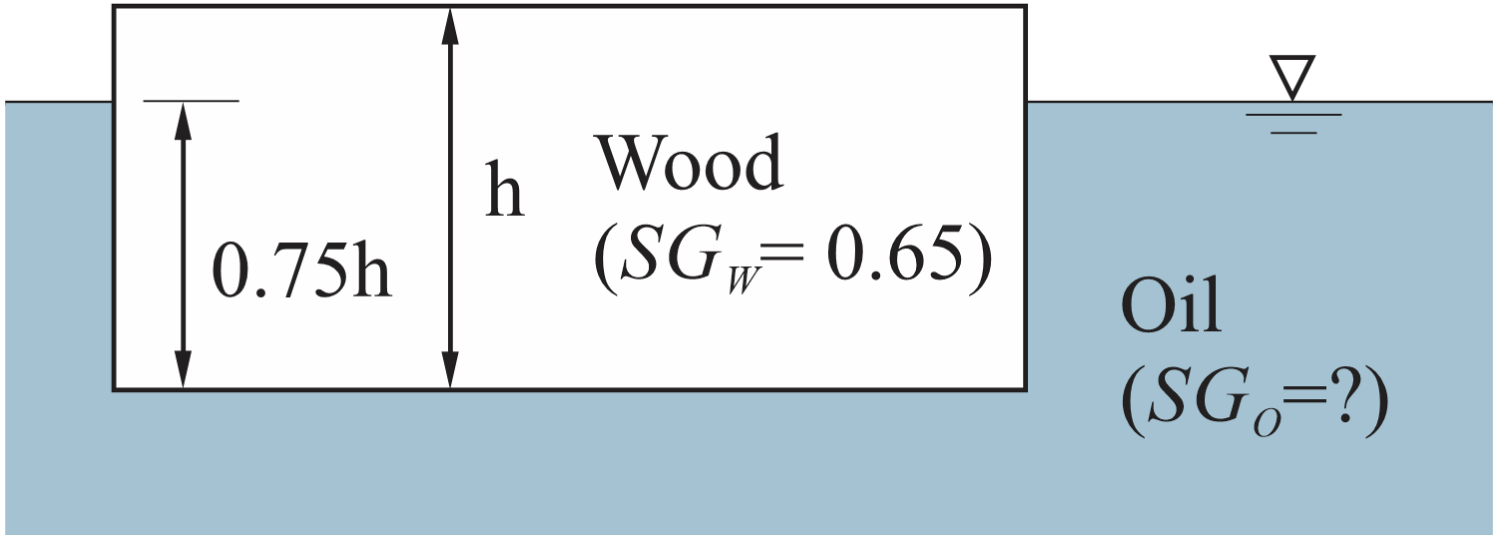
$$F_B = F_{Bot} - F_{Top} = \gamma_L (h + L)L^2 - \gamma_L h\ L^2 = \gamma_L\ L^3$$ Same as Archimedes' principle!

- **Key Concept**: Buoyancy is caused by the surface pressure distribution

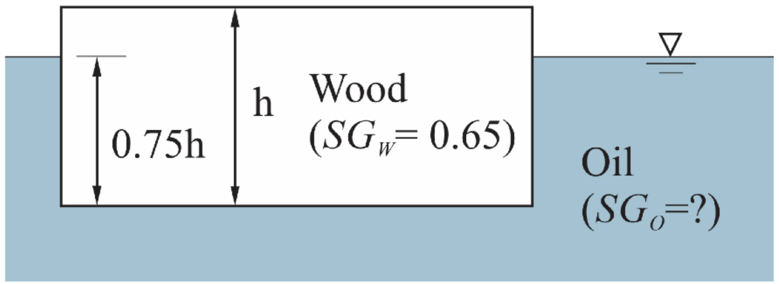
Example Problem (Midterm 2015)

A rectangular block of wood (SG_W = 0.65) floats on oil with 75 percent of its volume below the free surface i.e., 75% submerged.

What is the specific gravity the oil?

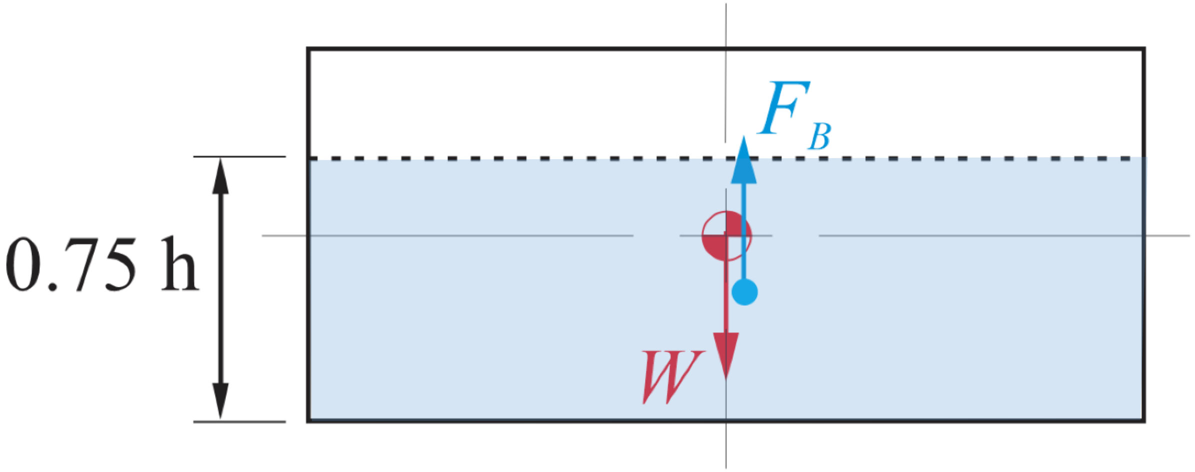


Example Problem



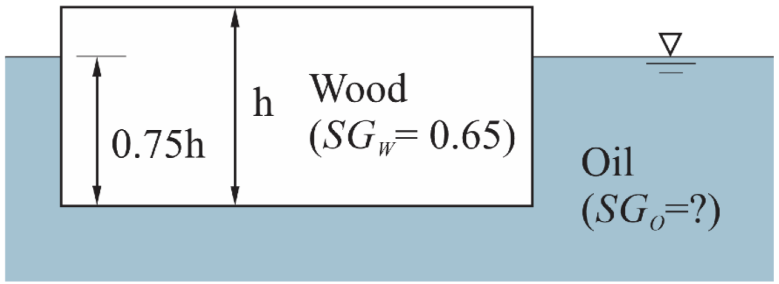
Solution

- What is the force balance for a floating object?
- Floating object: $F_B = \mathrm{W}$
- W acts at the centre of mass
- F_B acts at the centre of the displaced volume



Free Body Diagram

Example Problem



- Floating object: $F_B = W$

$$\gamma_o \forall_{displaced} = \gamma_w \forall_w$$

- Block is 75% submerged: $\forall_{displaced} = 0.75\ \forall_w$

- Thus: $0.75\ \gamma_o\ \forall_w = \gamma_w\ \forall_w$

- Divide by the specific weight of water: $0.75 \underbrace{\frac{\gamma_o}{\gamma_{water}}}_{SG_o} = \underbrace{\frac{\gamma_w}{\gamma_{water}}}_{SG_w}$

$$SG_o = \frac{SG_w}{0.75} = \frac{0.65}{0.75} = 0.867$$ Ans.

Stability of Floating Objects

- Not covering the mathematical details of stability. Engineers need to be aware of this issue. Naval architecture
- Stability is critical for the safe design of boats, barges, SAE mini-Baja vehicle, etc.
- Numerous accidents involving fishing boats capsizing

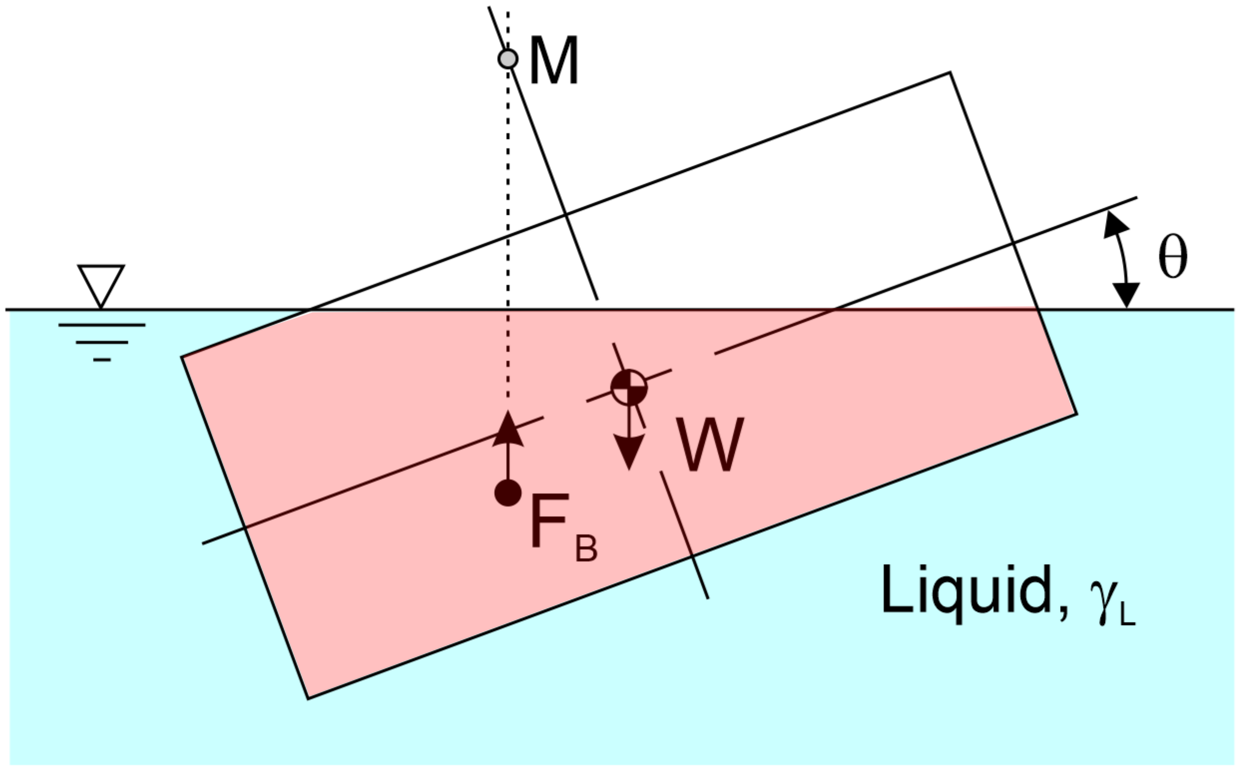
October 2015 near Tofino, British Columbia. Photo credit: Globe & Mail

Photo credit: Ryerson Baja

Stability of Floating Objects

Consider this floating object tipped at small angle θ from vertical:

- F_B will act at the centroid of the red area
- Clockwise restoring moment will act to reduce θ
- "Meta-centre" M is above CG
- This design will tend to be stable

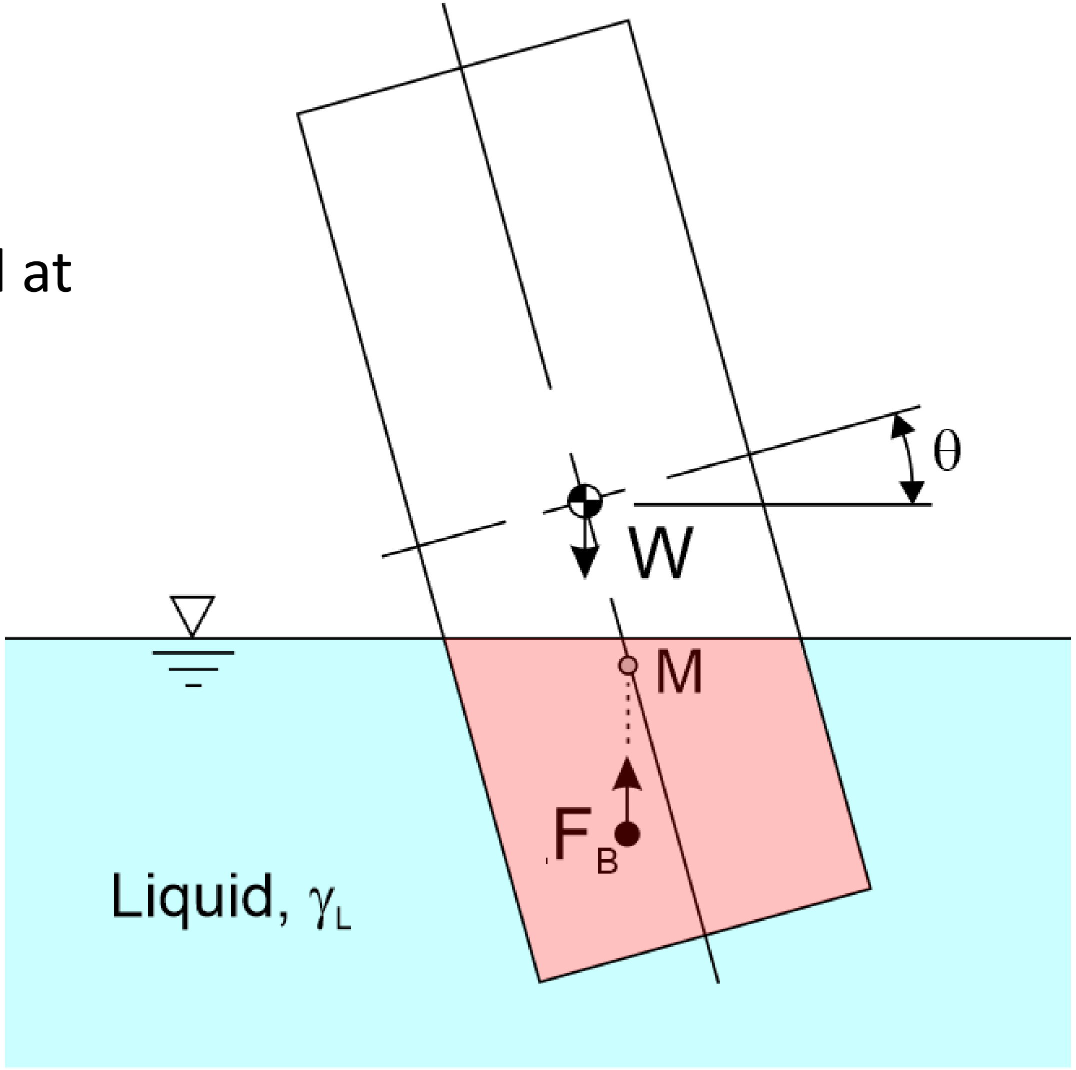


Stability of Floating Objects

- The previous floating object was wide and short

- Now consider a narrow/tall floating object tipped at small angle θ from vertical:

- "Meta-centre" M is below CG
- A counter clockwise moment will act to increase θ, i.e. to overturn
- This design will tend to be *unstable*



- Thus, a pencil will not float on water in an upright position -- unless you add a weight to the bottom to lower the CG
- Max. occupancy on the top deck of small fishing boats

Concept Question

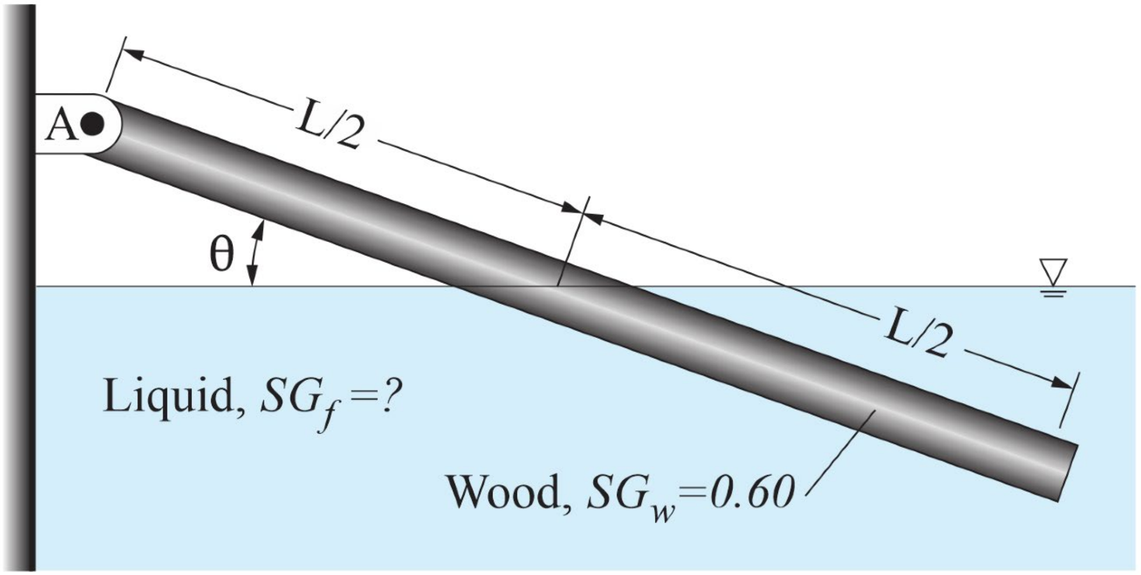
- Does buoyancy affect the reading on your bathroom scale or is this effect negligible?

Answer:

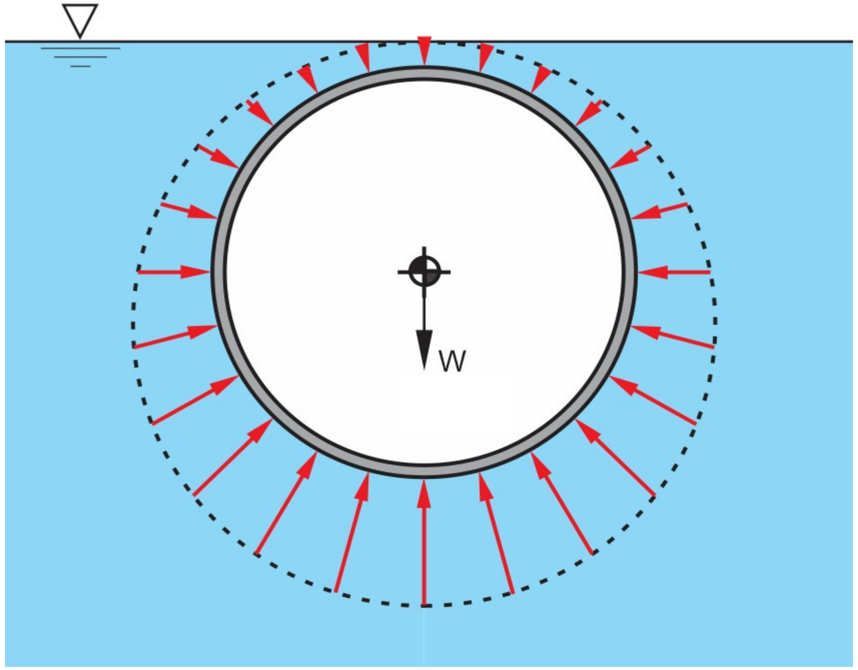
- If you could weigh yourself in a vacuum, you would weight a bit more
- Volume of a typical human body is ~60 liters. Displaces about 0.06 m^3 of air. Density of air is ~1.2 kg/m^3. So, the upward buoyancy force is ~0.07 kg or ~0.16 pounds. Small (~0.1%)…but not zero
- We often neglect this effect. But keep this approximation in mind, e.g. using a scale to get the mass of a piece of polystyrene → weight needs to be corrected

Example Problem

A long uniform wood rod is connected to a hinge and floats with half of its length in an unknown liquid, as shown in the sketch. The specific gravity of the wood is $SG_w = 0.60$. Calculate the specific gravity of the liquid.



Watch the Video Solution



Hydrostatic pressure distribution on ball

END NOTES

Presentation by Dr. David Naylor
Department of Mechanical and Industrial Engineering
Ryerson University, Toronto, Ontario
Canada