MEC516/BME516: Fluid Mechanics I

Chapter 2: Fluid Statics

RYERSON UNIVERSITY
Department of Mechanical
& Industrial Engineering

Hydrostatic Forces on Curved Surfaces

- Hydrostatic Forces on Curved Surfaces
  - Theory (use of FBDs)
  - Two solved examples

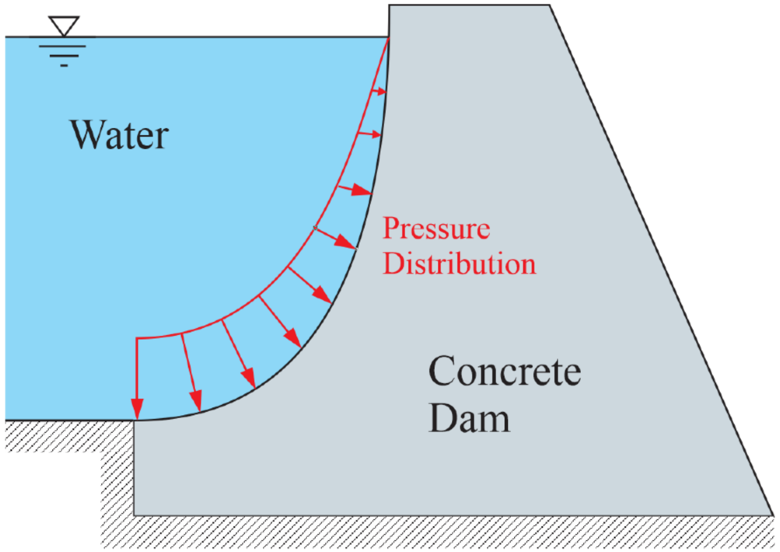
- Useful for the engineering design of:

  - Liquid containment structures
    (e.g. storage tanks, dams and levees)

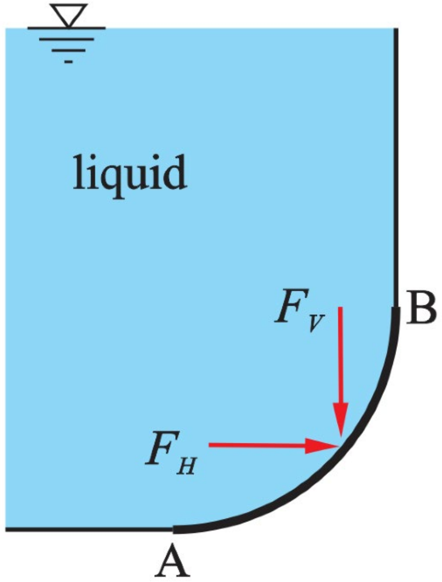
  - Ships, submarine vehicles

Hydrostatic Forces on Curved Surfaces

- Analysis is for ***liquids***:
  - Pressure increases linearly with depth
  - Gauge pressure distribution for incompressible fluids: $p = \gamma h$
  - Recall that pressure acts **normal** to bounding surface



- Goals of the analysis:
  1. Calculate the resultant forces on the surface: F_H, F_V
  2. Locate the line of action of each force

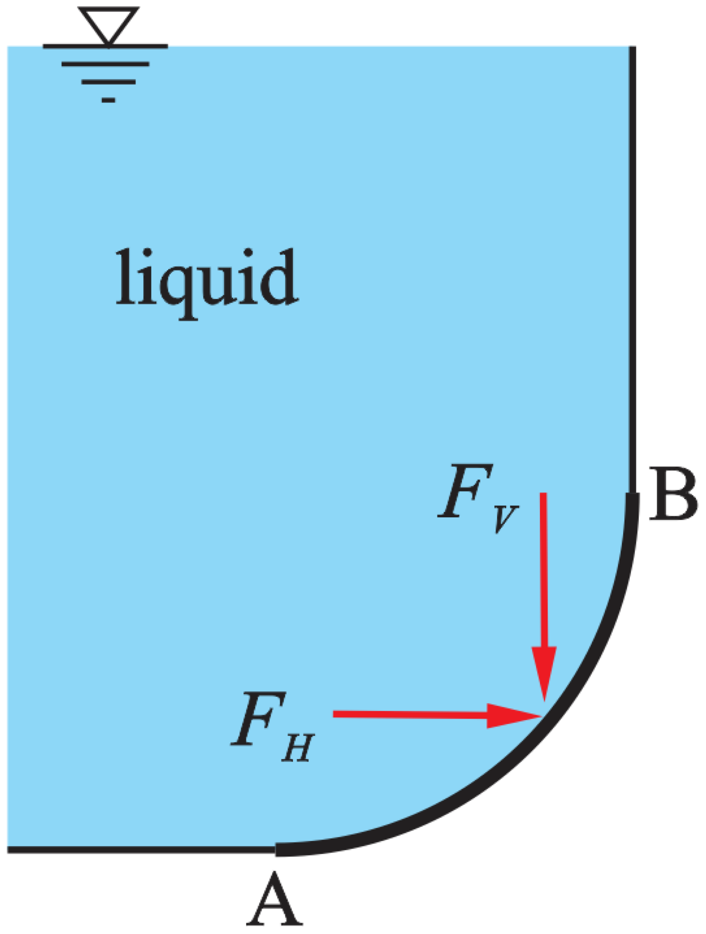


Hydrostatic Forces on Curved Surfaces

- While possible, integrating the local pressure distribution would be laborious

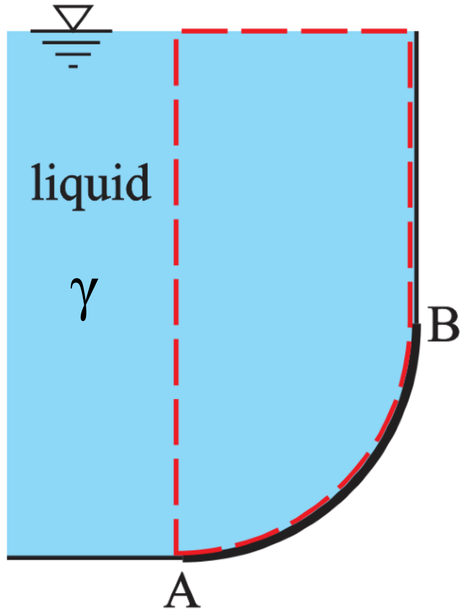
- Use a Free Body Diagram approach:

  I. Isolate a section of fluid adjacent to the surface. Draw a Free Body Diagram
  II. Decompose hydrostatic force into horizontal and vertical components
  III. Apply static equilibrium: $\sum F = 0$



Hydrostatic Forces on Curved Surfaces

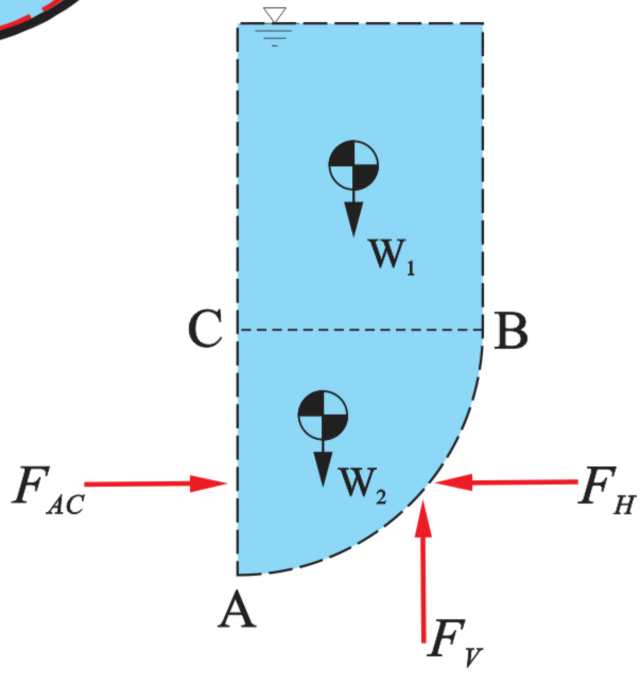
- Example: Consider the liquid above the curved gate (AB) in the diagram



Vertical Component: $\sum F_{vert} = 0$

$$F_v = W_1 + W_2 = \gamma\forall_1 + \gamma\forall_2$$

- Vertical component of the pressure force is equal to weight of the fluid above the surface



Free Body Diagram

- Note: FBD shows forces on the liquid

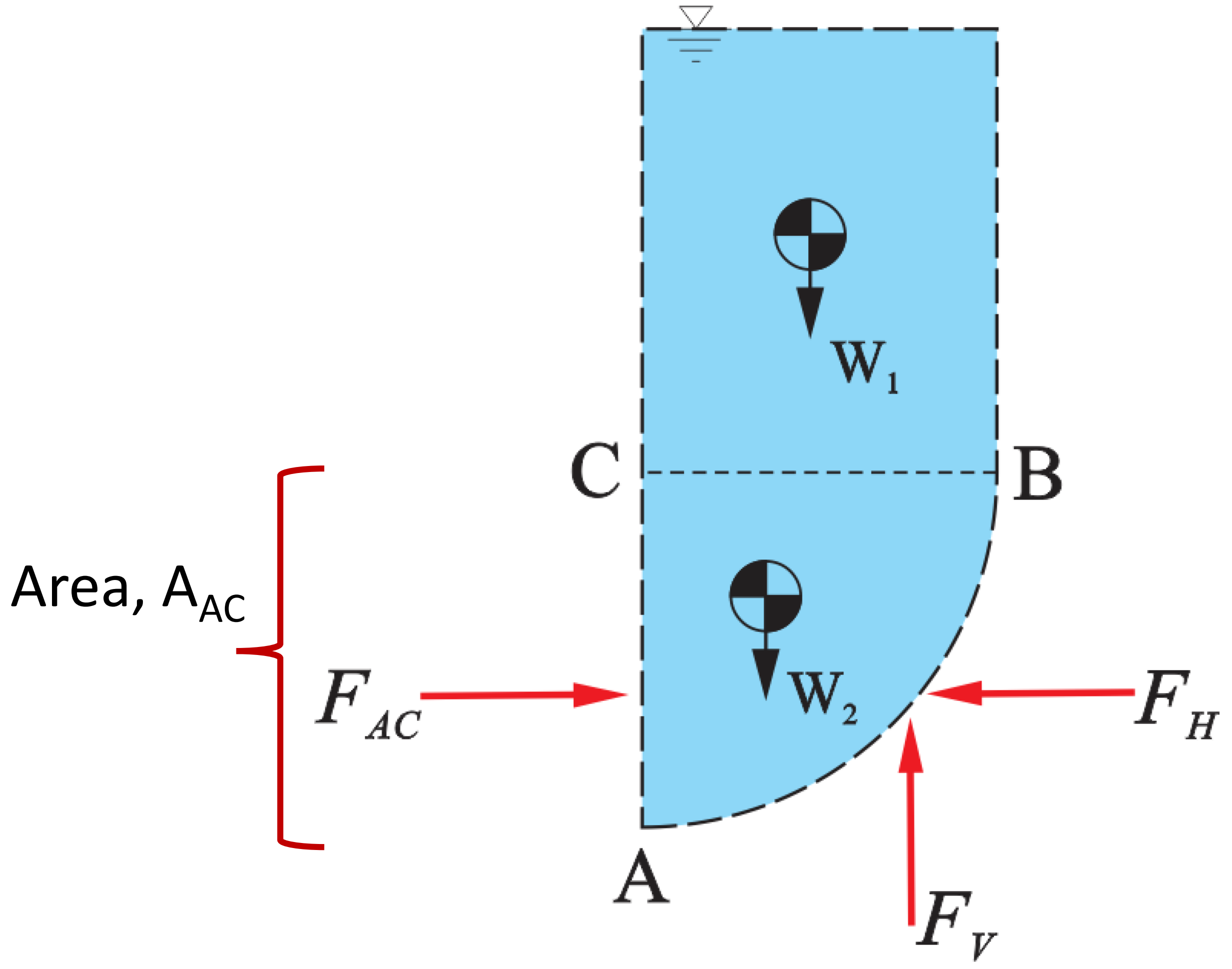
- The hydrostatic force of the liquid on the gate F_V

Hydrostatic Forces on Curved Surfaces

Horizontal Component: $\sum F_{horiz} = 0$

$$F_H = F_{AC} = \gamma h_{CG} A_{AC}$$

- Horizontal component of the pressure force equals the force on a projection of the curved surface into the vertical plane, AC
- F_H acts at the centre of pressure of the vertical face (AC)
- Plane gate methods to calculate F_H, y_{CP}



Example Problem (Midterm 2015)

Water (ρ=998 kg/m^3) is contained behind a semi-circular gate, AB with a radius of R=2.0m. The gate is hinged at point A. Neglect the weight of the gate.

Calculate:

(a) The horizontal and vertical components of the hydrostatic force on gate (AB) per unit depth (into the page). Clearly indicate the directions of the forces.

(b) The horizontal force (F_B) at point B needed to hold the gate in place (per unit depth)

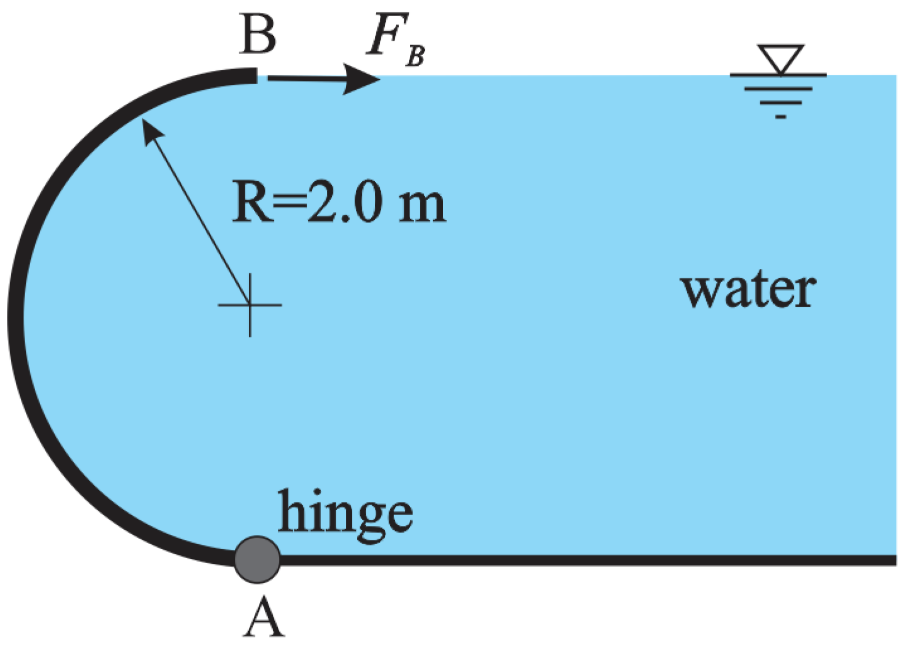
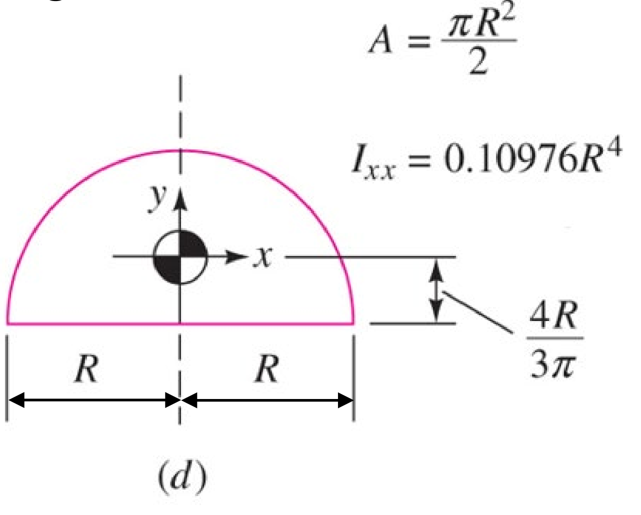
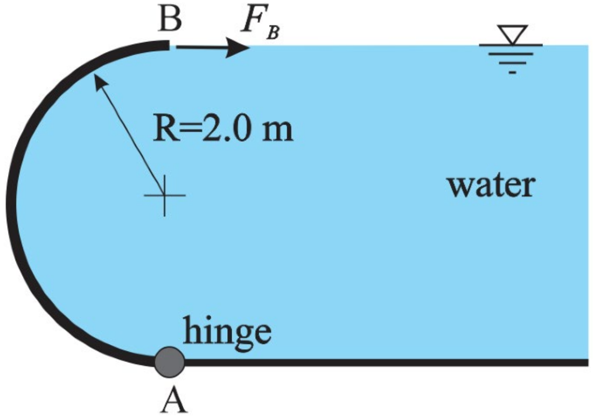


Figure 2.13

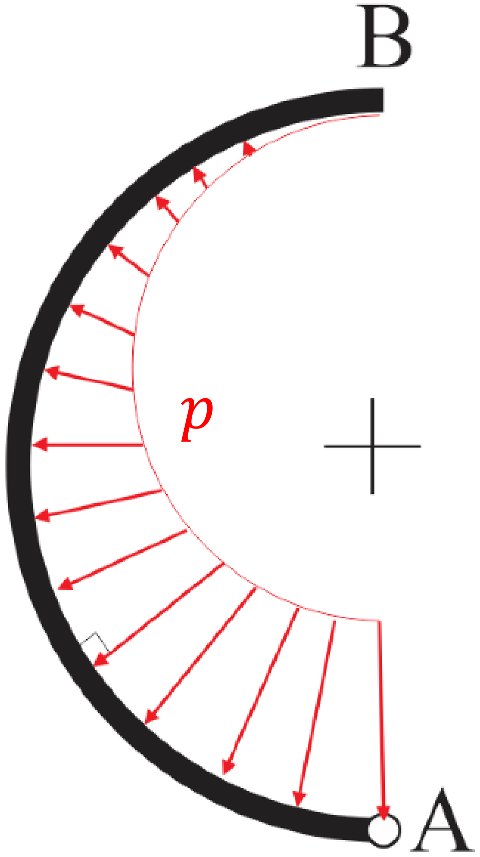


Example Problem (Midterm 2015)



Solution

(a) I recommend drawing the hydrostatic pressure distribution on the gate



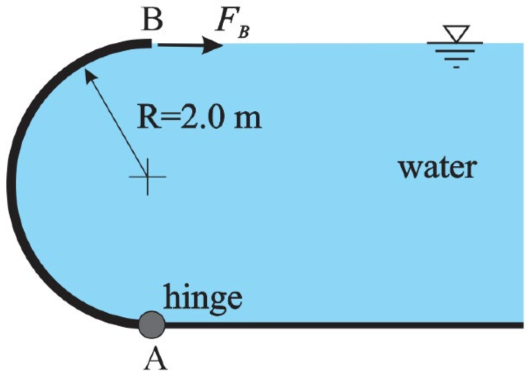
- The direction of the forces on the gate:

F_H ←

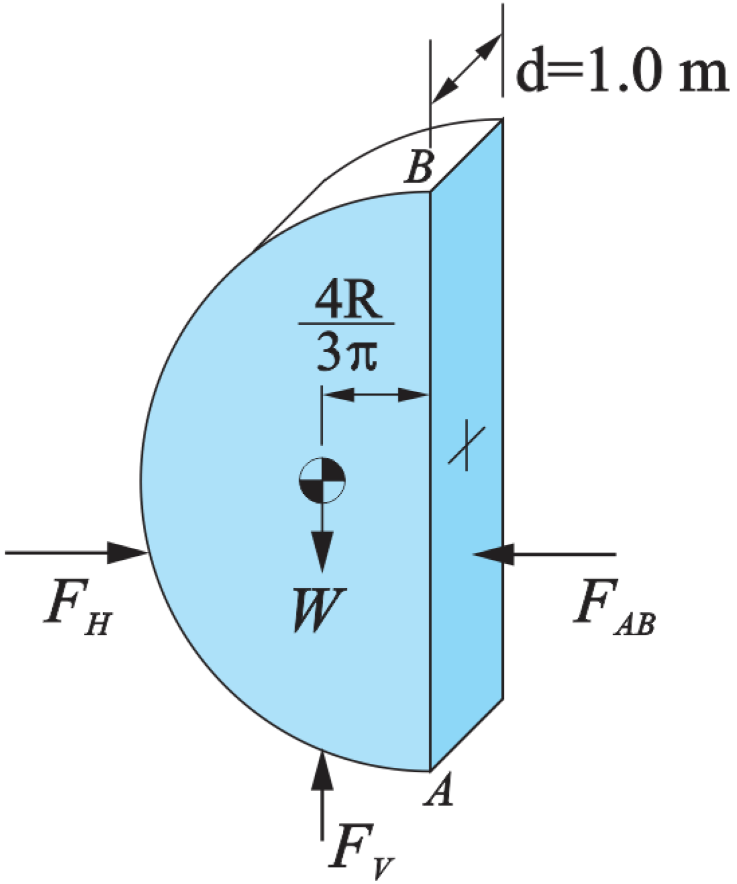
F_v ↓

Example Problem (Midterm 2015)

(a) Free body diagram of the water adjacent to the gate



- These are the forces on the water (opposite of forces on gate)

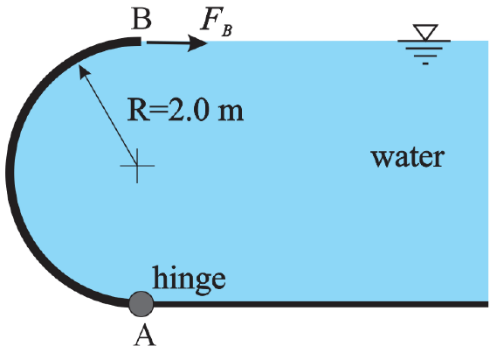


- Static equilibrium in vertical direction

$$\sum F_{vert} = 0 \quad F_V = W = \gamma \forall$$

$$F_V = \gamma \forall = \frac{\gamma(\pi R^2)d}{2}$$

Example Problem (Midterm 2015)



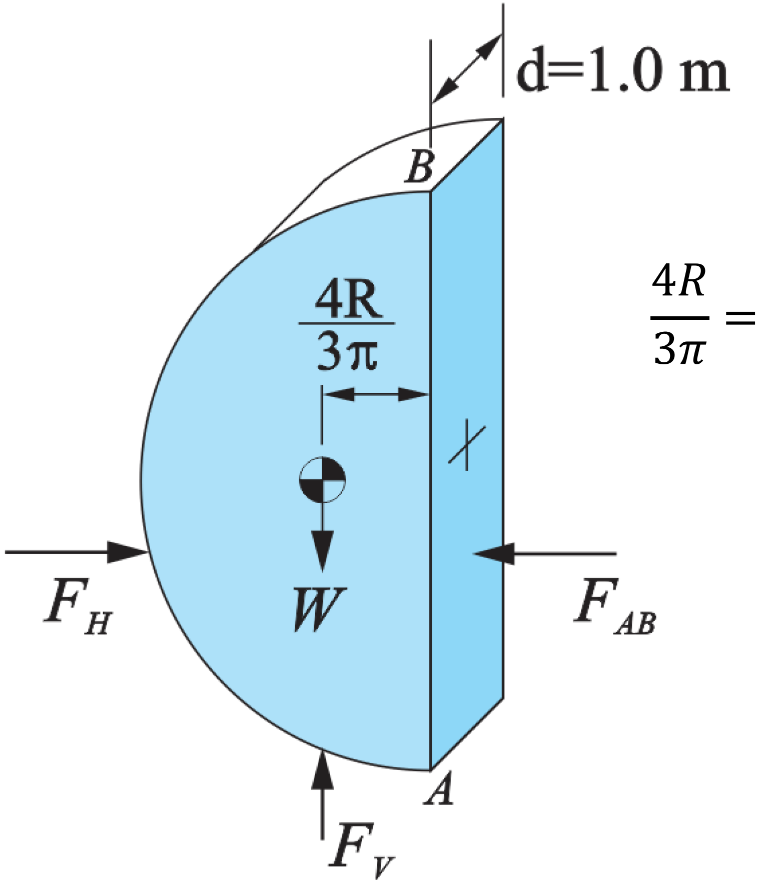
$$F_V = \gamma \forall = \frac{\gamma(\pi R^2)d}{2}$$

Thus, the vertical force on the gate is:

$$F_V = \frac{9790\,\frac{N}{m^3}\left(\pi(2m)^2\right)1m}{2} = 61.51\ kN\ \downarrow$$

- F_V acts in line with the weight at distance:

$$\frac{4R}{3\pi} = \frac{4\,(2.0m)}{3\pi} = 0.8488\ m$$



$$\frac{4R}{3\pi} = 0.8488\ m$$

Example Problem (Midterm 2015)

- Static equilibrium in horizontal direction

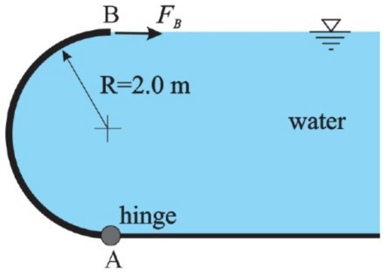
$$\sum F_{horiz} = 0 \quad F_H = F_{AB}$$

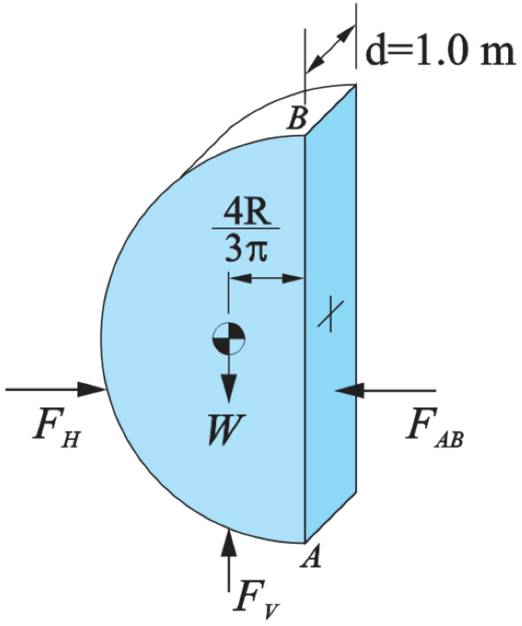
- Thus, F_H is equal to the force on vertical plane surface AB

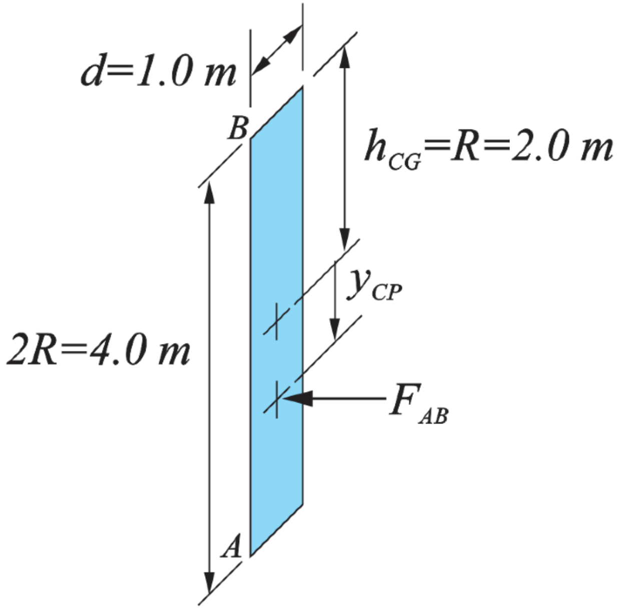
$$F_H = F_{AB} = \gamma\, h_{CG} A_{AB}$$

$$F_H = 9790 \frac{N}{m^3} (2.0\ m)\ 4.0\ m^2$$

$$F_H = 78.32\ kN\ \leftarrow$$







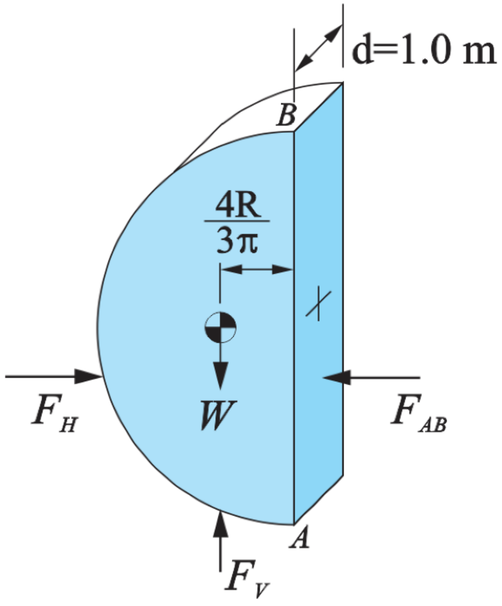
Example Problem (Midterm 2015)

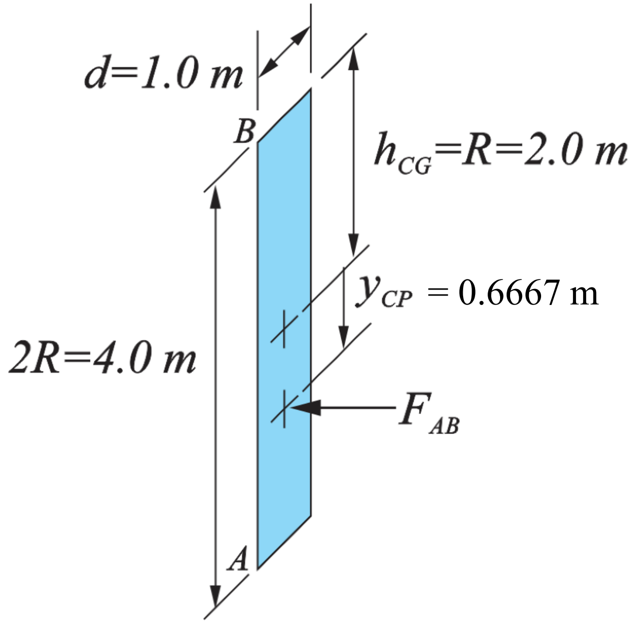
- Force F_H acts in line with F_{AB}
- F_{AB} acts below the centroid of surface AB:

$$y_{CP} = -\frac{I_{xx}\sin\theta}{h_{CG}A_{AB}}$$

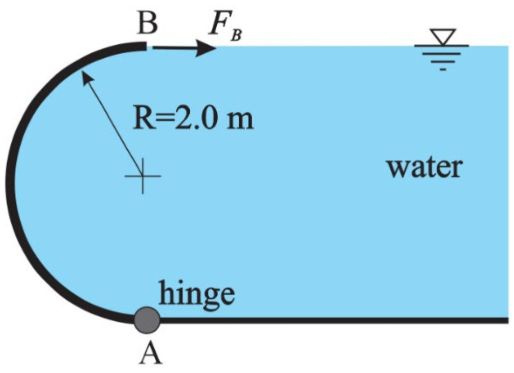
$$I_{xx} = \frac{d\,(2R)^3}{12} = \frac{1m(4m)^3}{12} = 5.333\,m^4$$

$$y_{CP} = -\frac{5.333\,m^4\sin(90^o)}{2.0m\,(4.0\,m^2)} = -0.6667\,m$$





Example Problem (Midterm 2015)



(b) Force (F_B) at point B needed to hold the gate in place

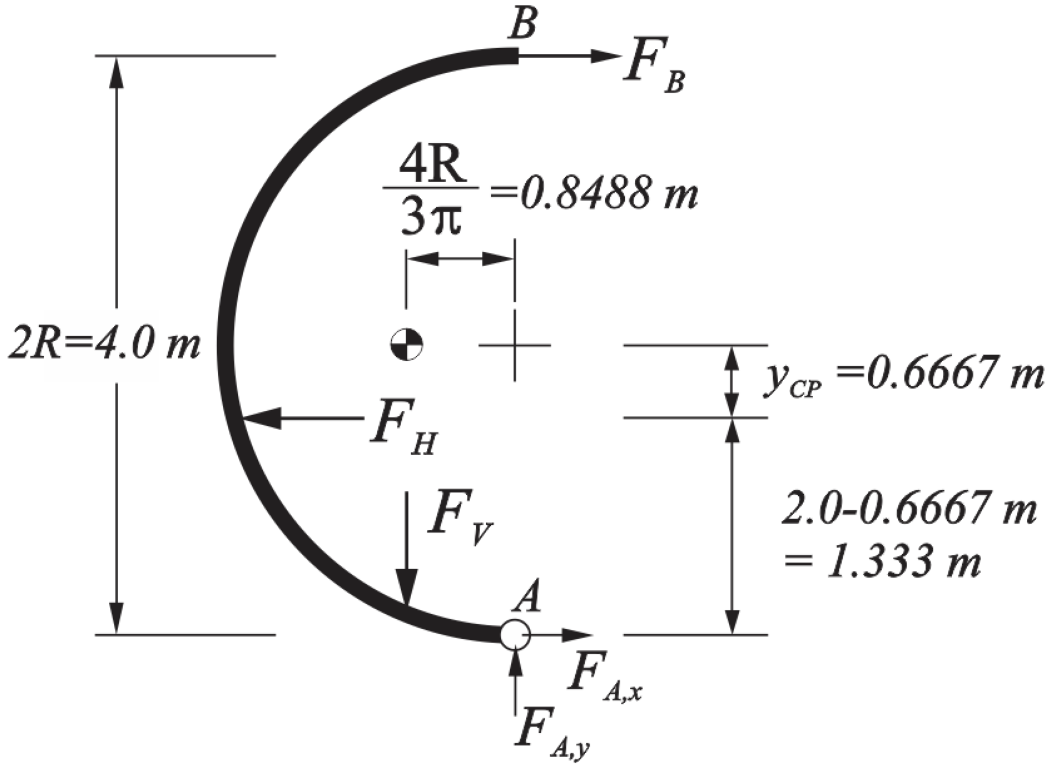
- Free body diagram for the gate

$$\sum M_A = 0 \quad \circlearrowright +$$

$$F_B(2R) - F_H(1.333m) - F_V(0.8488m) = 0$$

$$F_B = \frac{78.32\ kN(1.333m) + 61.51kN(0.8488m)}{4.0\ m}$$

$$F_B = 39.2\ kN \ \rightarrow$$ Ans.

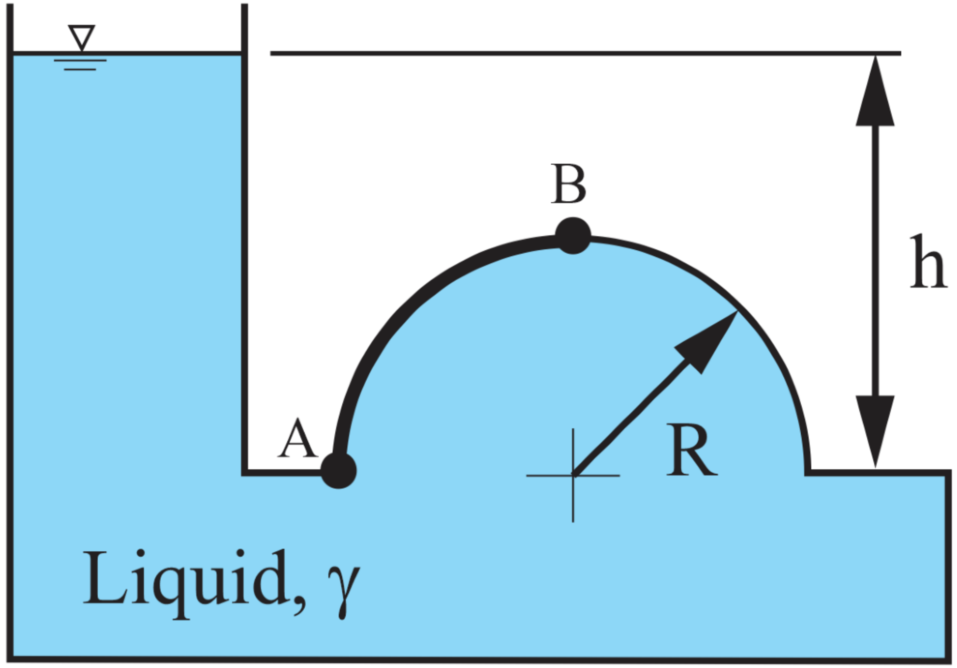


Example Problem

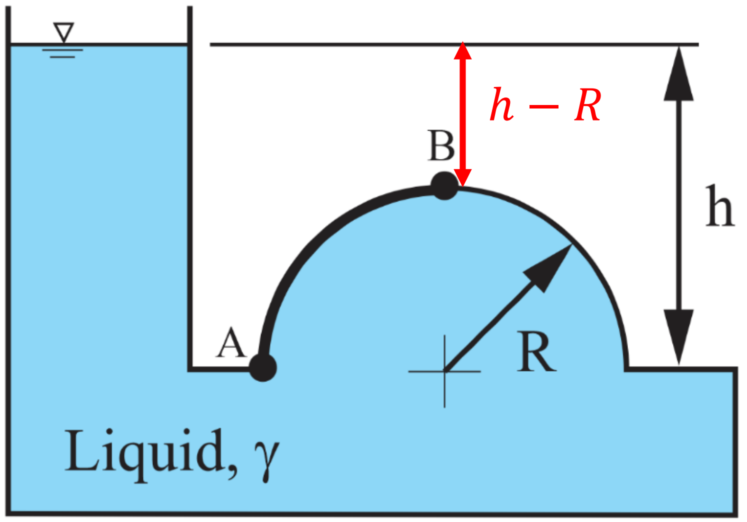
A liquid with specific weight (γ) is contained in a tank shown in the sketch. The tank has unit depth (into the page).

(a) Draw the hydrostatic pressure distribution on curved surface A-B

(b) Derive expressions for the vertical and horizontal hydrostatic forces on curved surface A-B. Clearly indicate the directions of the forces

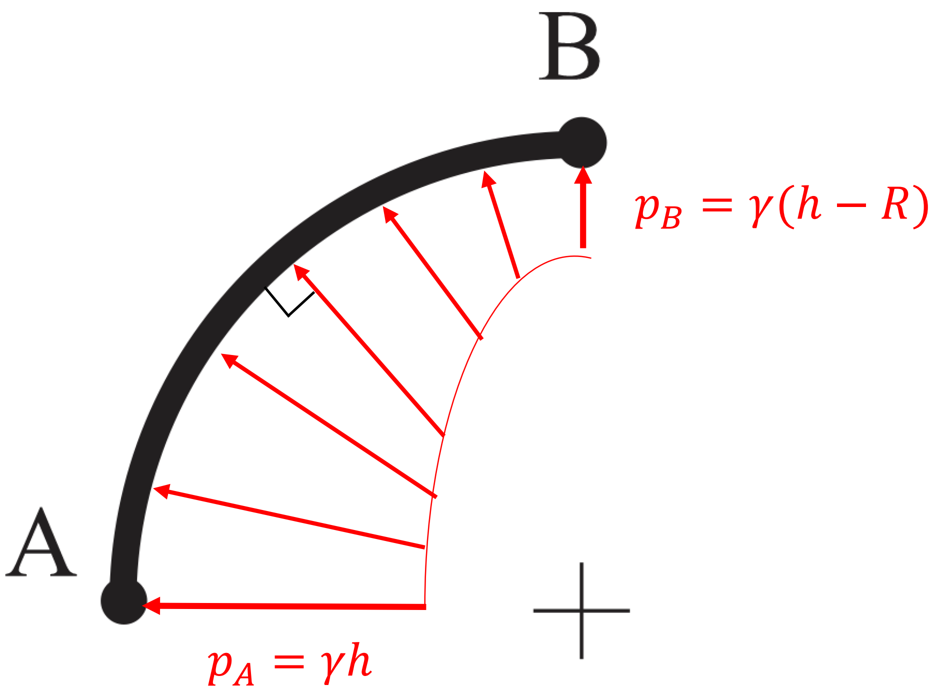


Example Problem



Solution

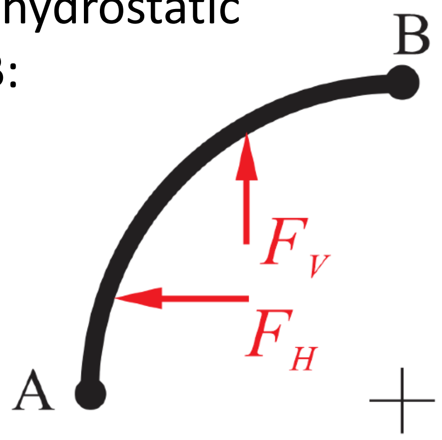
(a) hydrostatic pressure distribution on surface A-B



Sketch must have:

- $p_A > p_B$
- Arrow must be perpendicular to AB

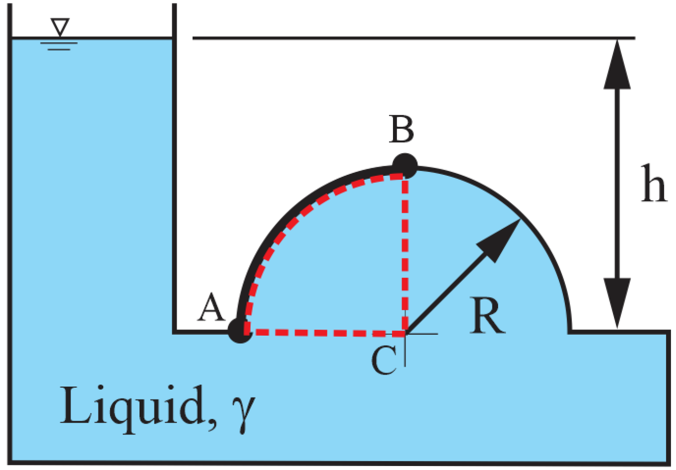
- Directions of the hydrostatic forces on gate AB:

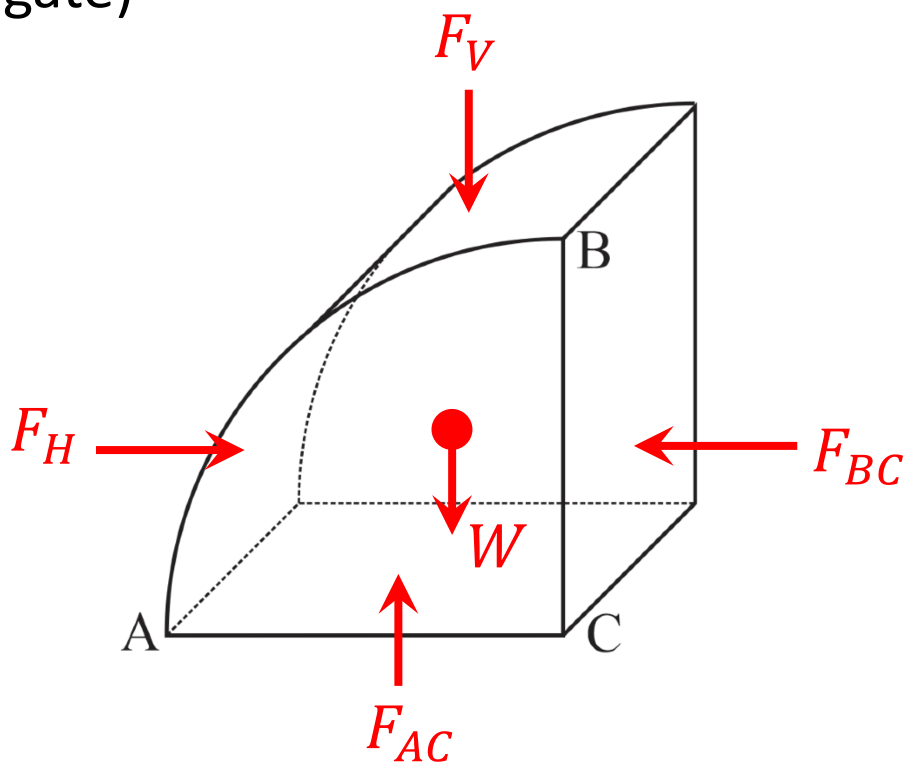


Example Problem

(b) Expressions for the vertical and horizontal hydrostatic forces

- Free body diagram of the forces on the water (under the gate)





Example Problem

- Static equilibrium: $\sum F_{horiz} = 0$

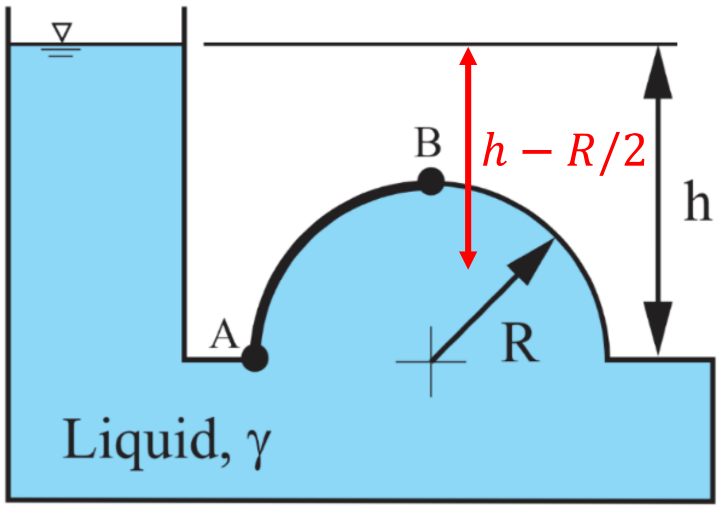
$$F_H = F_{BC} = \gamma h_{CG} A_{BC}$$

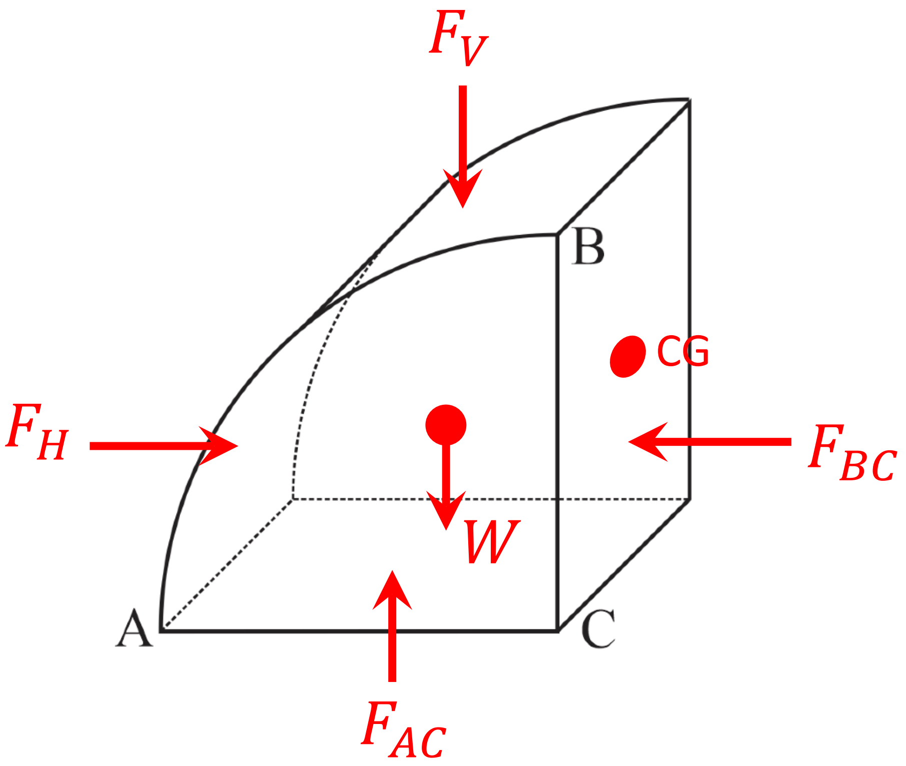
- For surface BC: $h_{CG} = h - \frac{R}{2}, A_{BC} = R(1)$

- Horizontal force on surface AB is:

$$F_H = \gamma \left(h - \frac{R}{2}\right) R \quad \leftarrow$$

Ans.





Example Problem

- Static equilibrium: $\sum F_{vert} = 0$

$$F_V = F_{AC} - W$$

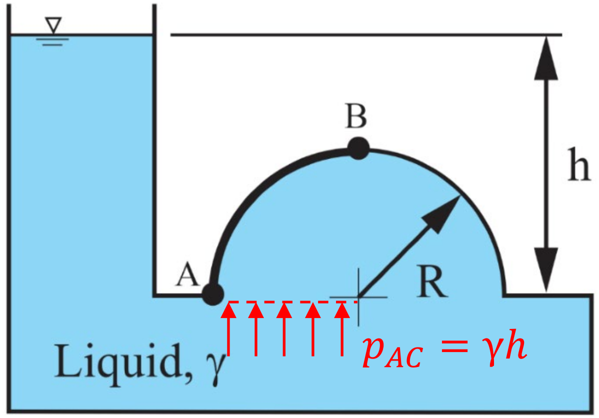
- For surface AC: $F_{AC} = \gamma h\, R(1)$

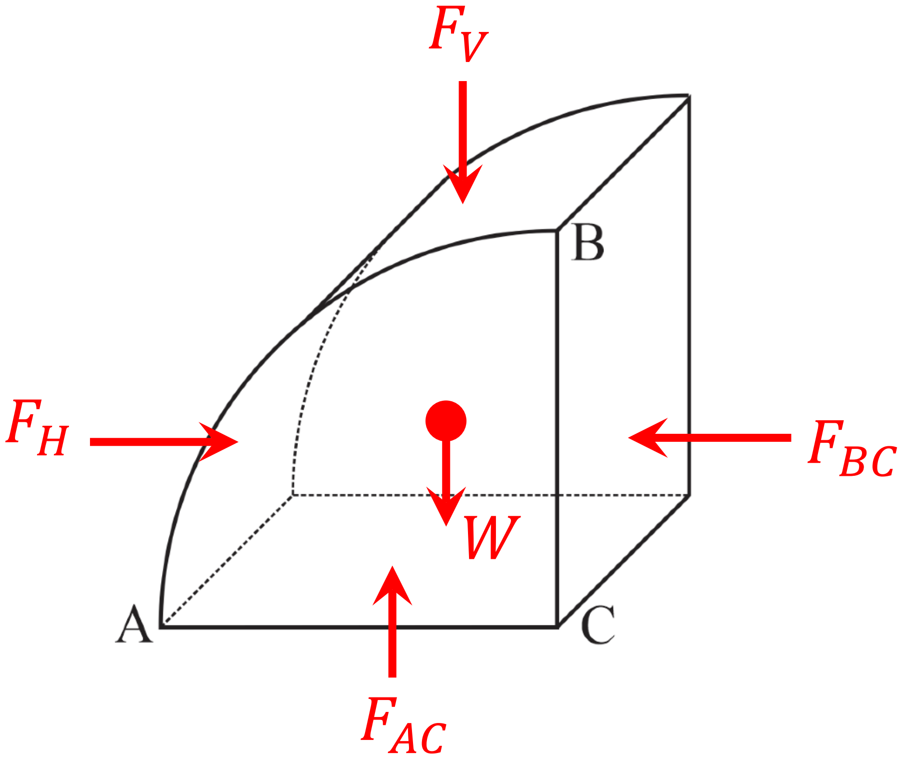
- Weight of the water: $W = \gamma\forall = \gamma \dfrac{\pi R^2}{4}(1)$

- Vertical Force on surface AB is:

$$\boxed{F_V = \gamma\, hR - \gamma \frac{\pi R^2}{4} \quad \uparrow}$$

Ans.

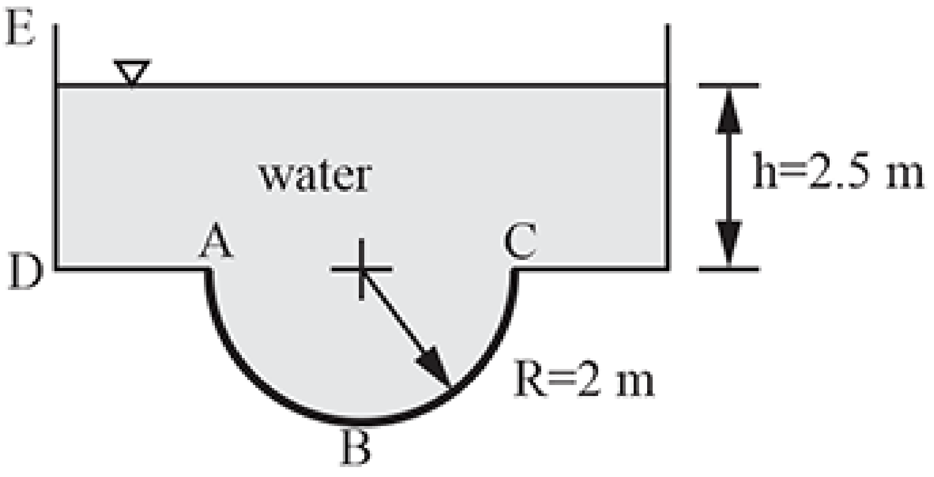


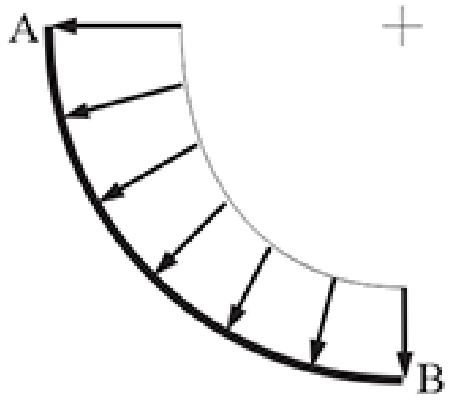


Pressure Distribution on a Curved Surface

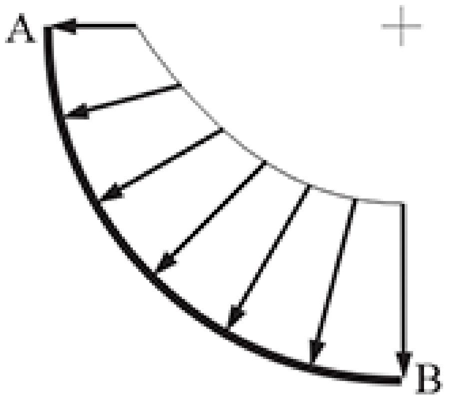
Which sketch of the pressure distribution is correct?

Answer: Distribution 2

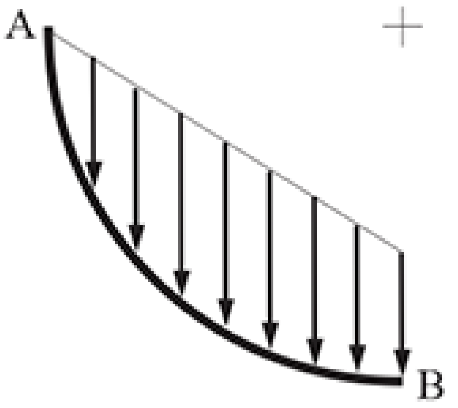




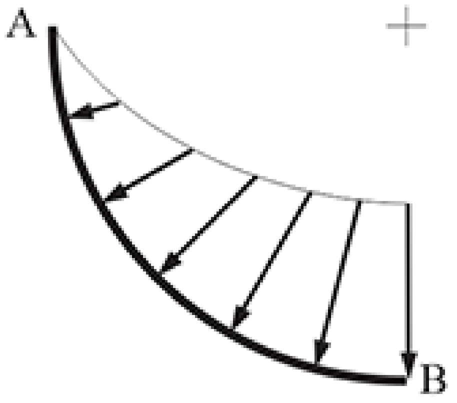
Pressure Distribution 1



Pressure Distribution 2



Pressure Distribution 3



Pressure Distribution 4

What is wrong with the others?

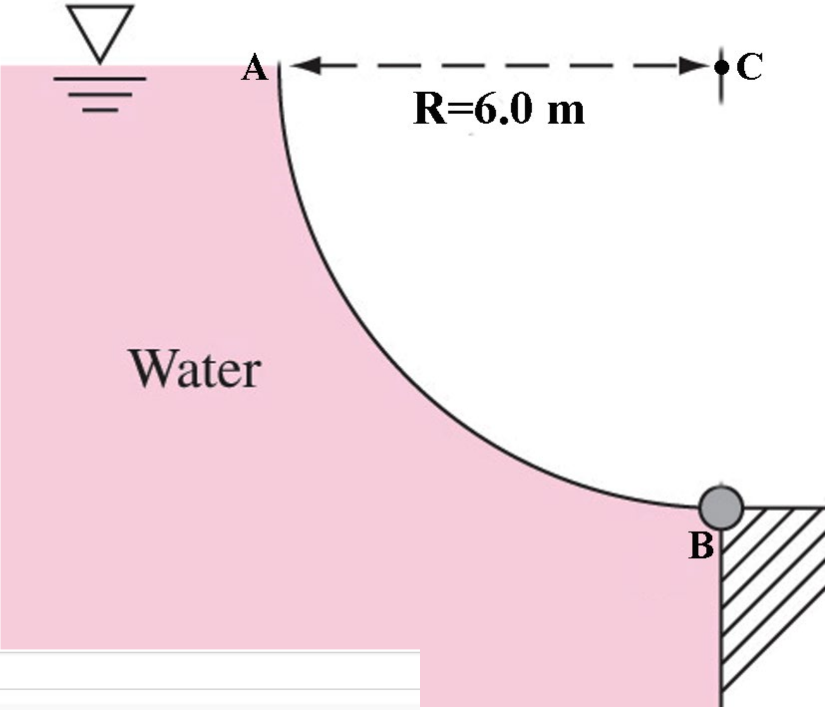
Example Problem

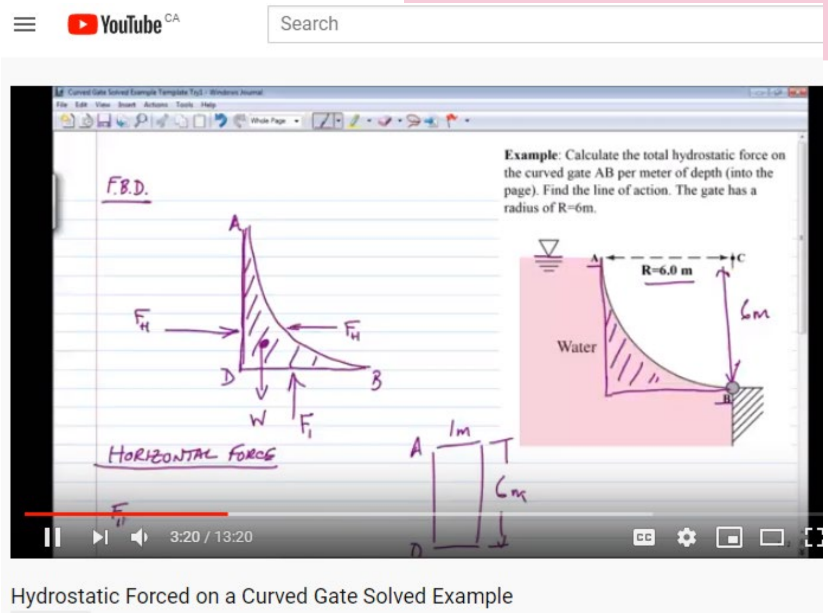
The curved gate (AB) shown in the sketch has a radius of R=6m.

(a) Calculate the total hydrostatic force on the curved gate AB per meter of depth (into the page)

(b) Find the line of action of the resultant force

Watch the Video Solution





END NOTES

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