MEC516/BME516: Fluid Mechanics I

Chapter 2: Fluid Statics

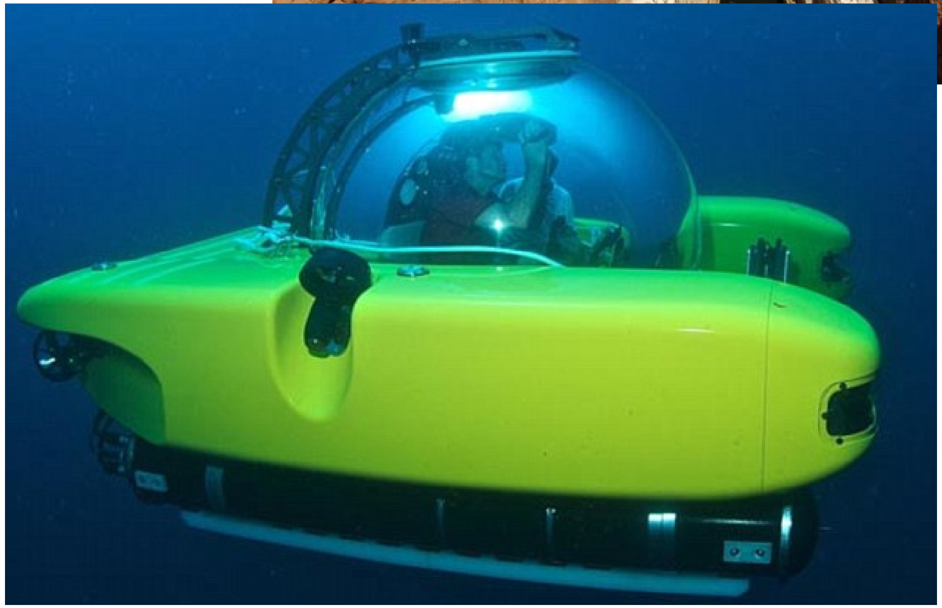
RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

Hydrostatic Forces on Plane Surfaces

- Plane surfaces, i.e. flat surfaces

- For the engineering design of:

  - liquid containment structures
    (e.g. storage tanks, dams and levees)
  - hulls of vessels
    (e.g. ships, submarine vehicles)



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Application: Water Tanks

- Water tanks are place on the top of buildings to supply domestic water
- Steel support bands are unevenly spaced
- Why?

Ans: Hydrostatic force increases (linearly) with depth

Domestic water storage tanks in New York City

Hydrostatic Forces on Plane Surfaces

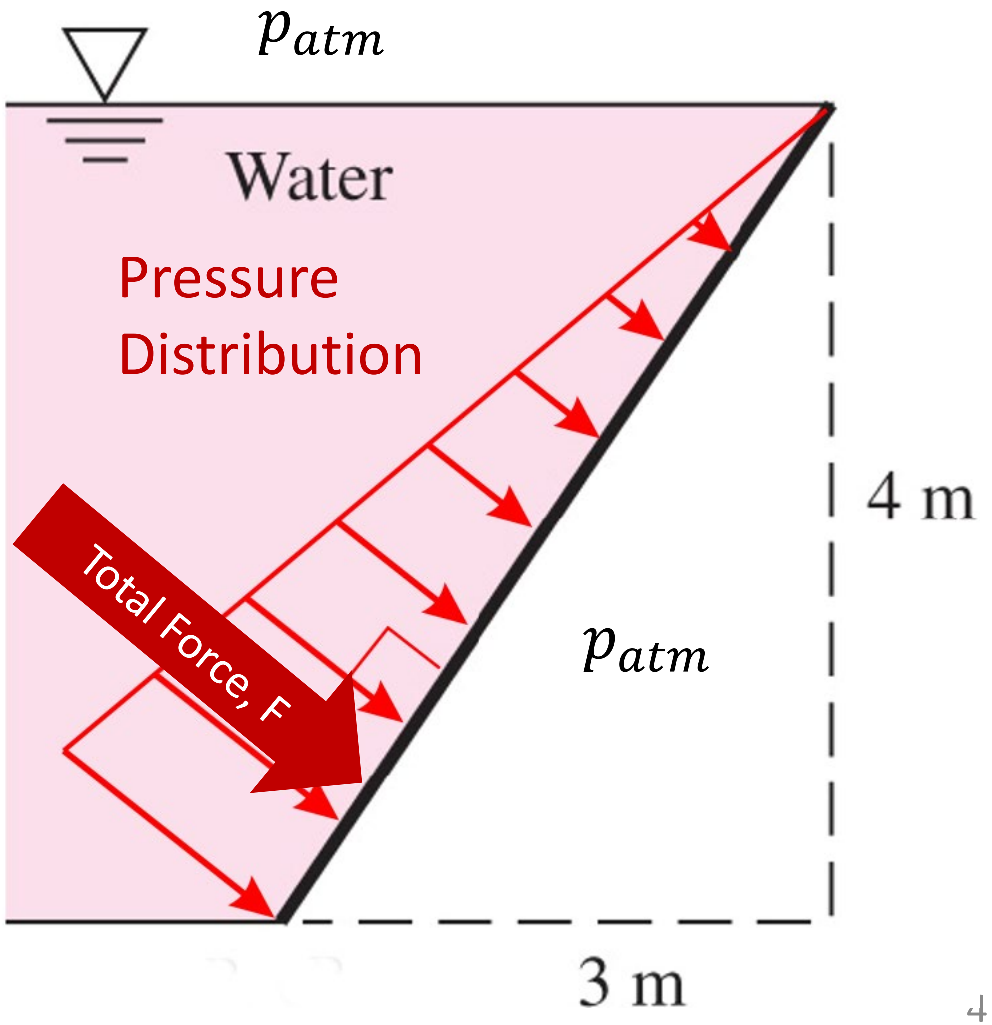
- Our analysis is for ***liquids***:
  - Pressure distribution for incompressible fluids

    $$p - p_{atm} = \gamma h \text{ (gauge)}$$

  - Pressure increases linearly with depth
  - Recall that pressure acts **normal** to bounding surface

- Goals of our analysis:
  1. Integrate pressure distribution to get total resultant force (F) on surface
  2. Locate the line of action of force, F (*Centre of Pressure*)

- Needed for stress analysis, plate thickness etc.



Hydrostatic Forces on Plane Surfaces

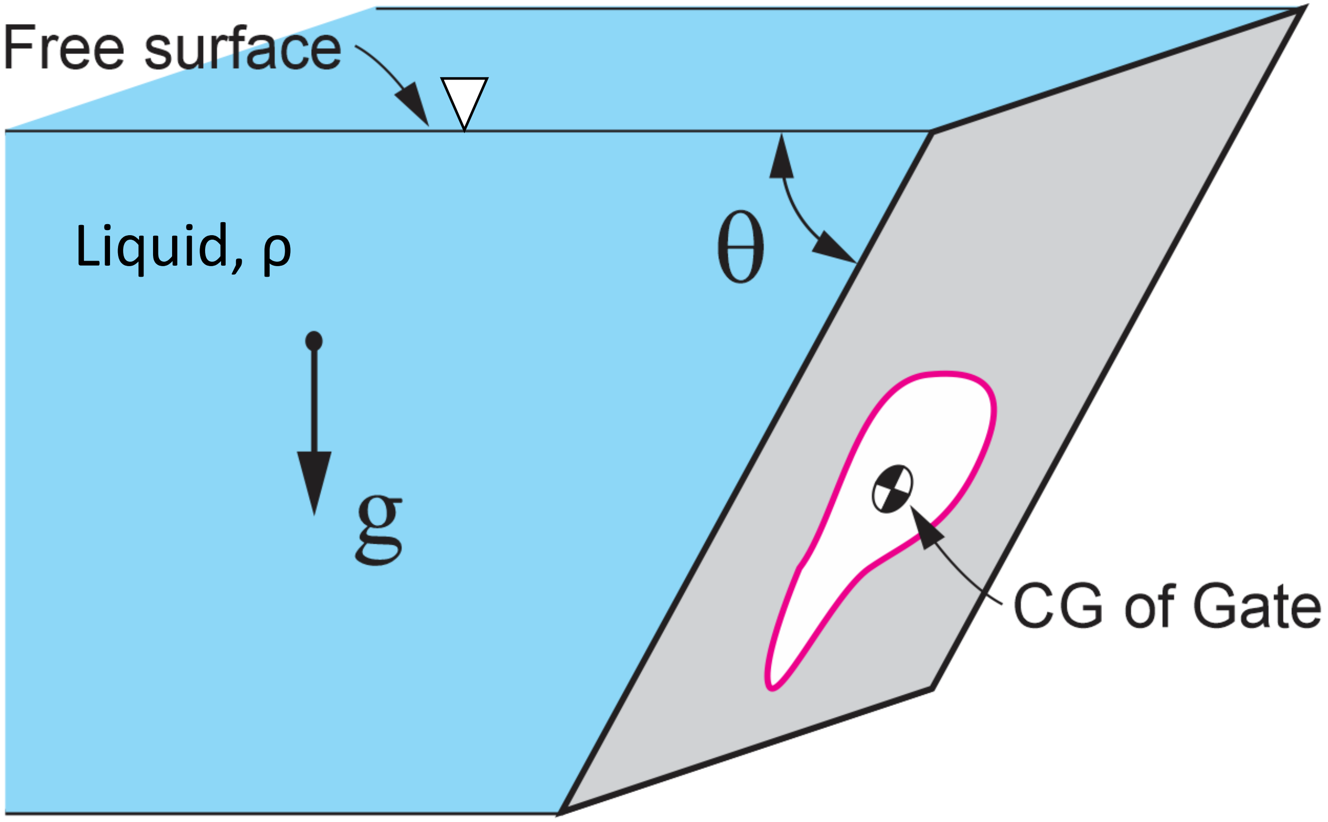
Problem Definition

- Flat surface at angle, θ
- Red outline is the gate
- CG: centre of *area* of gate

PCS211

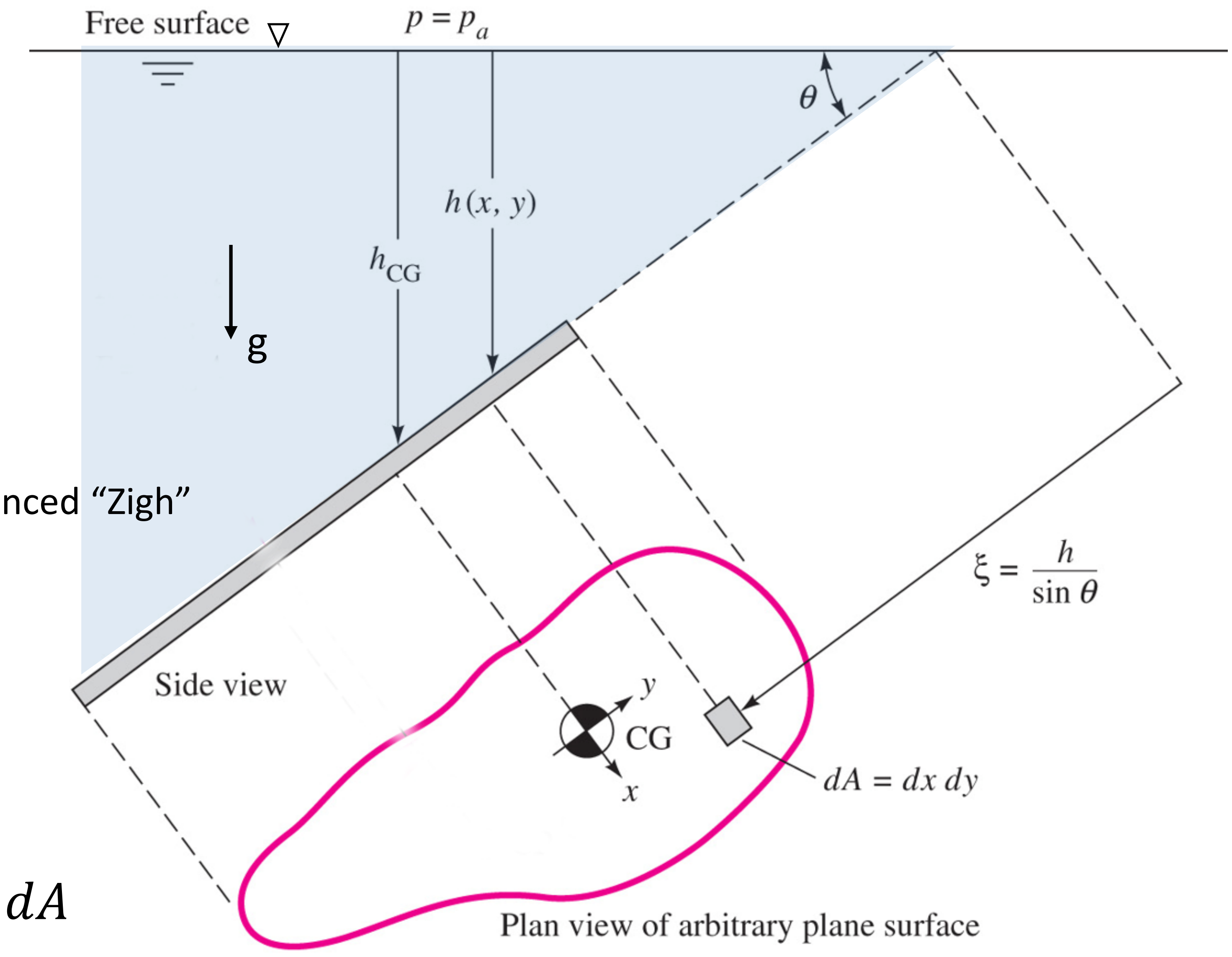
Analysis Objectives:

- Calculate the total force on the gate
- Find the location of the total force (Centre of pressure)



Oblique View of Gate Problem

Hydrostatic Forces on Plane Surfaces



Problem Definition

- Flat surface at angle, θ
- Red outline is plan (top) view
- CG, centre of ***area*** of gate
- Coordinate ξ measured from free surface, parallel to gate

Greek Xi, pronounced "Zigh"

Differential force:

$$dF = p\,dA = (p_a + \gamma\,h)\,dA$$

Integrate over gate area:

$$F = \int_A dF = \int_A (p_a + \gamma h(x,y))\,dA$$

constants

Hydrostatic Forces on Plane Surfaces

$$F = p_a A + \gamma \int_A h(x,y)\, dA$$

Put in terms of co-ord. ξ: $h = \xi \sin\theta$

$$F = p_a A + \gamma \sin\theta \int_A \xi\, dA$$

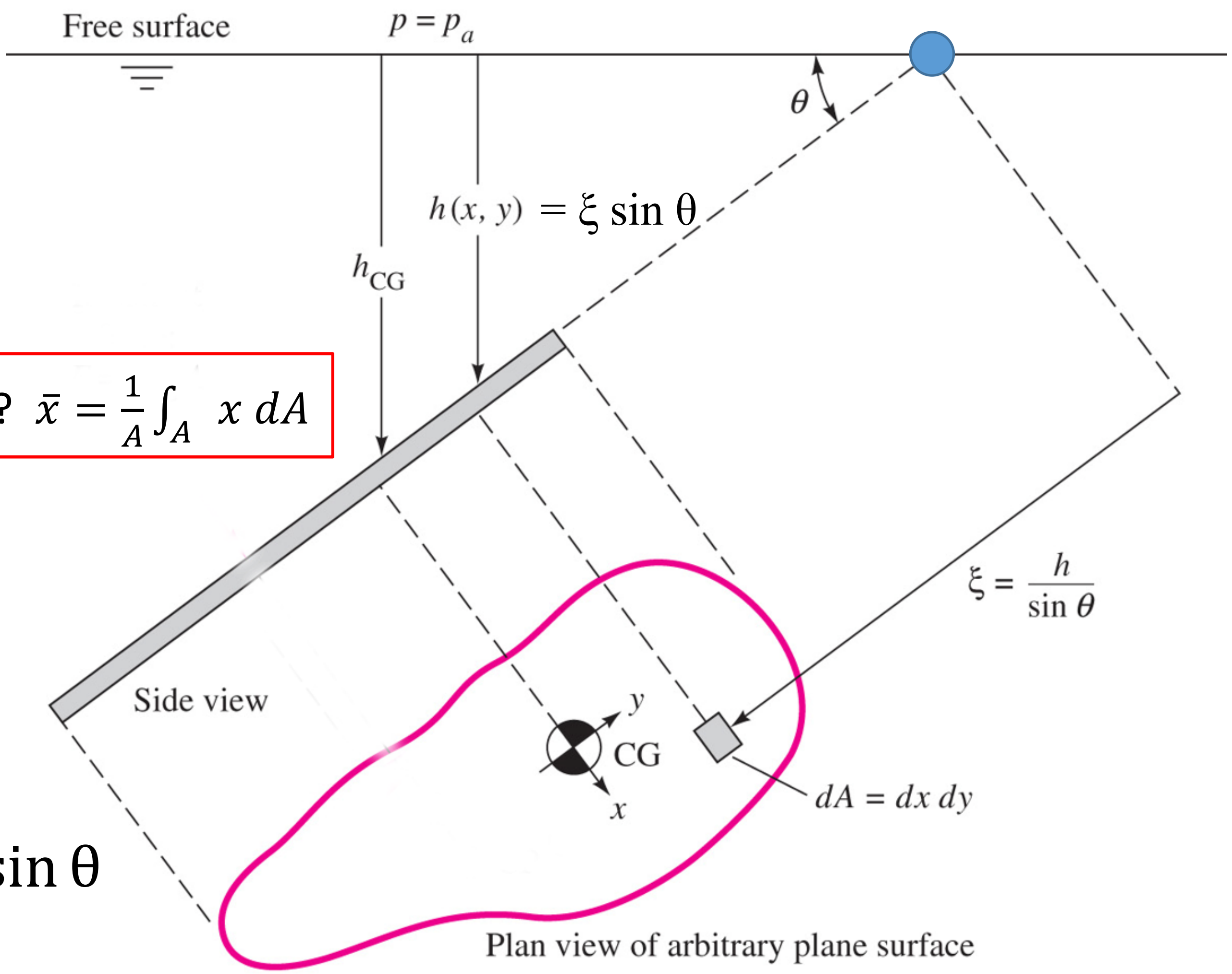
PCS211 Recall? $\bar{x} = \frac{1}{A}\int_A x\, dA$

Location of the centroid (centre of area):

$$\xi_{CG} = \frac{1}{A}\int_A \xi\, dA \rightarrow A\xi_{CG} = \int_A \xi\, dA$$

$$F = p_a A + \gamma\, A\, \xi_{CG} \sin\theta \quad \text{but } h_{CG} = \xi_{CG}\sin\theta$$

Thus, $$F = (p_a + \gamma\, h_{CG})\, A = p_{CG} A$$



Hydrostatic Forces on Plane Surfaces

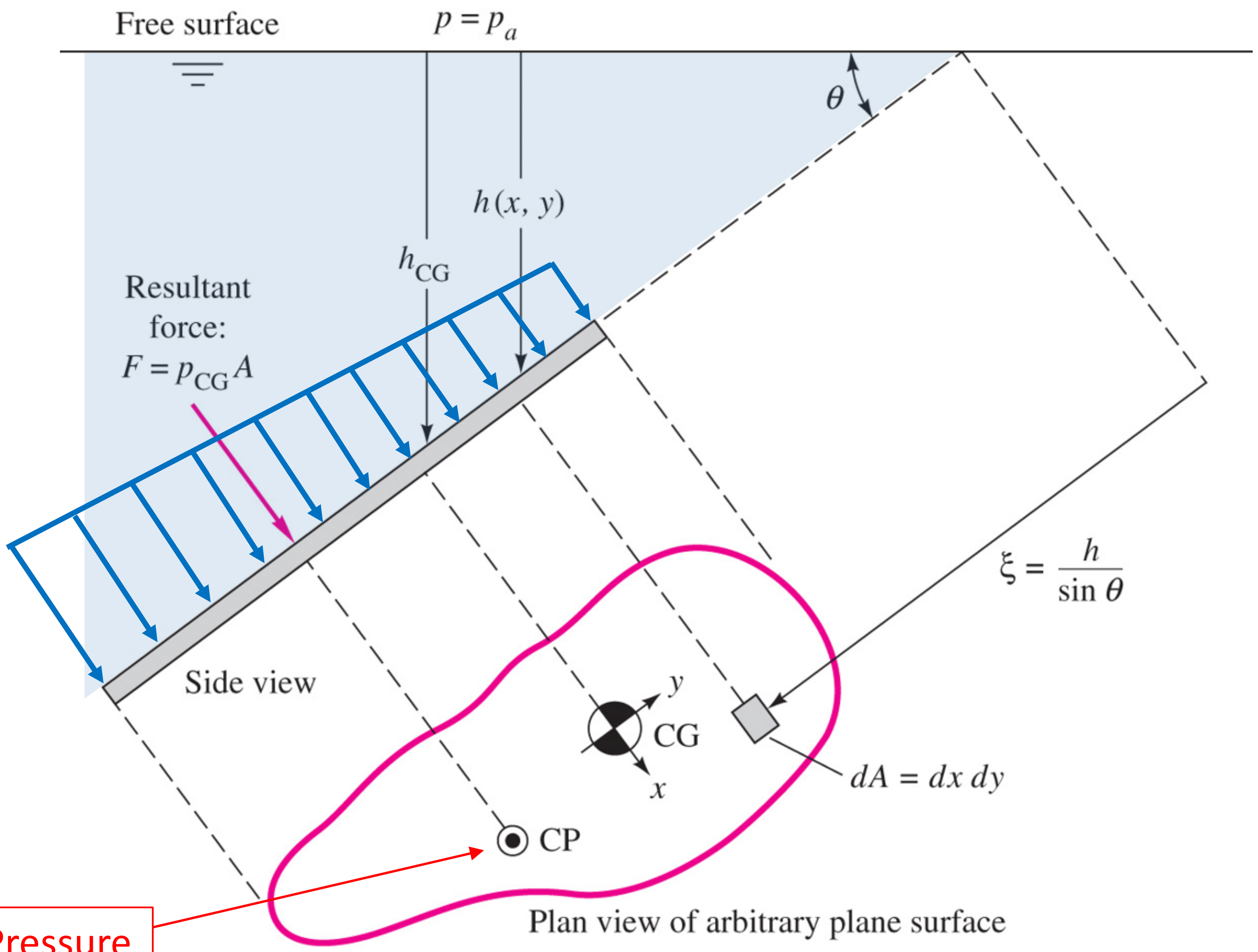
<u>Resultant Force</u>

$$F = (p_a + \gamma\, h_{CG})\, A = p_{CG} A$$

- Thus, p_{CG} is the average pressure on the gate

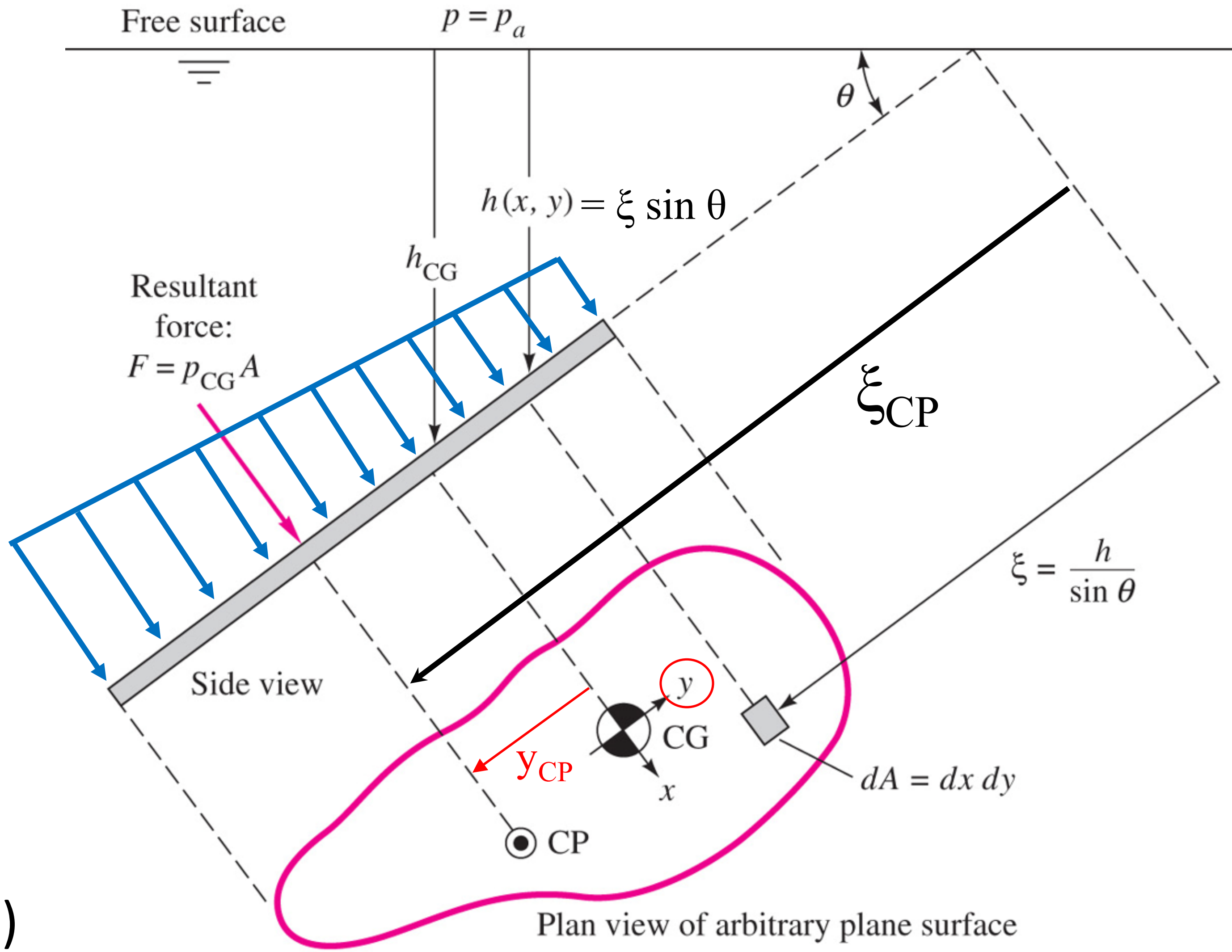
What is the line of action of F?

- **IMPORTANT! F does not act at CG**
  - F acts below centroid, at CP
  - To be explained…



CP: Centre of Pressure

Hydrostatic Forces on Plane Surfaces



Line of Action: Centre of Pressure, CP

- Moment of resultant F about y=0 must equal moment of pressure distribution about y=0

$$F\, y_{CP} = \int_A (p_a + \gamma\, h)\, y\, dA$$

Result: $$y_{CP} = -\gamma\ \sin\theta\ \frac{I_{xx}}{p_{CG} A}$$

- See textbook for details (straightforward)

Hydrostatic Forces on Plane Surfaces

<u>Line of Action: Centre of Pressure, CP</u>

$$y_{CP} = -\gamma \sin\theta \frac{I_{xx}}{p_{CG} A}$$

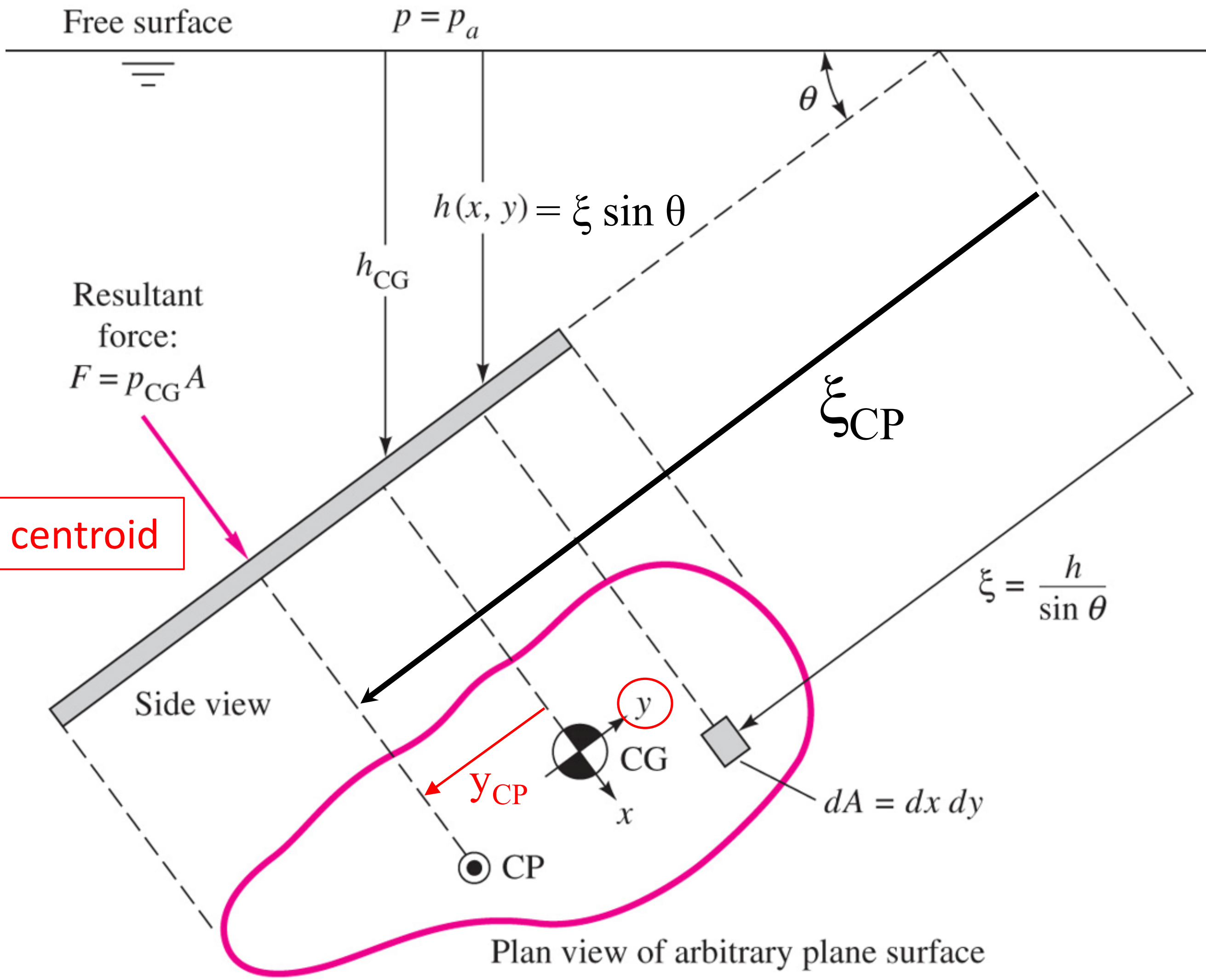
- y_{CP} is relative to the C of G, along the inclined gate

Always negative→ below centroid

- p_{CG} is the pressure at the C of G of the gate
- I_{xx} is the second moment of area of the gate:

$$I_{x,x} = \int_A y^2 \, dA$$

A geometric property



Hydrostatic Forces on Plane Surfaces

A similar analysis for the x-direction gives:

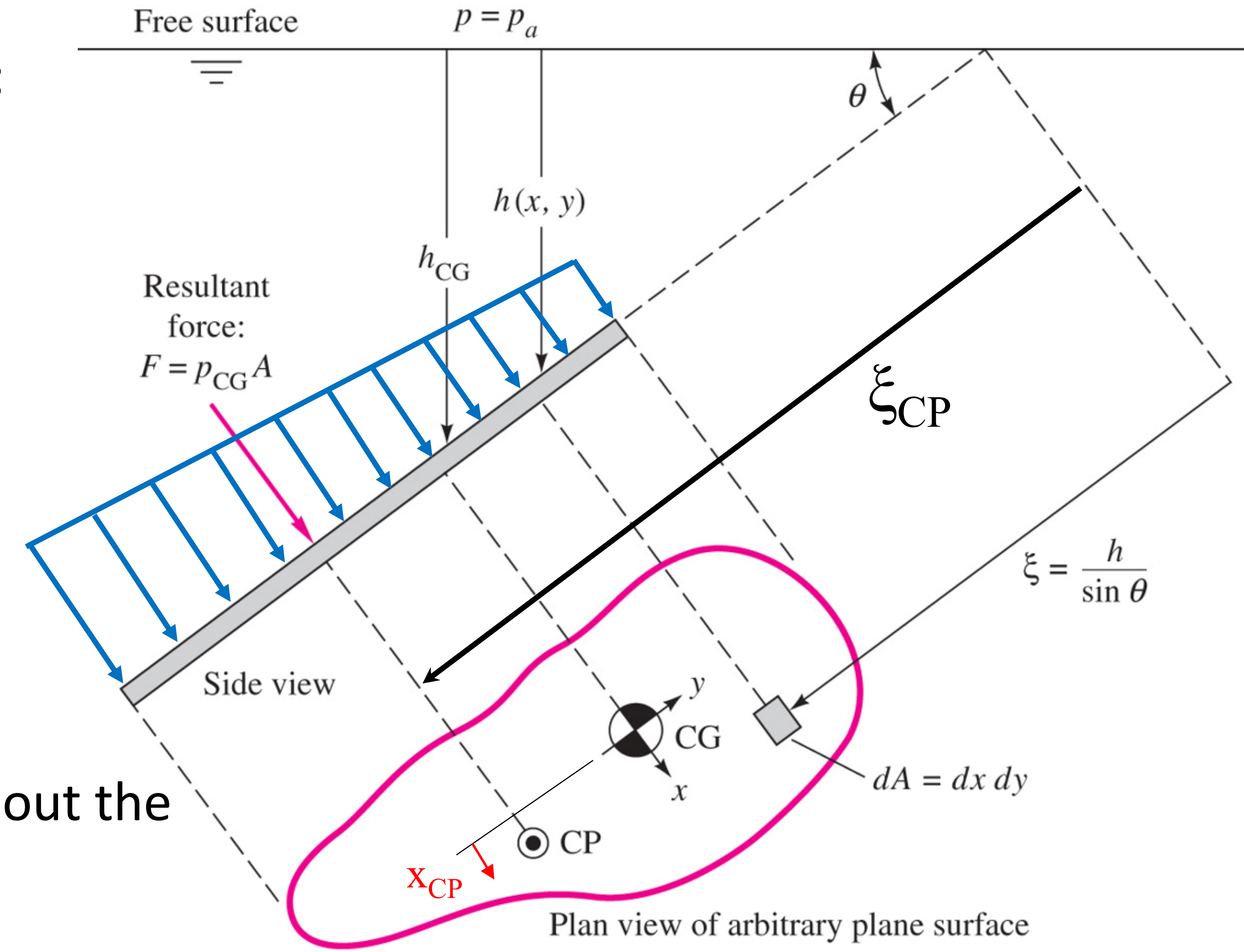
$$x_{CP} = -\gamma \sin\theta \frac{I_{xy}}{p_{CG}\,A}$$

where is the "product of area" about the centroid, CG

$$I_{x,y} = \int_A xy\,dA$$

I_{xy} is zero for gates about symmetrical about the y-axis → $x_{CP} = 0$

Centroid locations, I_{xx} & I_{xy} given in textbook



Second Moment of Area, I_{xx}

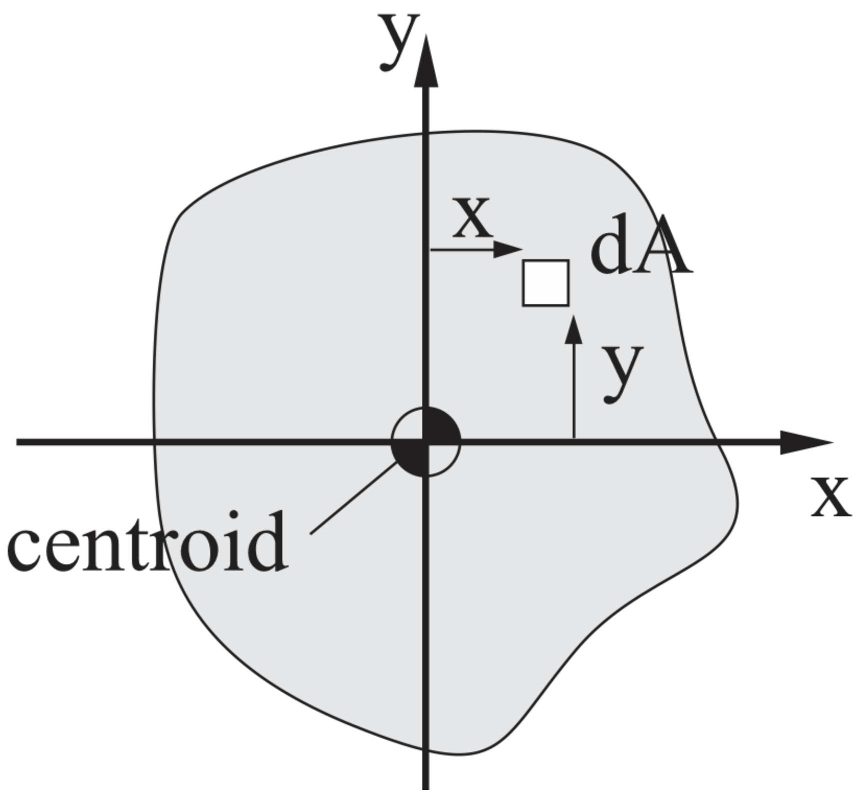
- A geometric property, analogous to moment of inertia (in physics, second moment of mass)

- 2^{nd} moment of area about the x-axis is defined as:

$$I_{xx} = \int_A y^2 dA$$ (units of m^4)

- This integral has been evaluated for common shapes

- See Figure 2.13 in textbook

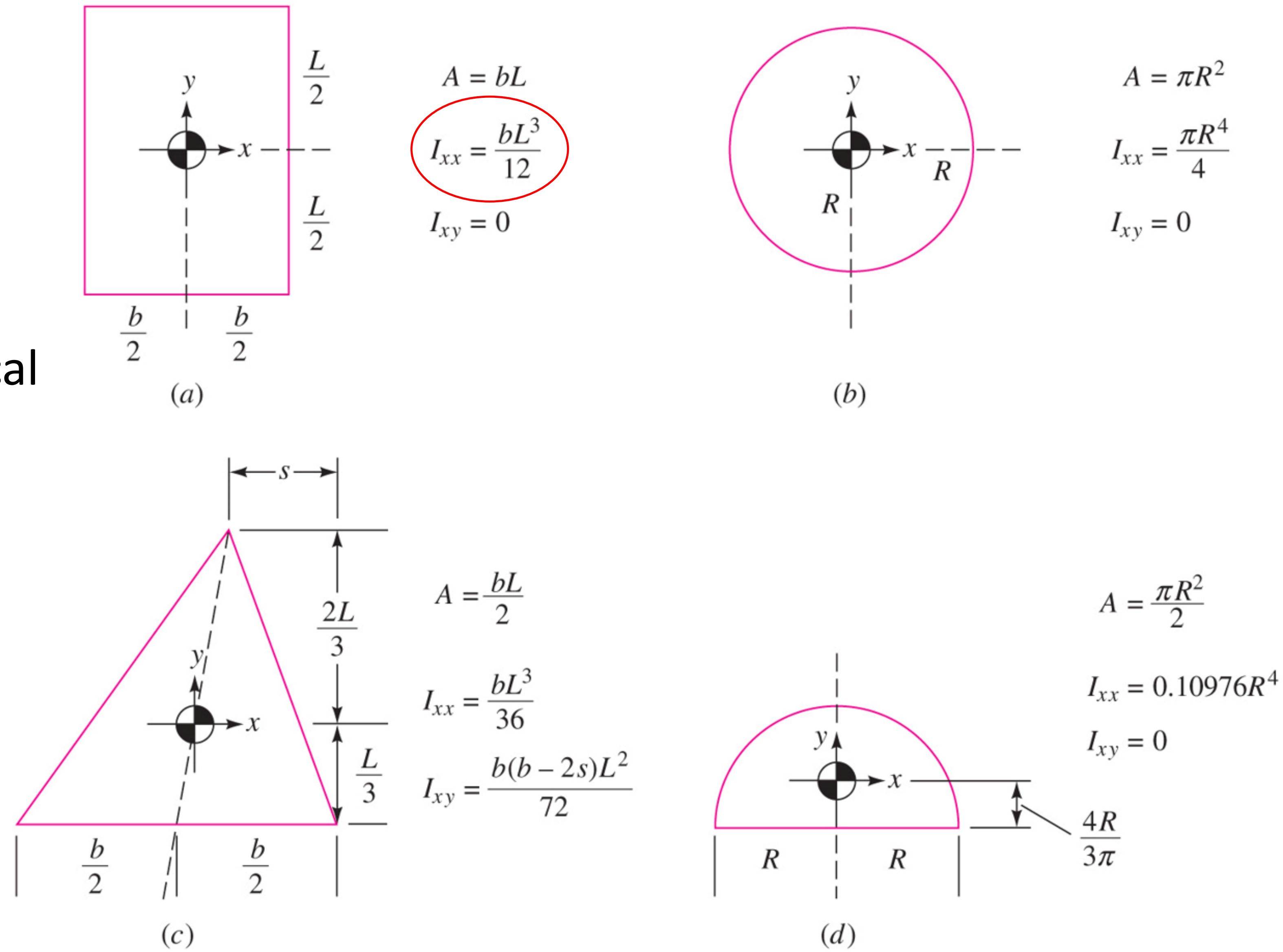


Recall Beam bending stress?
BME/MEC323:
$$\frac{M}{I} = \frac{\sigma}{y}$$

Figure 2.13: Centroids and Second Moments of Area

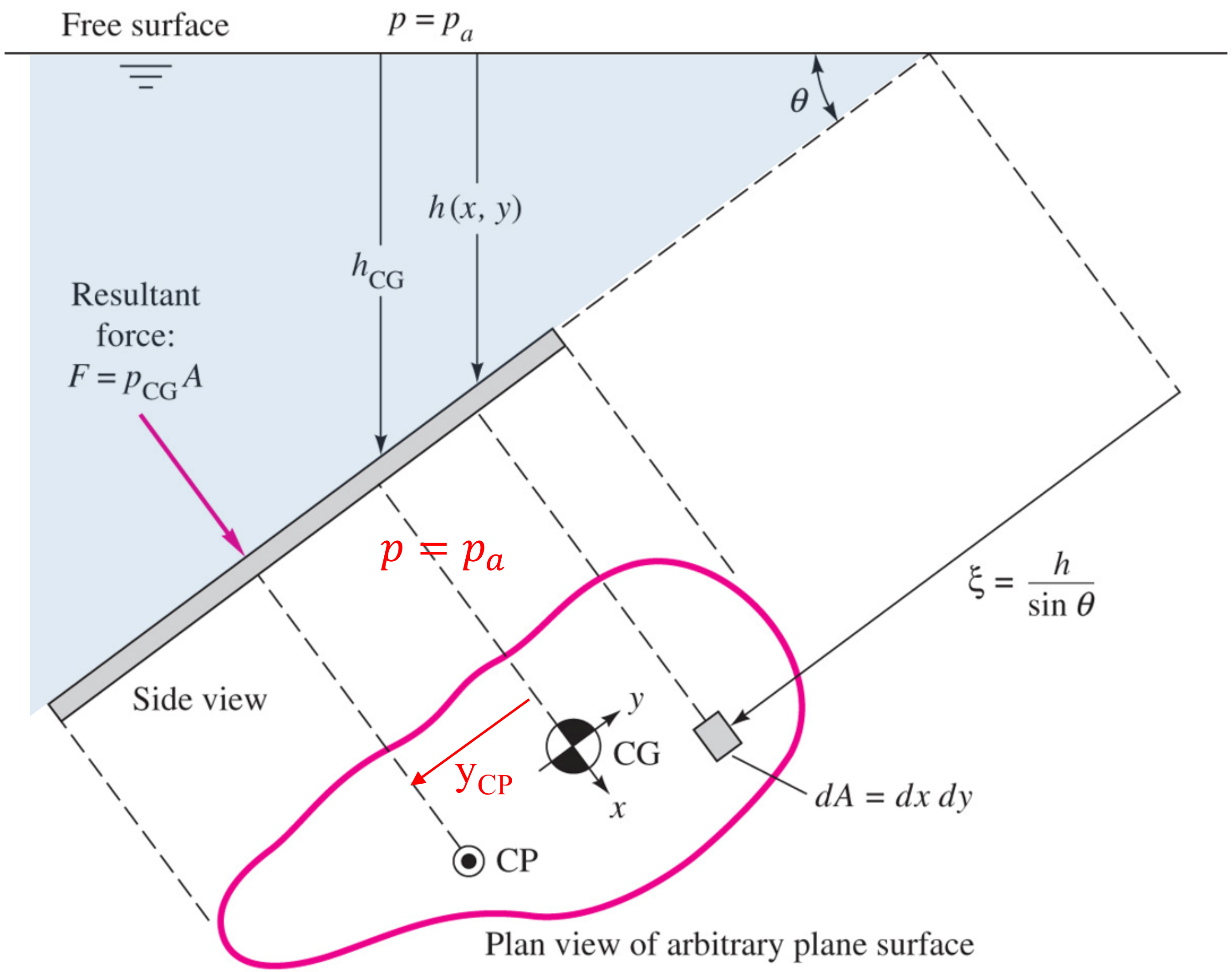
- Area, centroid location and I_{xx}
- I_{xy} is zero for symmetrical gates ($x_{CP=0}$)
- Note that for the rectangle:

$$I_{xx} = \frac{bL^3}{12}$$



Gauge Pressure Formulas

- In many cases p_a acts on both sides of the gate, cancels out
- Force F is only caused by weight of fluid



Simplified equations:

$$F = \gamma\, h_{CG}\, A$$

$$y_{CP} = \frac{-\, I_{xx} \sin \theta}{h_{CG} A}$$

y_{CP} is relative to an inclined axis passing through the gate's centroid

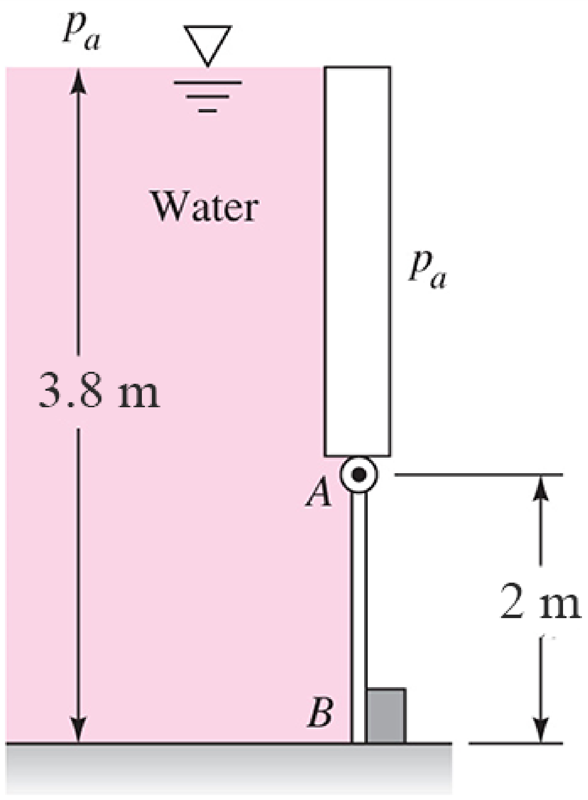
Example Problem

Consider a reservoir of water at 20°C. A vertical hinged gate (AB) holds back the water. The gate has a depth of 3.0 m (into the page).

(a) Sketch the hydrostatic pressure distribution on the gate (AB)

(b) Calculate the total force (F) on gate AB

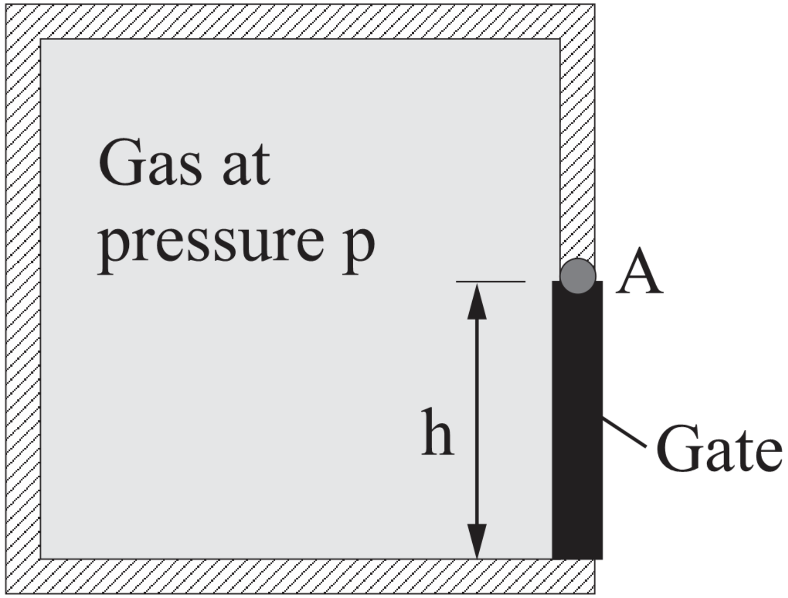
(c) Find the distance of the line of action of force (F) from the hinge at point A

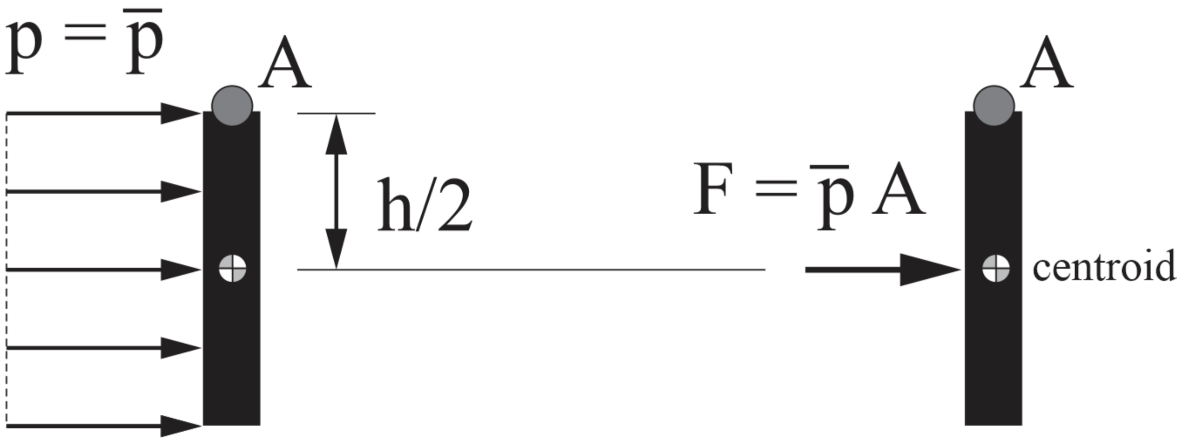


Why is the C of P below the Centroid?

- Consider the pressure force on a gate in a chamber of gas
- Uniform pressure distribution (Why?)

Density of gases O(10^3) less dense liquids.
Variation of pressure with height is negligible





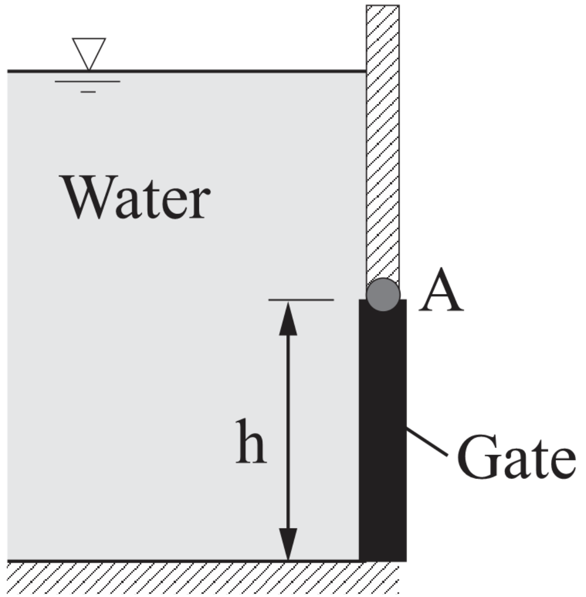
Pressure Distribution

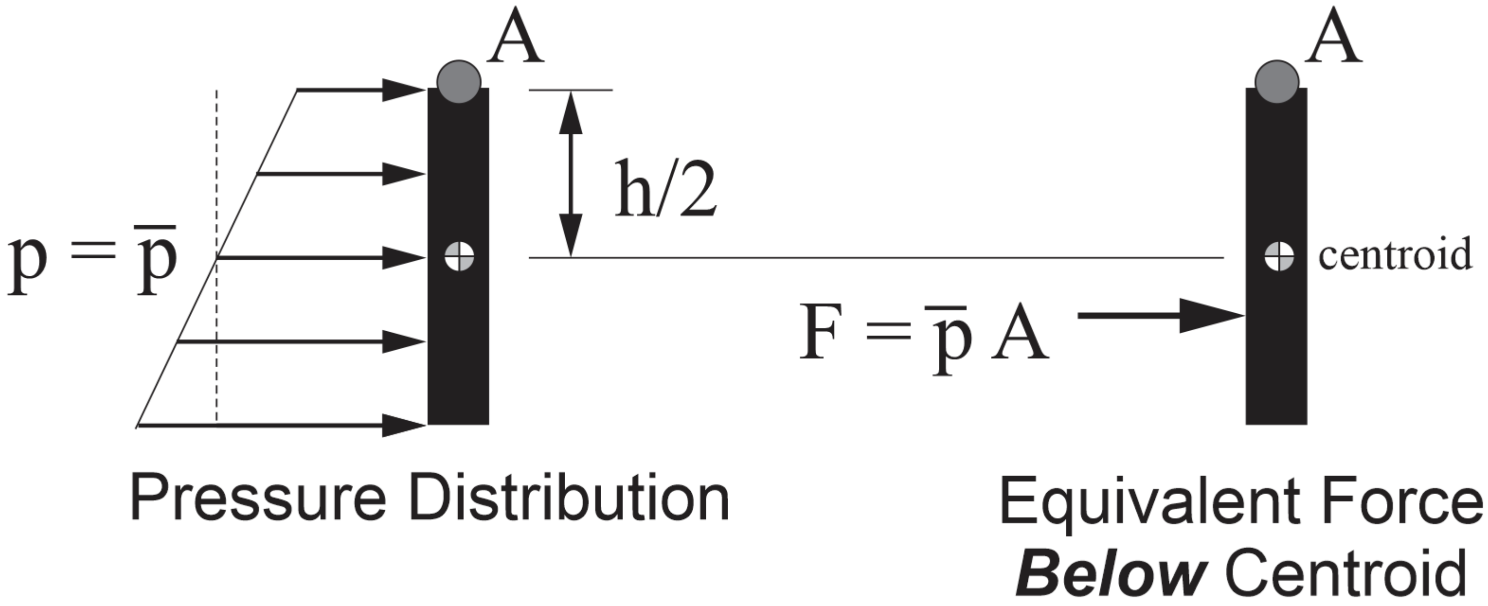
Equivalent Force at Centroid

- C of P is at the centroid
- With F located at the centroid, both systems have the same moment about the CG

Why is the C of P below the Centroid?

- Non-uniform pressure distribution (hydrostatics)
- Pressure at the centroid is the average pressure on gate





- To have the same moment about CG, F must act below the centroid (by amount, y_{CP})
- Like a ramped distributed load (statics)

Example Problem

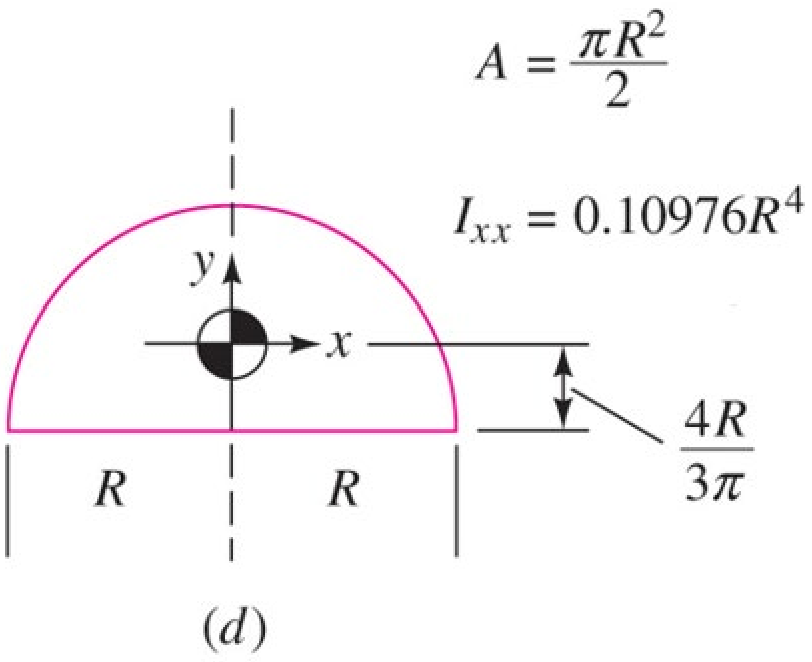
Watch the Video Solution

Example: Hydrostatic Force on a Plane Gate

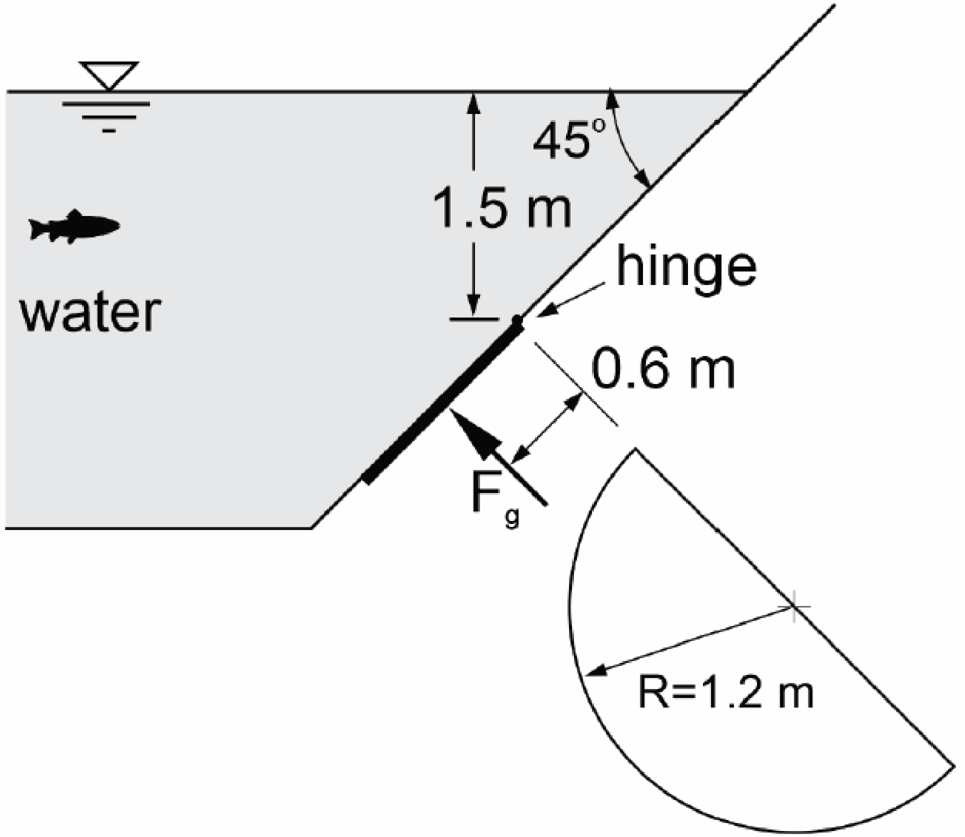
A semi-circular gate is held closed by force F_g applied 0.6 m from the top edge. The gate is hinged along the upper straight edge. Calculate the minimum force F_g necessary to keep the gate closed against the hydrostatic force of the water.

Neglect the mass of the gate

Figure 2.13



20 °C

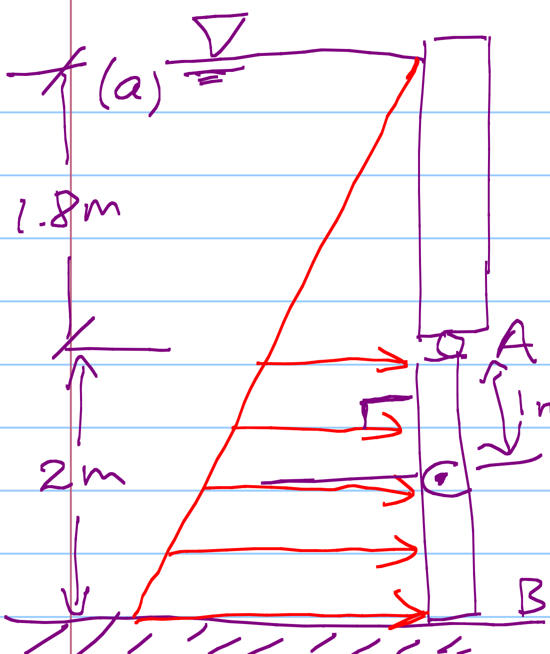


END NOTES

Presentation by Dr. David Naylor
Department of Mechanical and Industrial Engineering
Ryerson University, Toronto, Ontario
Canada

Ryerson University

(a)



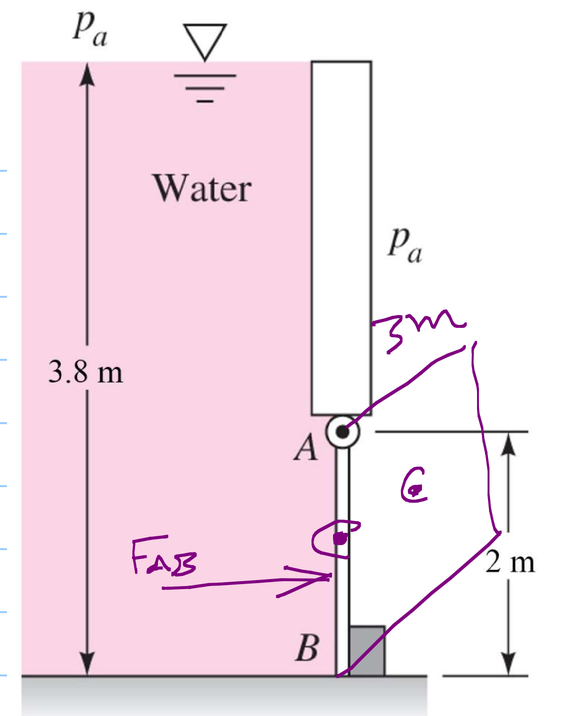
(b) $F_{AB} = \gamma_w h_{CG} A$

$\gamma_w = \rho_w g = 998\ kg/m^3 (9.81\ m/s^2)$

$= 9790\ N/m^3$

$A = 6.0\ m^2$

$h_{CG} = 1.8\ m + 1.0\ m = 2.8\ m$

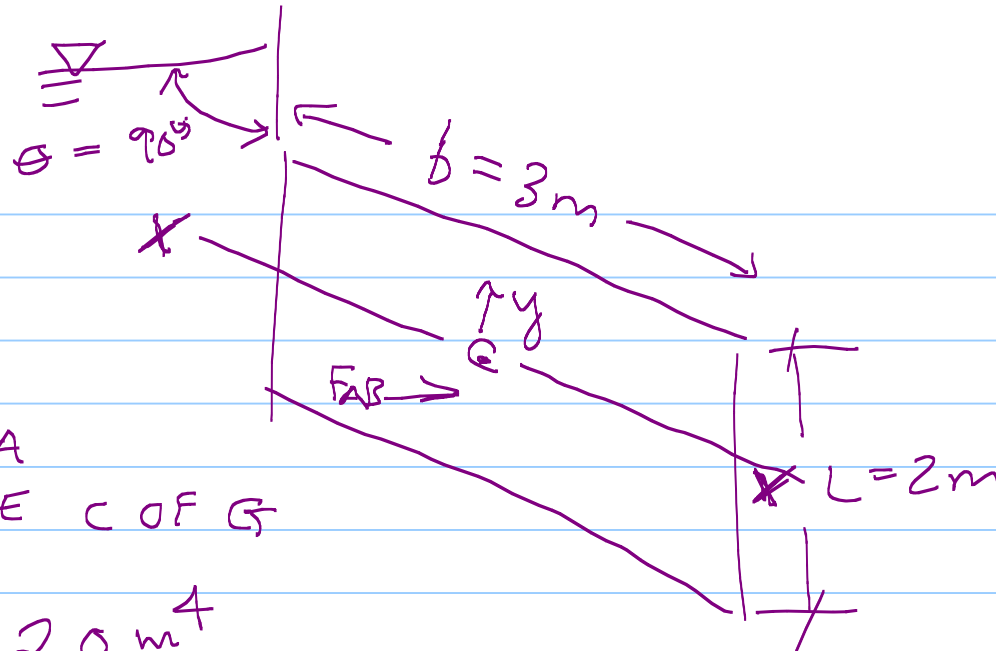


$F_{AB} = 9790\ N/m^3 (2.8\ m)\ 6.0\ m^2 \simeq 164{,}500\ N = \underline{164.5\ kN}$ ⊳

(c) LINE OF ACTION

BELOW C OF G

$$y_{cp} = -\frac{I_{xx} \sin\theta}{h_{CG} A}$$

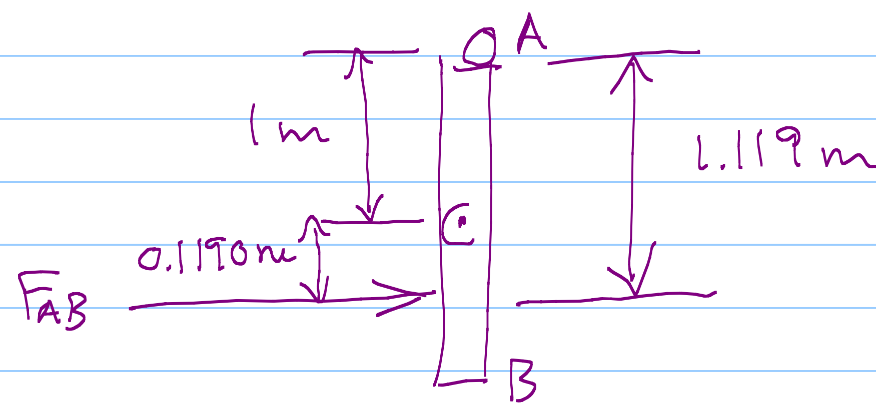


2ND MOMENT OF AREA ABOUT A HORIZONTAL AXIS THROUGH THE C OF G

$$I_{xx} = \frac{bL^3}{12} = \frac{(3m)(2m)^3}{12} = 2.0\,m^4$$

$$y_{cp} = \frac{-2.0\,m^4 \sin 90°}{~~(1.8m)~~\; 6.0\,m^2} = -0.1190\,m$$

Oops. Typo.
Should be 2.8 m



(c) ✓