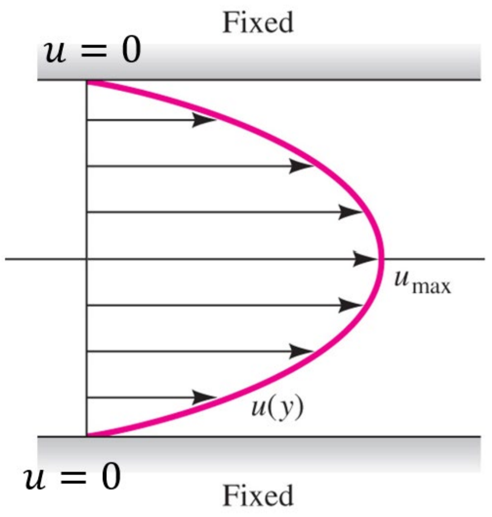
MEC516/BME516: Fluid Mechanics I

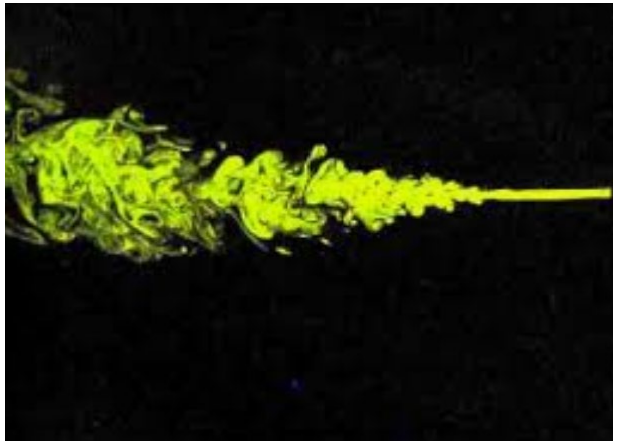
Chapter 1: Introduction

RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

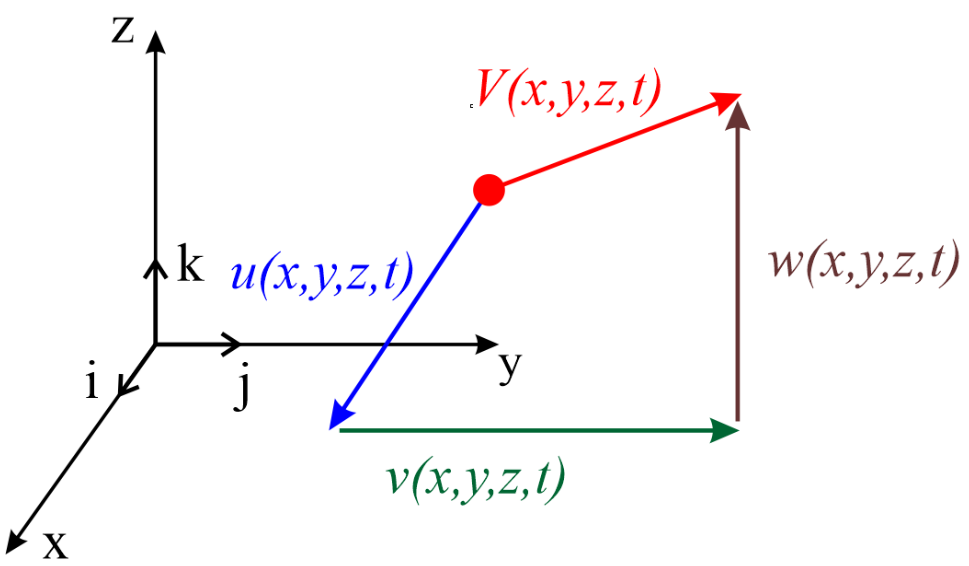
Overview

- Velocity Vector Field
  - The No-slip Condition
- Fluid Properties:
  - Density: ρ   Specific gravity: SG
  - Specific weight: γ
  - Viscosity: μ, ν
- Newton's Law of Viscosity
  - Non-Newtonian fluids
- Laminar and Turbulent Flow
  - The Reynolds number





Velocity Field



- Fluid velocity is a vector field

- The velocity vector is a function of position (x,y,z) and time (t):

$$\boldsymbol{V} = \mathbf{i}\,u(x,y,z,t) + \mathbf{j}\,v(x,y,z,t) + \mathbf{k}\,w(x,y,z,t)$$

where u, v and w are the velocities in the x, y, and z directions

- If V depends on time (t), the flow is said to be *transient* or *unsteady*

No-slip Condition

- **One of the most important concepts in fluid mechanics!**
- At a solid surface, fluid “sticks” to the surface
- Experimentally observed fact: Fluid has the same velocity as a solid surface
- If the surface is stationary: $u=0$ at $y=0$
- Thus, at a solid stationary surface:

$$u = v = 0$$ (for a solid impermeable surface)

no slip (→ u)

Impermeability (→ v)

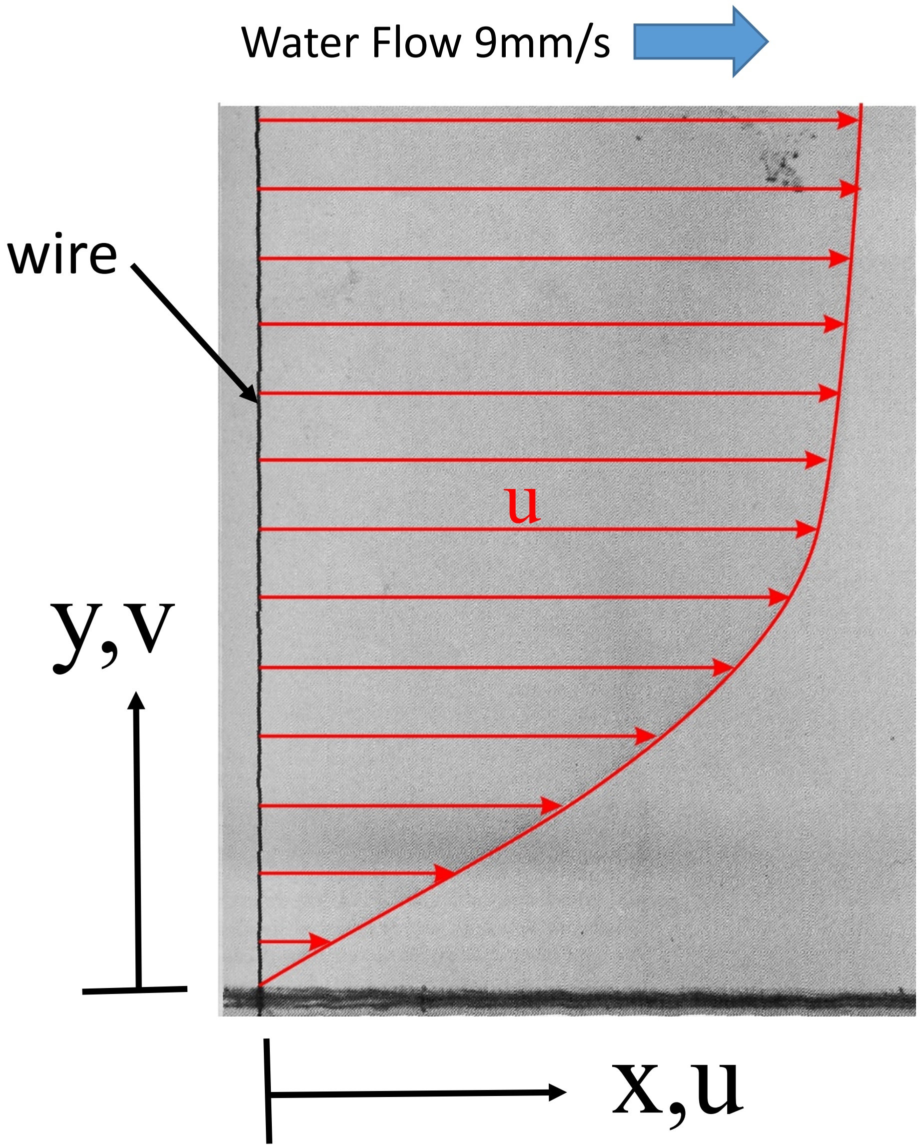
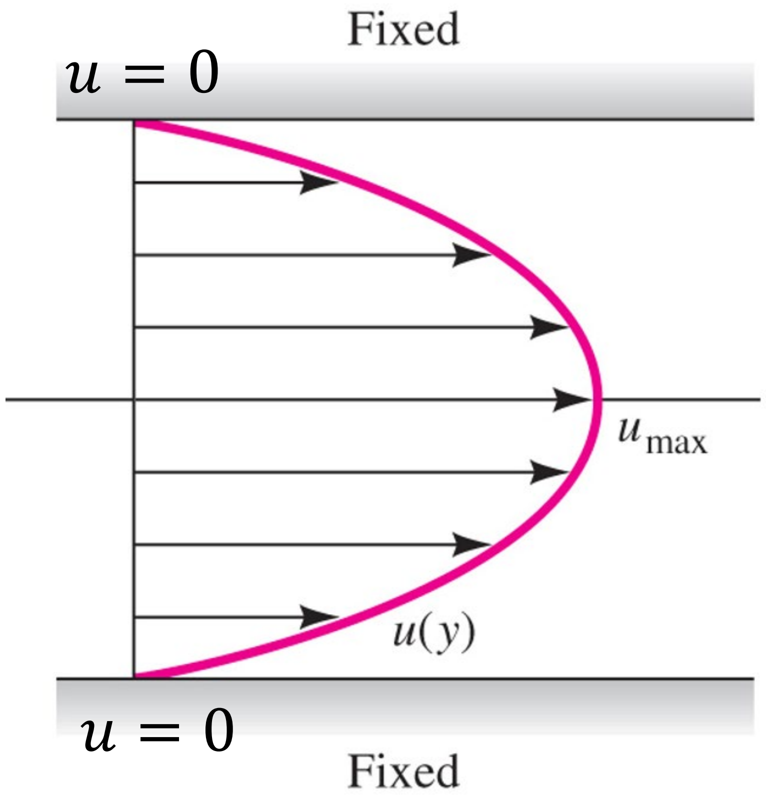


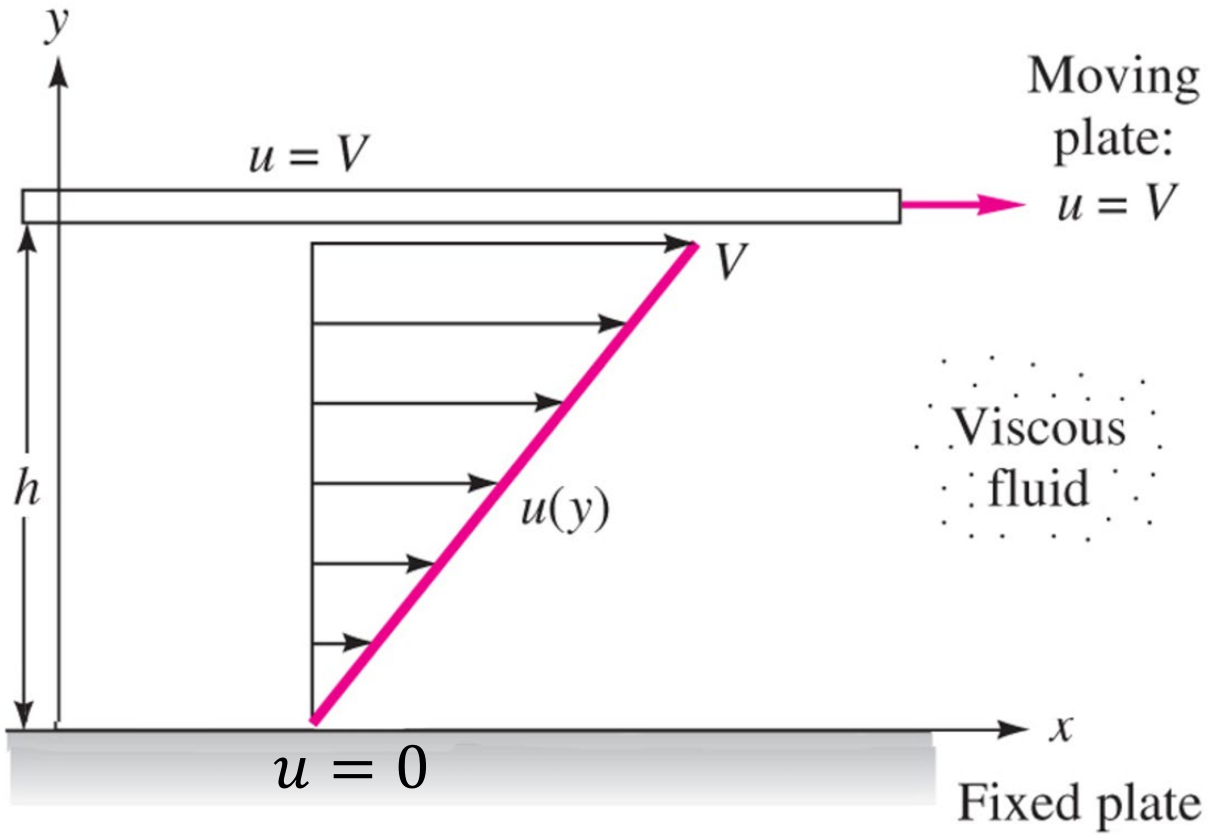
Photo: Album of Fluid Motion

No-slip Condition

- The no-slip condition provides the *boundary condition* for mathematical solutions of fluid flow (Chapter 4)



Poiseuille flow



Couette flow

No-slip Condition

- Every day consequences of the no-slip condition:

- Difficult to getting the last of the ketchup out of the bottle

- Your car is still dirty after driving fast. Dust particles are in a low velocity region due to no-slip

https://www.youtube.com/watch?v=cUTkqZeiMow

Fluid Properties

Table A.1 Density of liquid water as a function of temperature.

T, °C	ρ, kg/m^3
0	1000
10	1000
20	998
30	996
40	992
50	988
60	983
70	978
80	972
90	965
100	958

Density, ρ

- Mass per unit volume: units kg/m^3, $slugs/ft^3$
- Liquids can be considered *incompressible* for most engineering applications
  - Liquid density is independent of pressure: $\rho \neq f(p)$
  - Liquid density is a weak function of temperature (due to thermal expansion)

Fluid Properties

- Density of gases at high temperatures and low pressures ($T \gg T_{critical}$ and $P \ll P_{critical}$) can be calculated using the ideal gas equation:

$$\rho = \frac{p}{RT}$$

where R is the gas constant

For air: R=287 J/kgK

- For example, the density of air at room conditions ($p = 100\ kPa$ and 20°C) is:

$$\rho = \frac{p}{RT} = \frac{100x10^3 \cancel{N}/m^2}{287 \frac{\cancel{N}m}{kg\cancel{K}}(20+273)\cancel{K}} = 1.19\ \frac{kg}{m^3}$$

Check that units balance!

R for other gases in Table A.4

Be sure to use absolute temperature

Fluid Properties

Specific Gravity, *SG*

- Ratio of the density to the density of water (at 4°C)

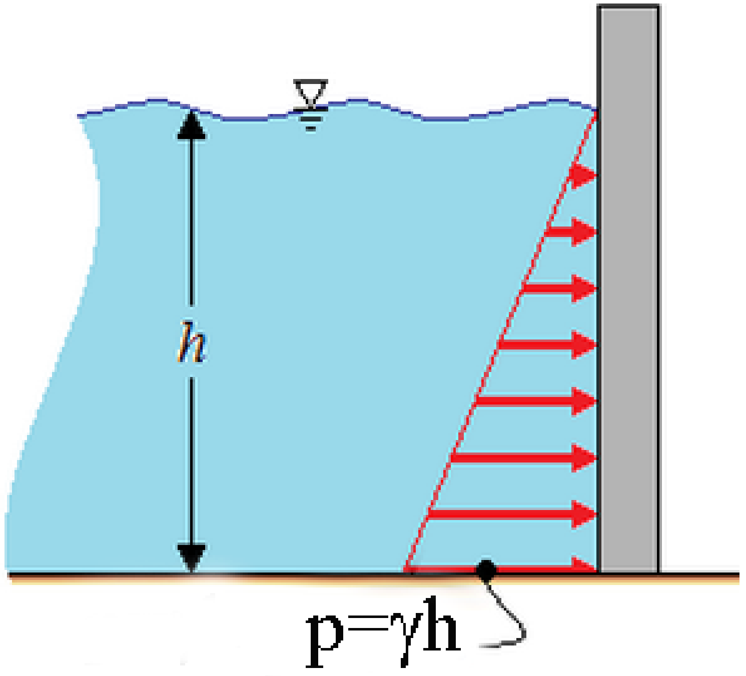
$$SG_{liquid} = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\rho_{liquid}}{1000\ kg/m^3}$$

- *SG* is dimensionless, no units

- *SG* indicates if substance will sink or float on water. If SG is less than 1.0, it will float, e.g. most oils

Olive oil on water

Fluid Properties



Specific Weight, γ

- Weight per unit volume; units N/m³, lb/ft³

$$\gamma = \rho g$$

e.g. Liquid water at 20 °C: $\rho = 998\,\frac{kg}{m^3} = 1.937\,\frac{slug}{ft^3}$

$$\gamma_{water} = 998\,\frac{kg}{m^3}\left(9.81\,\frac{m}{s^2}\right) = 9790\ N/m^3$$

$$\gamma_{water} = 1.937\,\frac{slug}{ft^3}\left(32.17\,\frac{ft}{s^2}\right) = 62.3\ lb/ft^3$$

- We will use these values to calculate hydrostatic forces in Chapter 2

Fluid Properties

Dynamic Viscosity*, μ (Greek lower case mu)

- Units, kg/(m s) or slug/(ft s)
- Viscosity is the fluid's resistance to flow, i.e. the resistance to applied shear stress
  - High viscosity fluids, e.g., honey, engine oil
  - Low viscosity fluids, e.g. water, air

www.freeimages.com

Kinematic Viscosity, ν (Greek lower case nu)

- Units, m^2/s or ft^2/s

$$\nu = \frac{\mu}{\rho}$$

*Note: Some textbooks call μ *absolute viscosity*

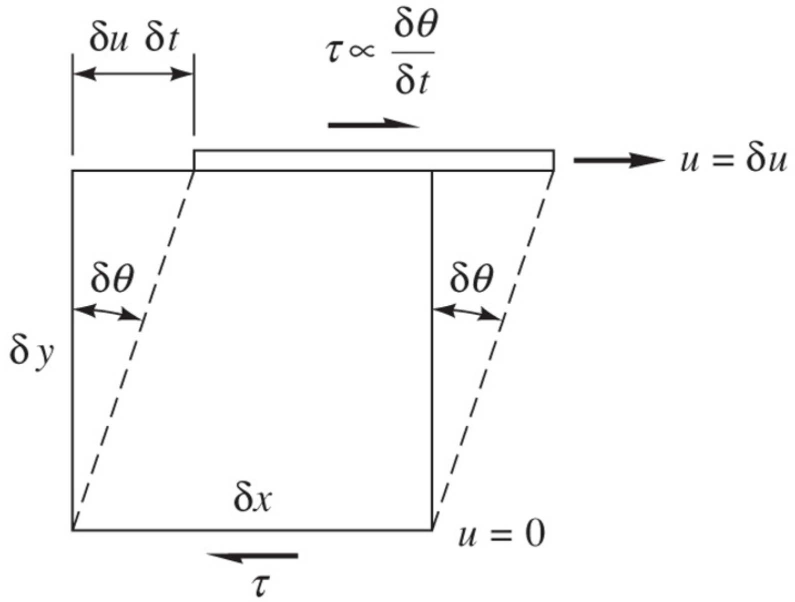
Fluid Properties

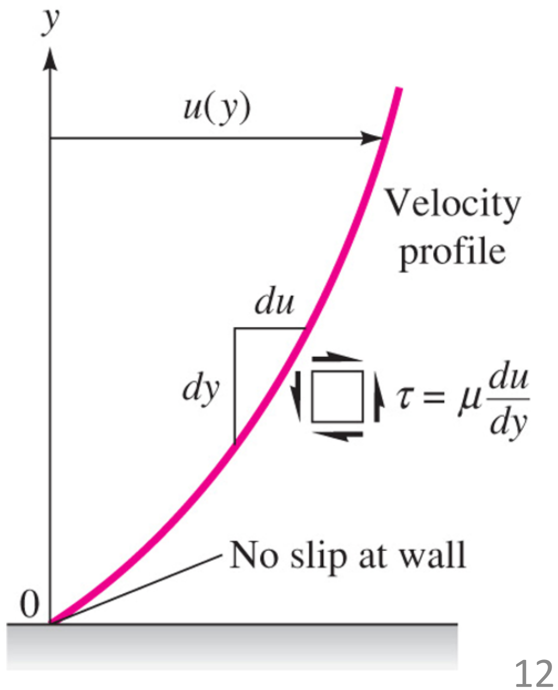
Viscosity

- For most fluids, the shear stress (τ) is linearly proportional to the rate of shear strain of the fluid element
- Shear strain rate is equal to the local velocity gradient:

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Using arc length formula: $\delta\theta\delta y = \delta u\delta t$





- For most fluids the relationship is linear:

$$\tau \backsim \frac{du}{dy}$$

Fluid Properties

- Dynamic viscosity (μ) is the constant of proportionality

$$\tau = \mu \frac{du}{dy}$$

Shear stress
N/m^2

Local velocity gradient
$m/(s \cdot m) \equiv 1/s$

- Sometimes called "Newton's Law of Viscosity"

Viscosity of selected fluids at 20 °C

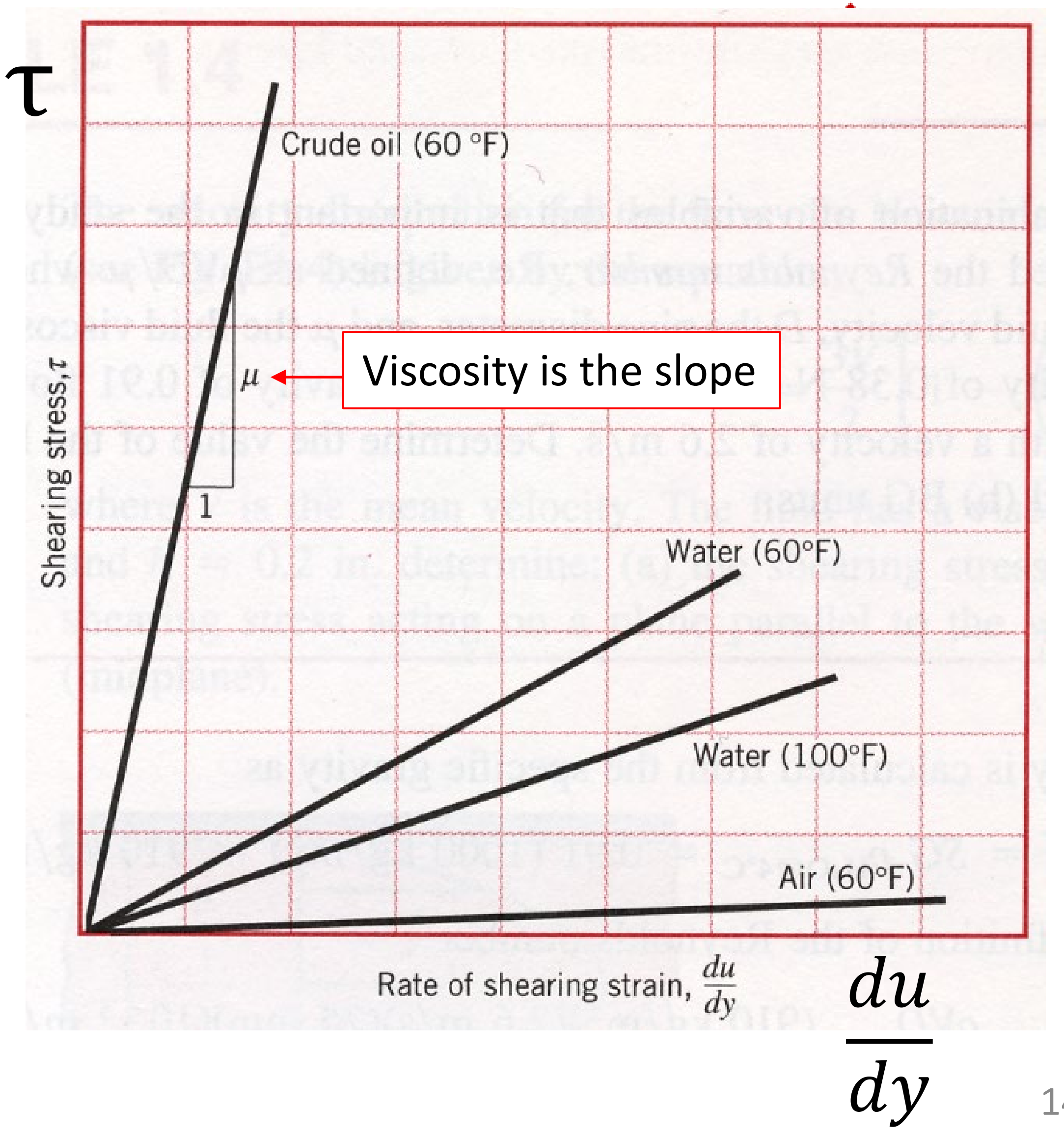
Fluid	μ, kg/(m · s)†
Hydrogen	9.0 E–6
Air	1.8 E–5
Gasoline	2.9 E–4
Water	1.0 E–3
Ethyl alcohol	1.2 E–3
Mercury	1.5 E–3
SAE 30 oil	0.29
Glycerin	1.5

Fluid Properties

$$\tau = \mu \frac{du}{dy}$$

Dynamic Viscosity, μ

- For common fluids (air water, oils) viscosity is a constant; independent of shear rate
- Viscosity (μ) is the slope
- Such fluids are called *Newtonian fluids*
- In this course, we will assume that fluids are *Newtonian*



Fluid Properties

<u>Dynamic Viscosity</u>, μ

- Very weak function of pressure
- Viscosity is a function of temperature:
  - viscosity of liquids *decrease* as temperature increases
  - viscosity of gases *increase* as temperature increases

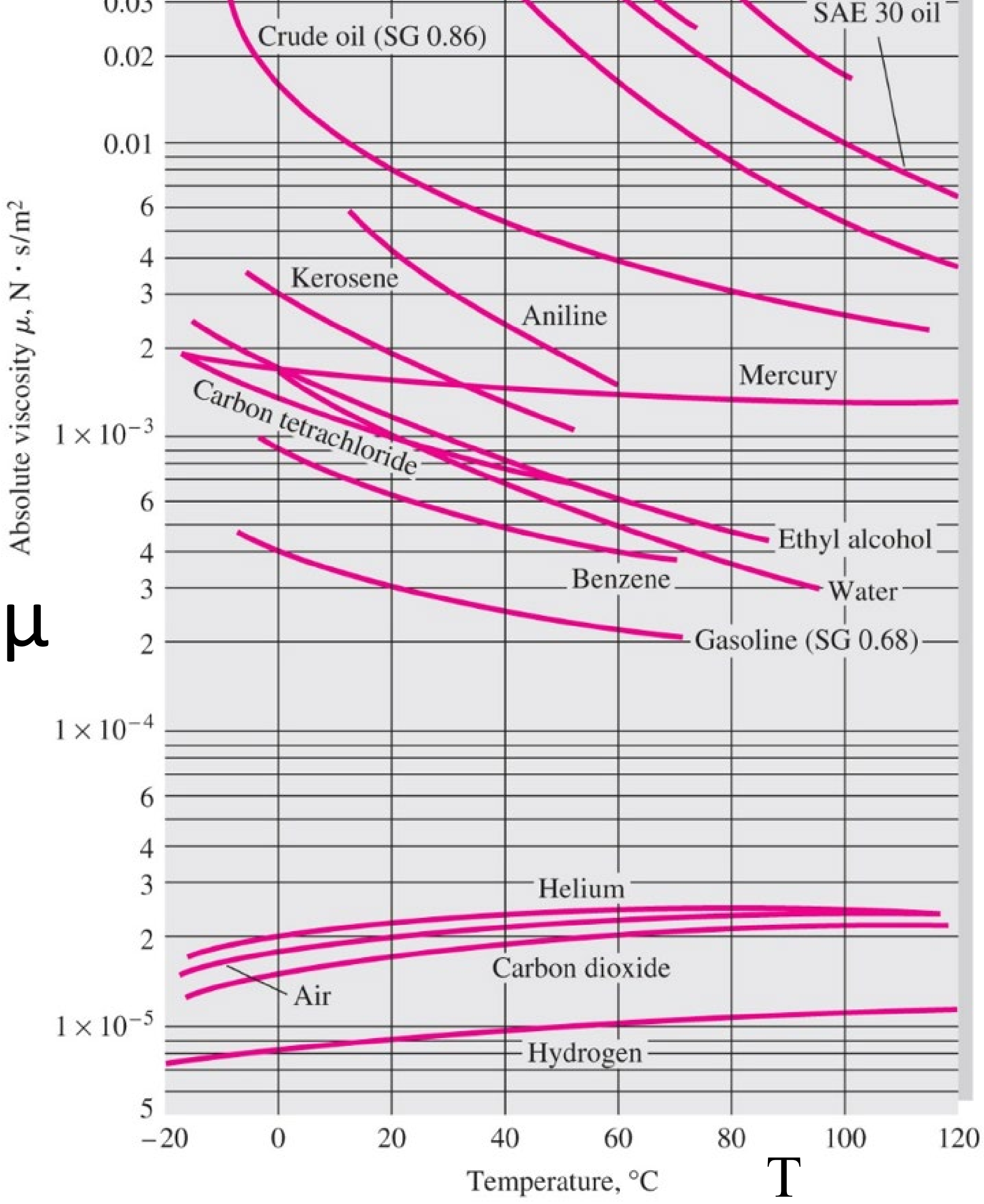


Figure A2: Absolute viscosity at 1 atm.

Measurement of Dynamic Viscosity (μ)

- Commercial instruments (~$5000) measure the torque required to turn a spindle at a known speed

- Rotary viscometer

- Measures in "Centipoise" (cP)

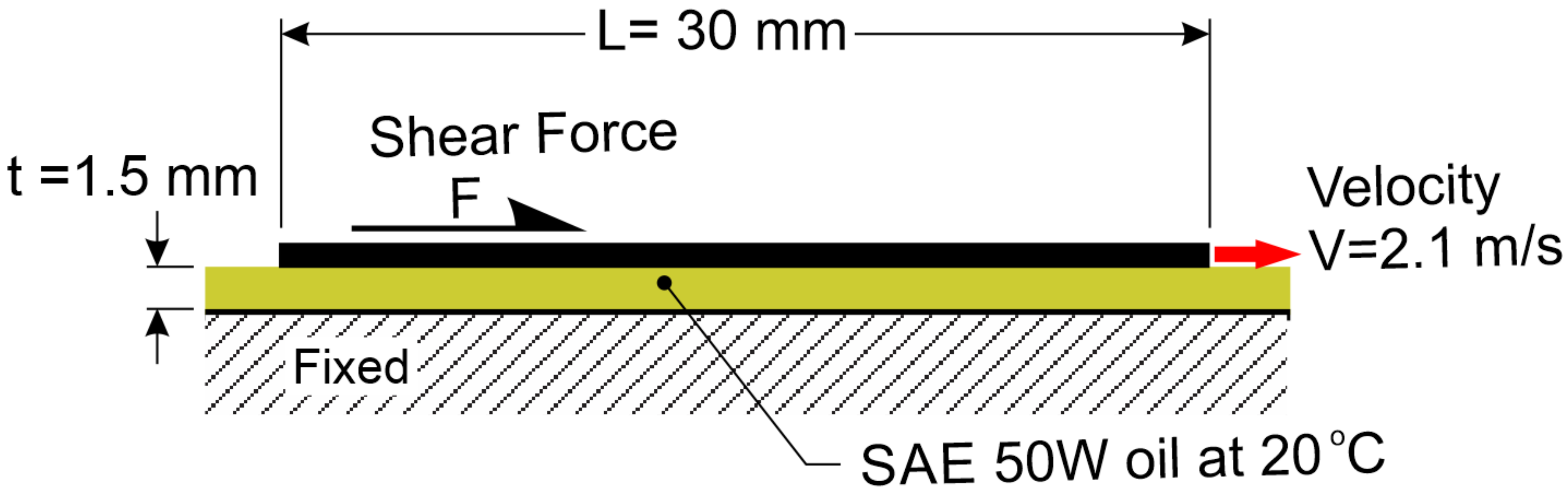
$$1 \text{ cP} = 10^{-3} \text{ kg/(m s)}$$

(After Jean Poiseuille, studied blood flow in 1800s)

Bigger spindles are for less viscous fluids

Example: The Viscous Shear Force

- A plate in a machine is lubricated by a film of SAE50W oil at 20 °C with thickness t=1.5 mm. The plate has length L=30 mm and depth w=130 mm (into the page). Calculate the shear force (F) required to slide the plate at a velocity of V=2.1 m/s.



Solution

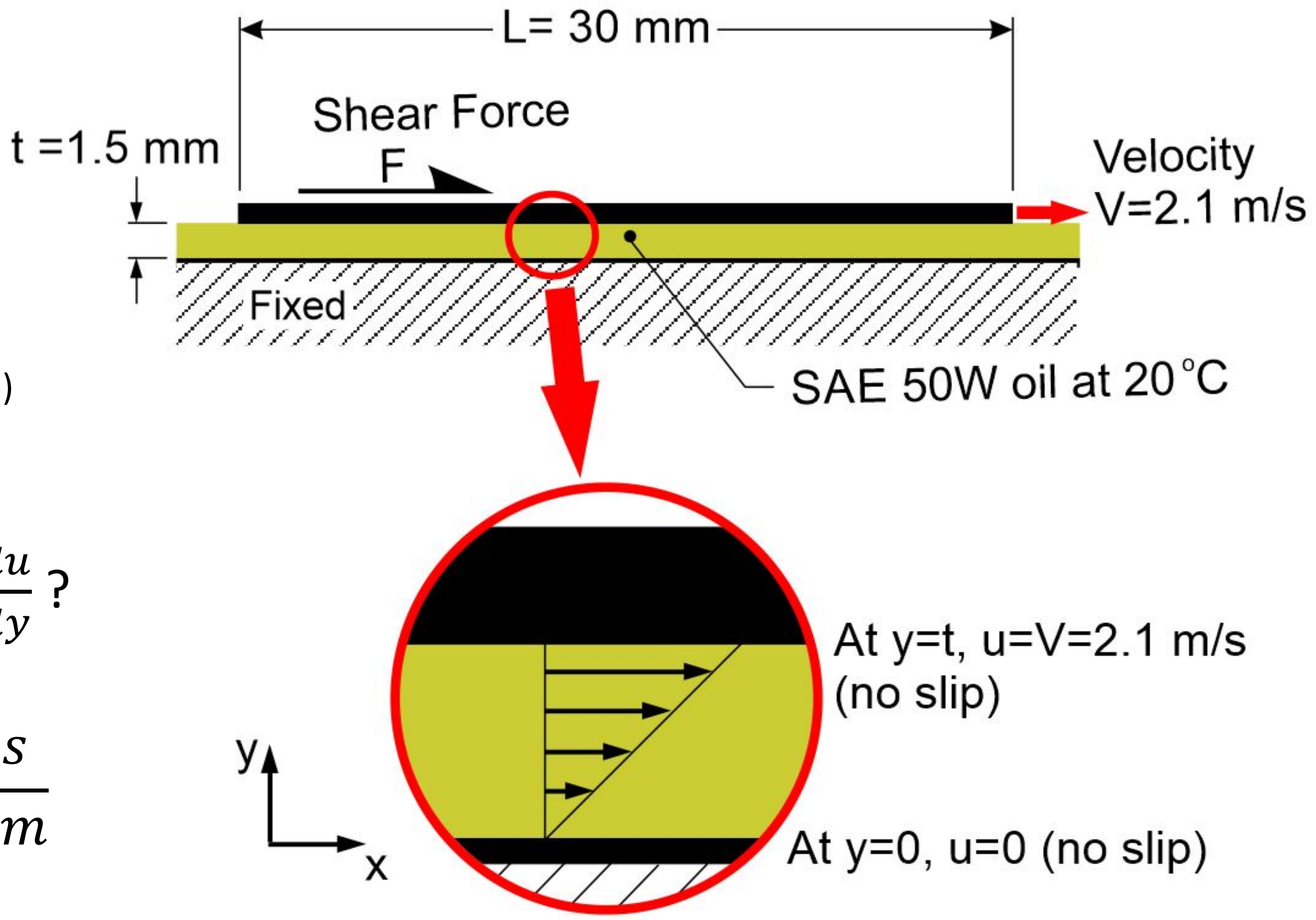
- Shear stress in a fluid:

$$\tau = \mu \frac{du}{dy}$$ (Engine oil is Newtonian)

- What is the velocity gradient, $\frac{du}{dy}$?

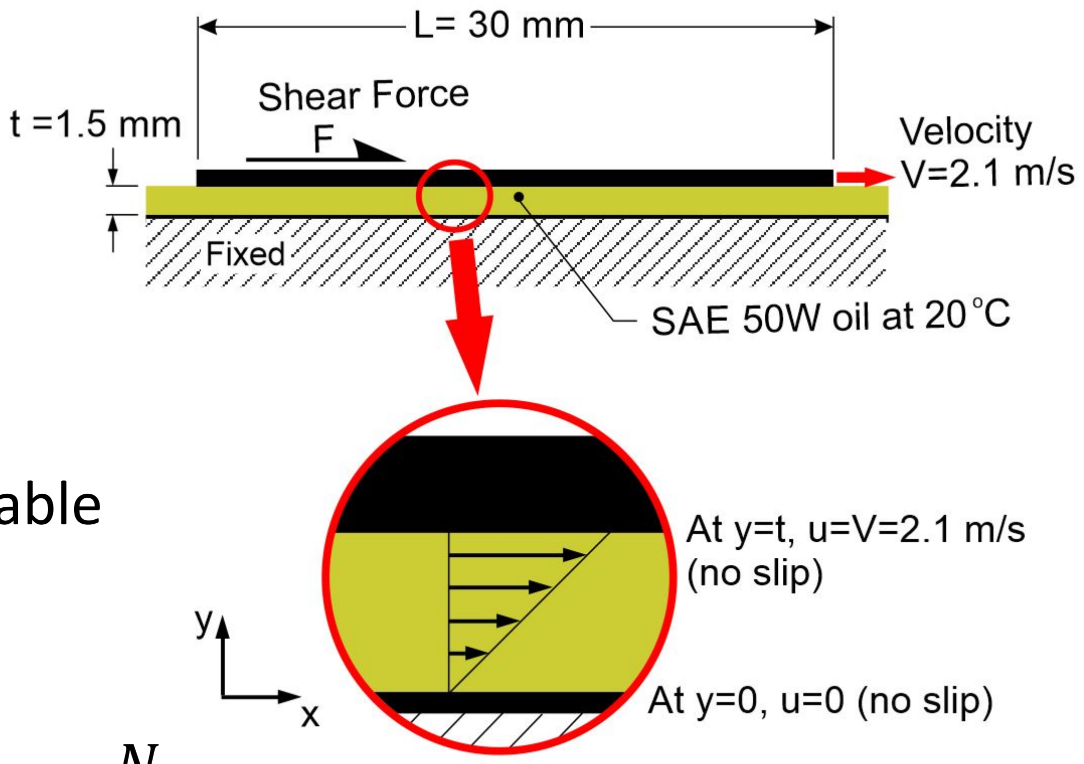
$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{V - 0}{t} = \frac{2.1 m/s}{0.0015 m}$$

$$\frac{du}{dy} = 1400\ s^{-1}$$



Solution

$$\frac{du}{dy} = 1400\ s^{-1}$$



- Dynamic viscosity of SAE50W oil at 20 °C from Table A.3: $\mu = 0.86\ \frac{kg}{m\ s}$

$$\tau = \mu \frac{du}{dy} = 0.86\ \frac{kg}{m\ s}\left(1400\frac{1}{s}\right) = 1204\ \frac{kg}{ms^2} = 1204\frac{N}{m^2}$$

$$\tau = \mu \frac{du}{dy}$$

- From F=ma: $N \equiv kg\frac{m}{s^2}\ \ or\ \ kg \equiv \frac{Ns^2}{m}$

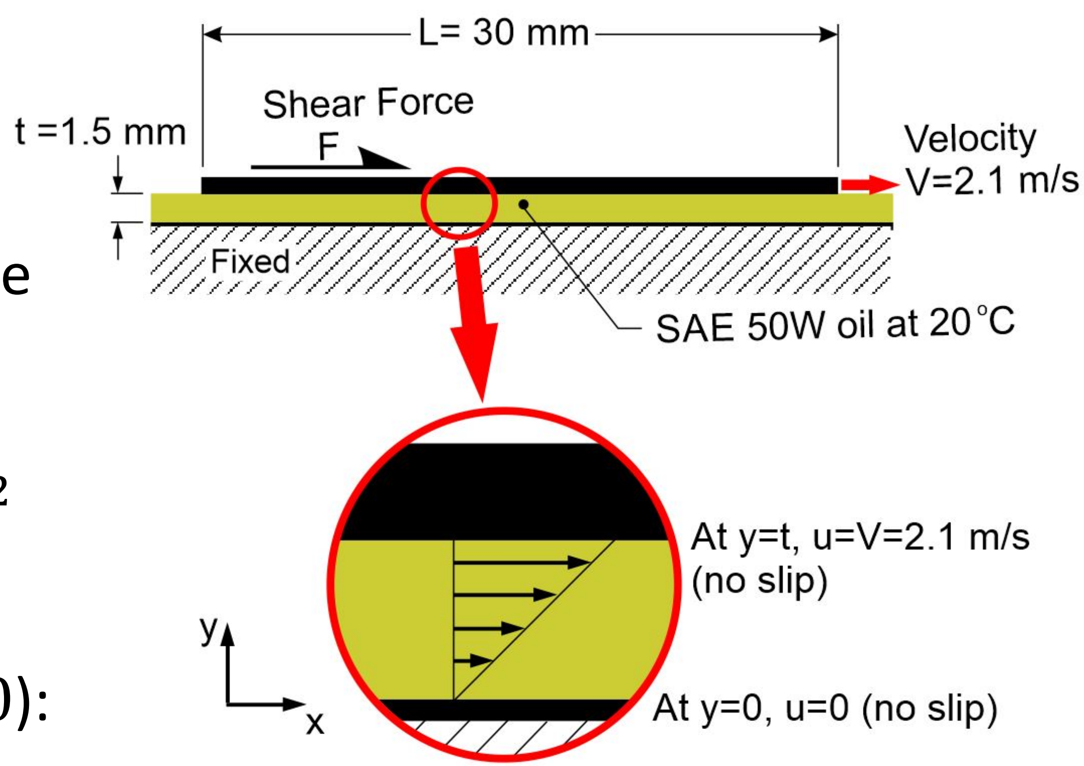
- Direction of the fluid shear stress **<u>on the plate</u>**?

Fluid viscosity opposes motion, acts like fluid "friction"

Solution

$$\tau = \mu \frac{du}{dy} = 1204\ \frac{kg}{ms^2} = 1204\ \frac{N}{m^2} \leftarrow$$

Plate depth (into page) w=130 mm

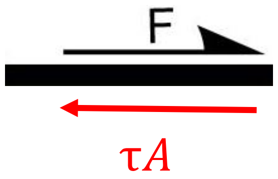


- This shear stress acts over the surface area of the plate (at fluid plate interface):

$$A = Lw = 0.030m(0.130\text{m}) = 3.9x10^{-3}m^2$$

- Force (F) balances the fluid shear force ($\sum F = 0$):

$$\text{F} = \tau\ A$$



$$F = 1204\ \frac{N}{m^2}\left(3.9x10^{-3}m^2\right) = 4.70\ N$$

Ans.

Non-Newtonian Fluids

- Some fluids, viscosity depends on shear rate and/or time

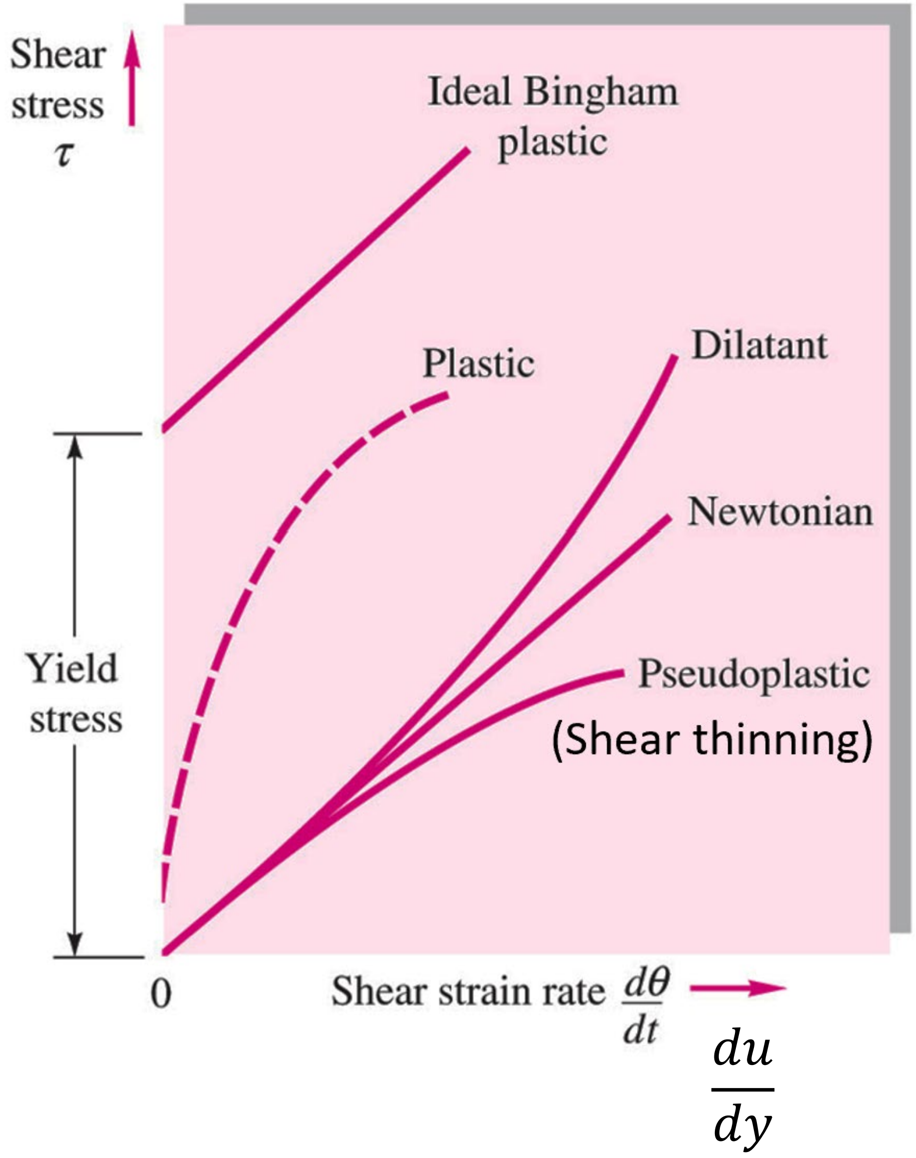
$$\tau \neq \mu \frac{du}{dy}$$

More complex models are required

- Viscosity is not constant
- Such fluids are called non-N*ewtonian fluids*

- *Examples:*

- *Blood is shear thinning*

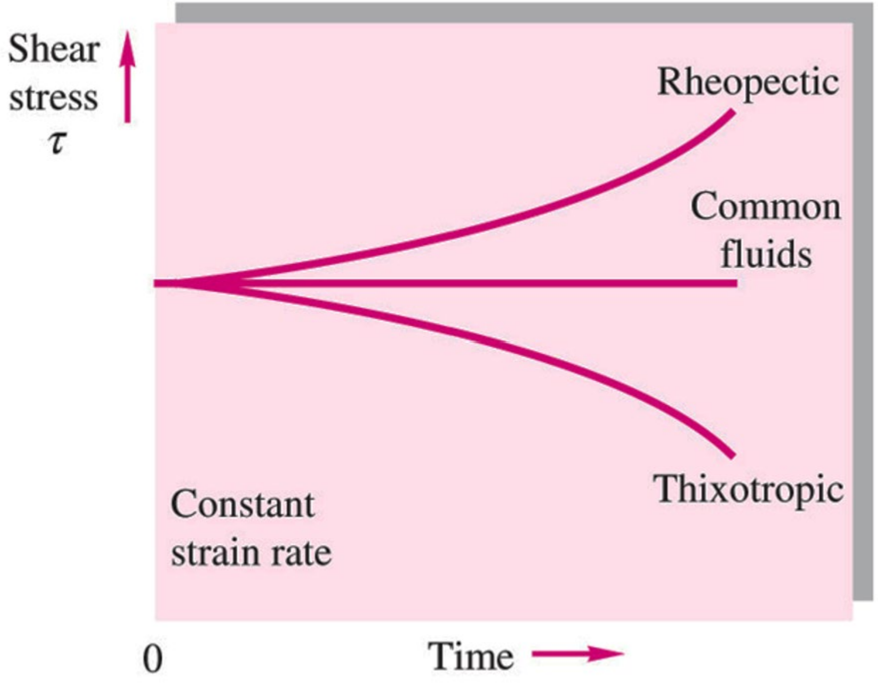


Non-Newtonian Fluids

$$\tau \neq \mu \frac{du}{dy}$$

- *Examples:*

- *Ketchup is thixotropic* (viscosity decreases with time)
- *Egg whites are rheopectic*



- Study of these unusual fluids is called *Rheology* (beyond current scope)

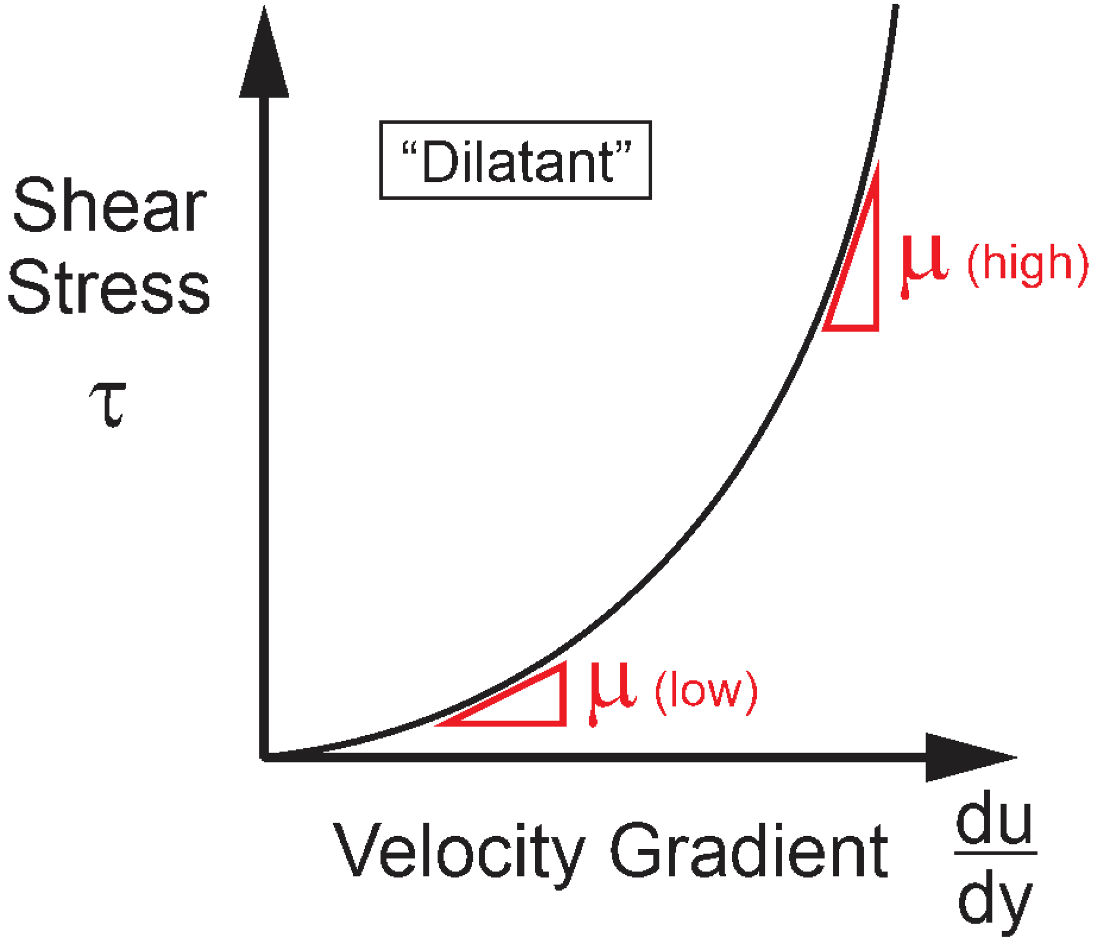
Corn Starch and Water

- A Non-Newtonian fluid
  - Recommend trying this a home

Corn Starch and Water Demo

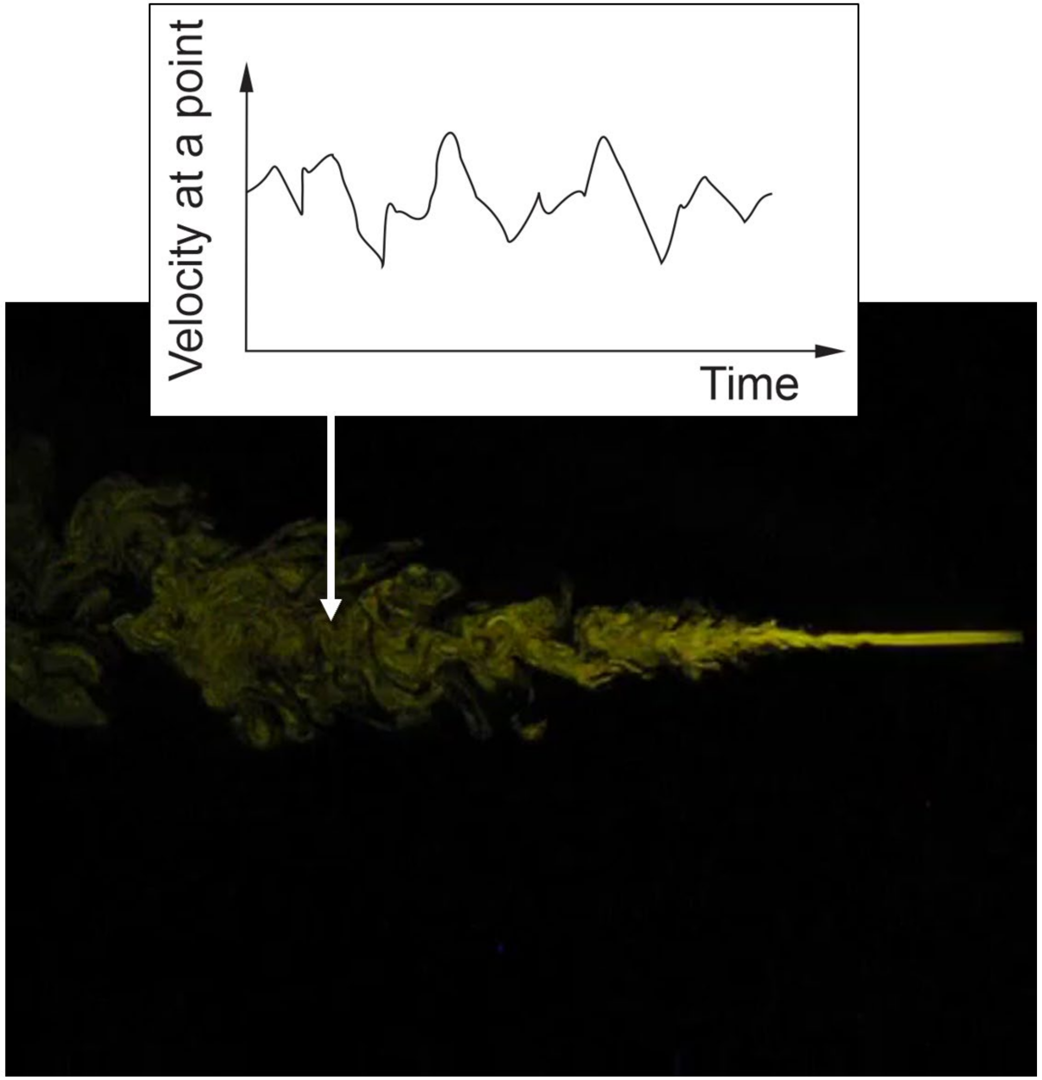
Corn Starch and Water

- What does the shear stress-velocity gradient curve look like?
- What type of non-Newtonian fluid is corn starch and water?
- Viscosity increases with shear rate: *Dilatant*



Laminar & Turbulent Flows

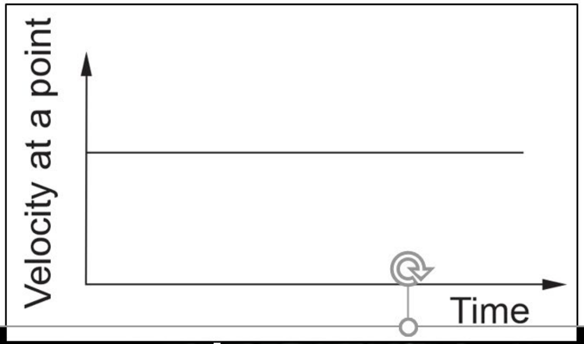
- In turbulent flow, fluid "particles" move in irregular paths
- The velocity at a fixed point varies randomly (Turbulent flow is a *stochastic process*; requiring statistical analysis)
- Enhances convective heat transfer, good mixing
- Turbulent flows are most common
- While there are some good engineering models, turbulence remains an unsolved problem in modern physics



Video: Turbulent Jet Flow

Laminar & Turbulent Flows

- In *laminar flow* the fluid "particles" move along smooth paths (from *laminae*, meaning thin layers)
- Velocity at a point is constant in steady flow
- Laminar flow occurs a low fluid velocities, over small objects or in highly viscous fluids
- Fluid viscosity dampens out the eddies (vortices) associated with turbulent flow



Video: Laminar flow over an inclined airfoil

Transition from Laminar to Turbulent Flow

- The character of a flow, laminar or turbulent, depends mainly upon a dimensionless parameter called the ***Reynolds number (Re):***

$$Re = \frac{\rho V D}{\mu}$$ (e.g. Re<2300 laminar pipe flow)

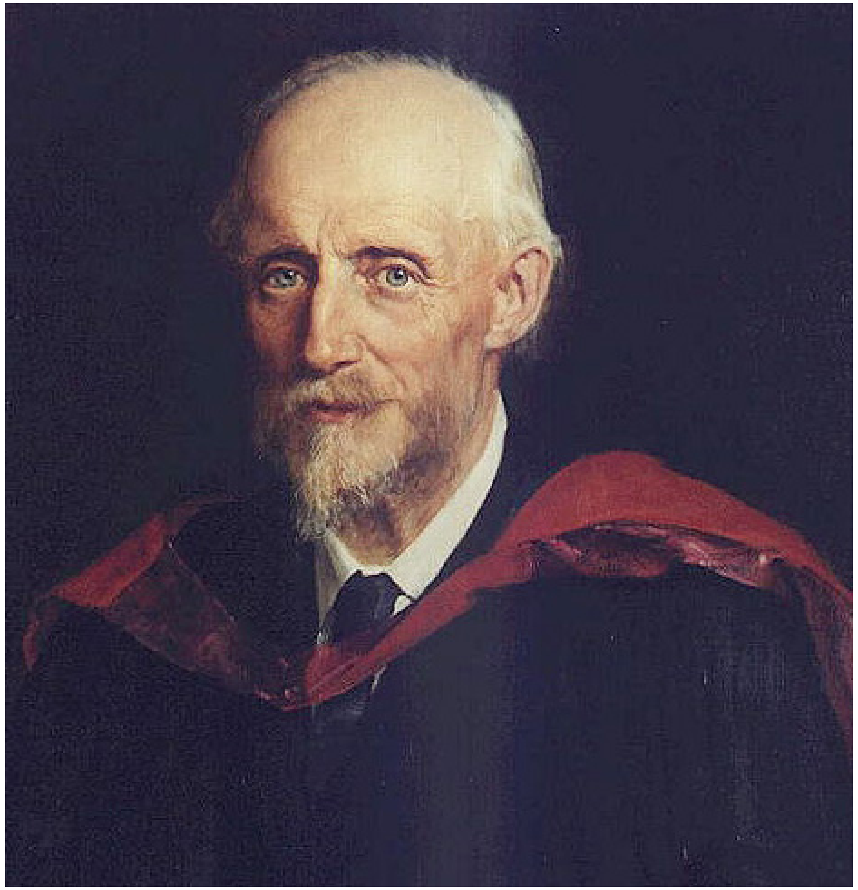
ρ *fluid density*

μ *fluid viscosity*

V *fluid velocity*

D *pipe diameter*

- You will learn more about the Reynolds number, later in this course and in Lab 2
- We will show this famous result in Chapter 5



Osborne Reynolds (1842-1912)
Irish a pioneer in fluid dynamics

Example: Dimensional Consistency

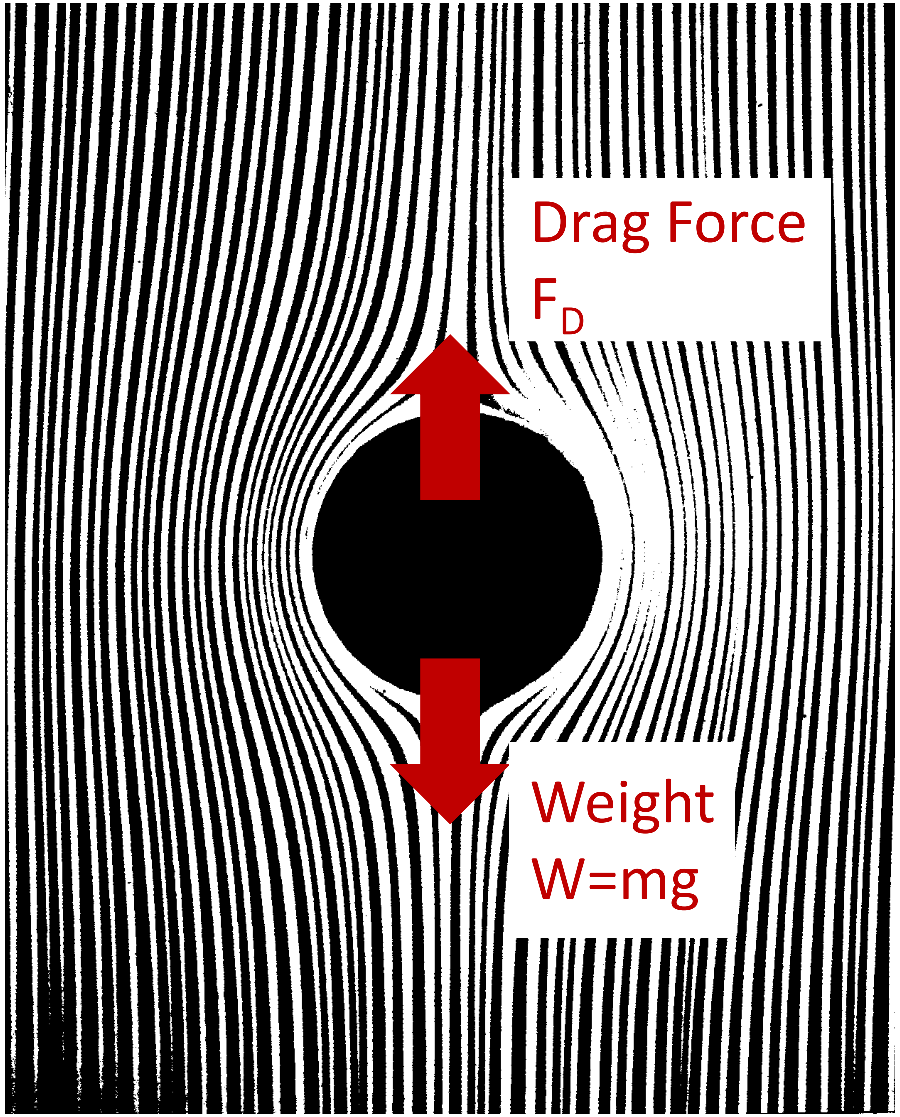
- The dynamic viscosity (μ) of an oil is calculated by measuring the terminal velocity (V) of small spheres falling under the action of gravity (g)

- For very slow flow ("Stokes Flow"):

$$\mu = \frac{D^2 g\,(\rho_{sphere} - \rho_{oil})}{18\,V}$$

Confirm the dimensional consistency of this equation

Watch the Video Solution



Streamlines for Laminar Flow Over a Sphere

Example: Viscous Shear Stress in a Boundary Layer

When a fluid flows over a surface, the velocity is reduced near the surface due to the action of viscosity. The velocity profile is approximately:

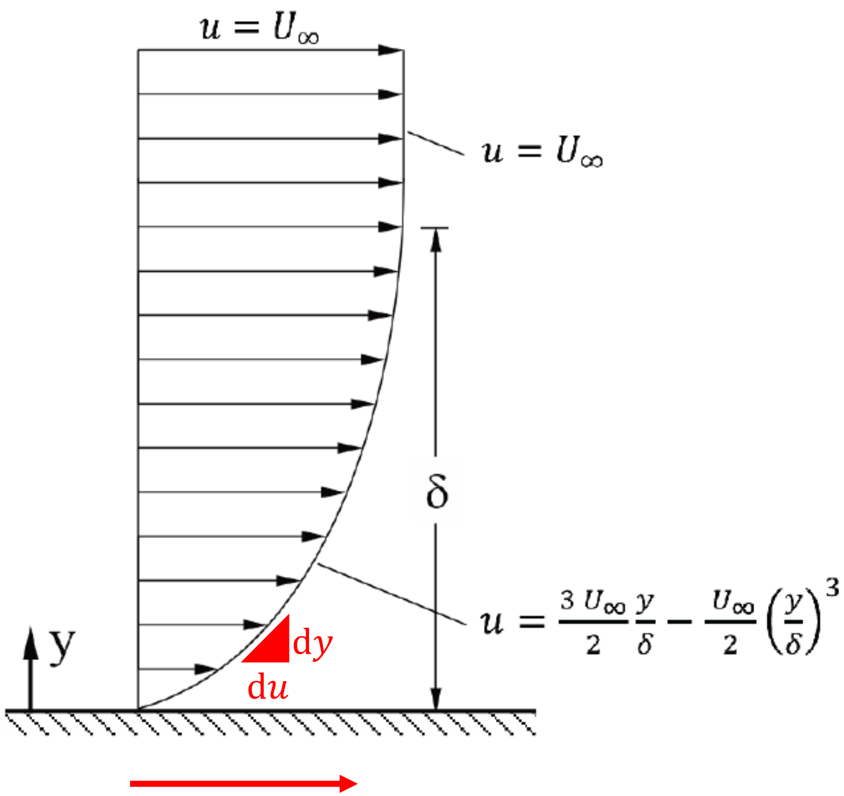
$$u = \frac{3}{2}\frac{U_\infty}{}\frac{y}{\delta} - \frac{U_\infty}{2}\left(\frac{y}{\delta}\right)^3 \qquad 0 \le y \le \delta$$

$$u = U_\infty \qquad y > \delta$$

$$\tau_{wall} = \mu \left.\frac{du}{dy}\right|_{y=0}$$

U_∞ is called the freestream velocity, the uniform velocity far from the surface

δ is called the boundary layer thickness



Shear Force on Surface $F = \tau A$

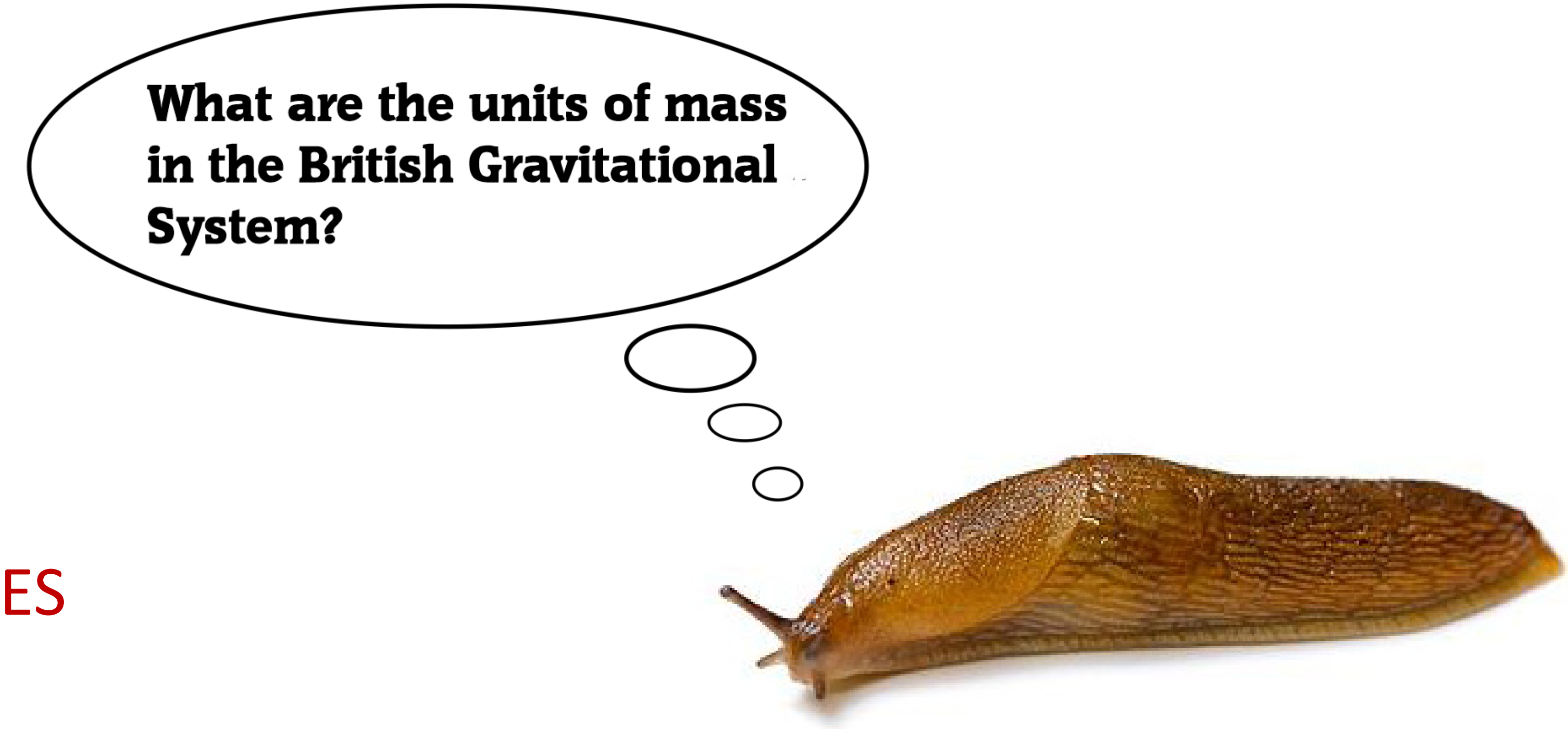
Calculate:

(a) The fluid shear stress **on** the surface. In what direction does it act?

(b) The fluid shear stress at edge of the boundary layer, $y = \delta$.

(c) For 20°C air flowing at $U_\infty = 11\ m/s$, calculate the total shear force on a surface with area (one side) A=15 m^2. Assume the boundary layer thickness is δ=5.0 mm and constant over the entire surface.

Watch the Video Solution


What are the units of mass in the British Gravitational System?

END NOTES