

*MEC516/BME516:
Fluid Mechanics I*

Chapter 1: Introduction

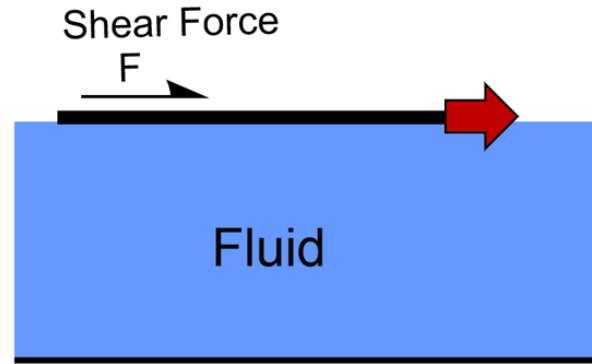
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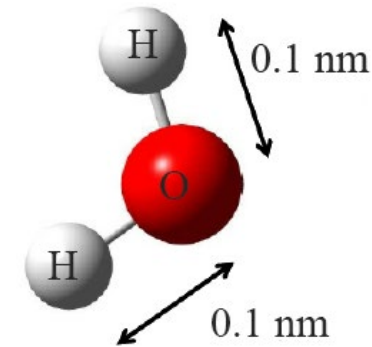
Overview

- Basic Definitions:
 - Definition of a fluid
 - Temperature and pressure



- The Continuum Approximation

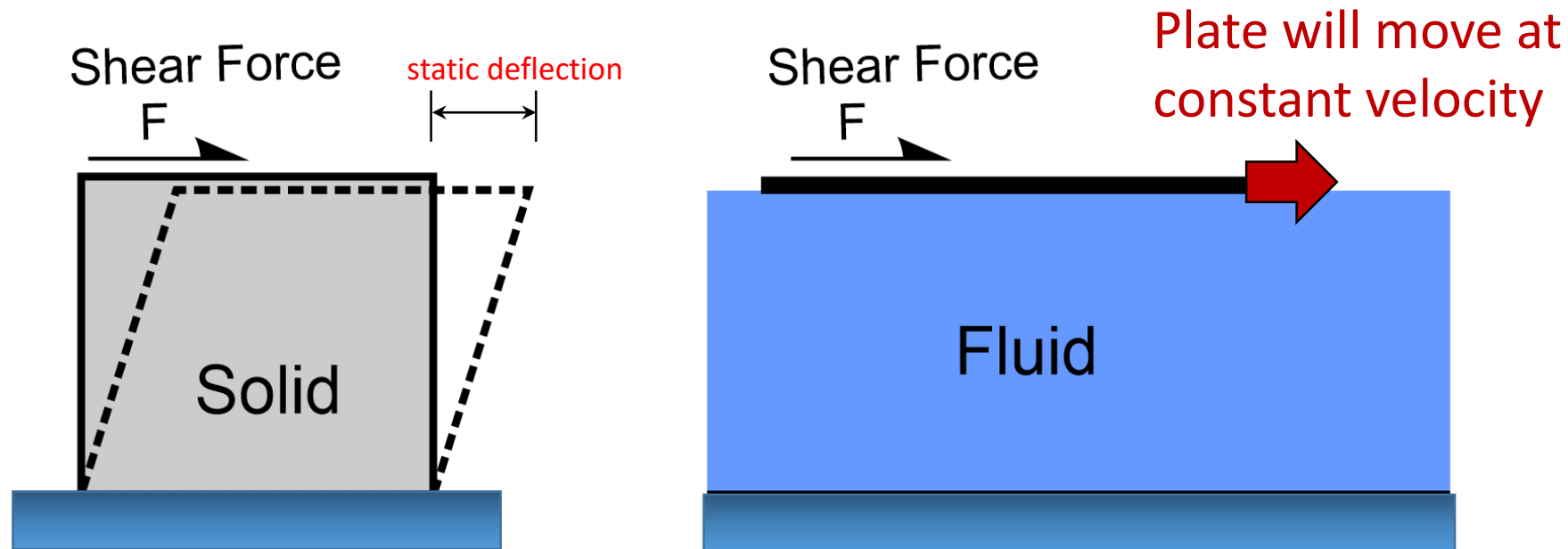
- Dimensions, Units & Dimensional Homogeneity



$$J \equiv \frac{\text{kg m}}{\text{s}^2} \text{ m} = \frac{\text{kg m}^2}{\text{s}^2} \quad \left\{ \frac{M L^2}{T^2} \right\}$$

Fluid Mechanics is the study of fluids at rest (*fluid statics*) and in motion (*fluid dynamics*)

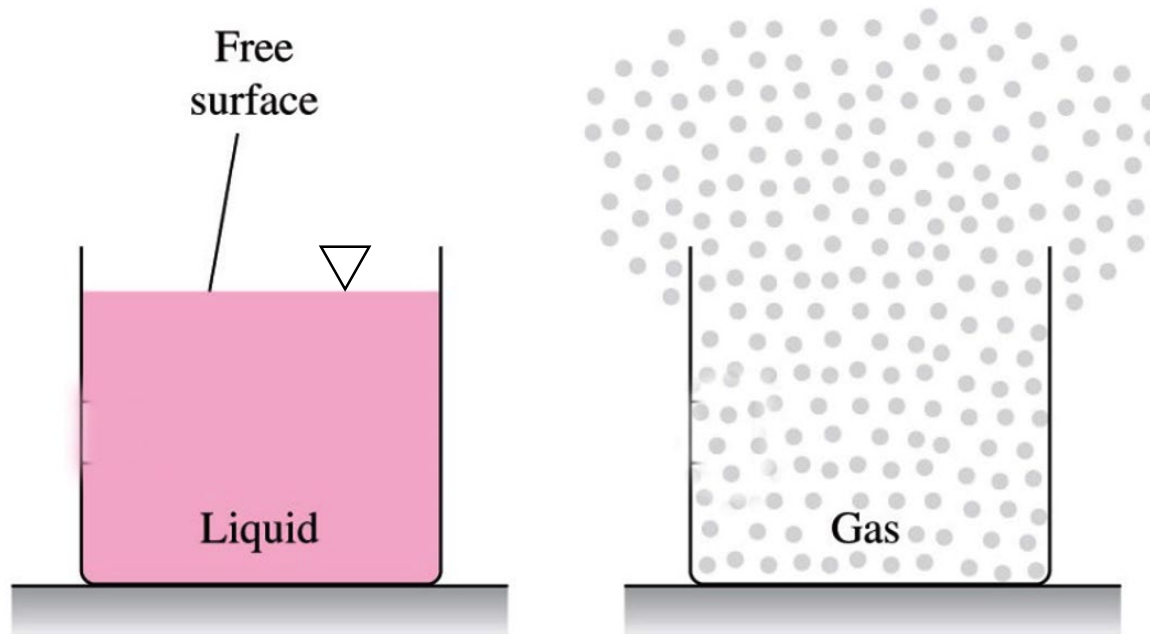
- What is a fluid? (How does a fluid differ from a solid?)



- Unlike a solid: **A fluid cannot resist shear stress**
- A fluid will deform continuously for any applied shear force, no matter how small

There are two main classes of fluids: **Gases** and **Liquids**

- Gases expand to fill their container – molecules are widely spaced
- Liquids retain their volume and form a free surface – molecules are more closely spaced

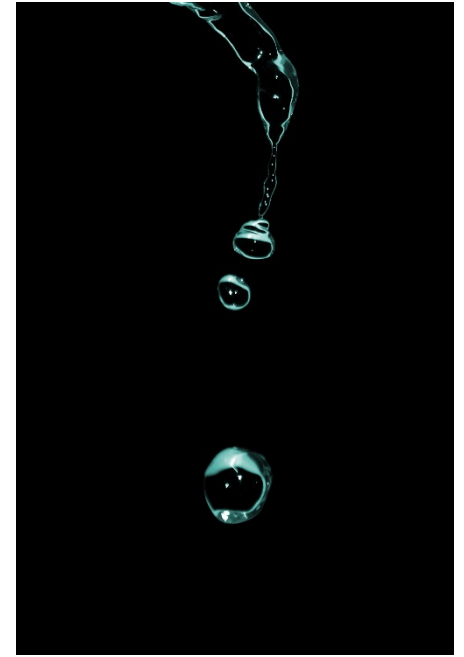
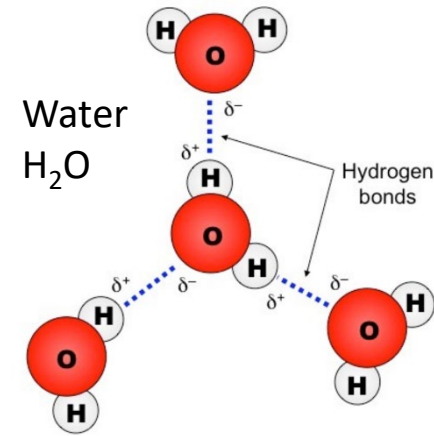


Glass contains **two** fluids: water and air

Not all fluids are liquid!

Air is a fluid
CO₂ gas is a fluid
Water vapor is a fluid

- Closely space molecules in liquids have strong cohesive forces → free droplets tend to form spheres due to **surface tension**



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- Most liquids are about $O(10^3)$ times more dense than gases at ambient conditions:

Density of ambient air (near STP): $\rho_{\text{air}} \approx 1.2 \text{ kg/m}^3$

Density of liquid water: $\rho_{\text{water}} \approx 1000 \text{ kg/m}^3$

Concept of a Fluid

- We know: Fluids cannot resist shear stress

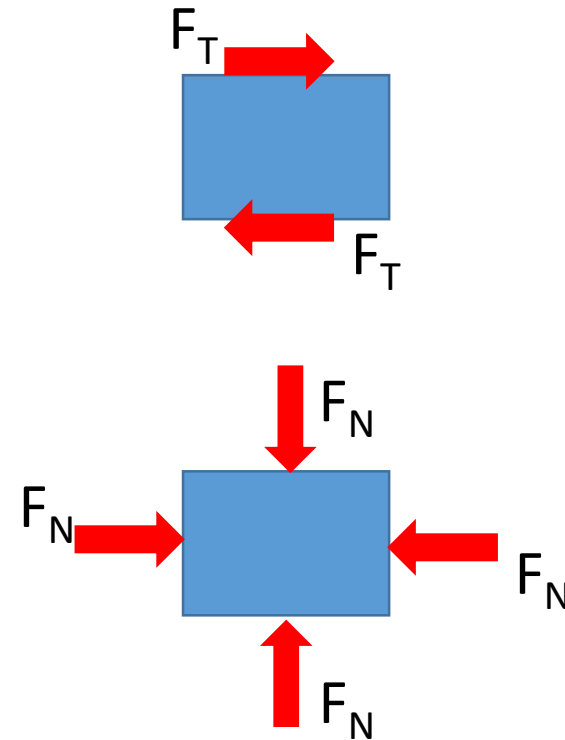
Can a fluid resist normal stress?

- (a) Yes, but only liquids
- (b) Yes, but only gases

Ans. (c) Yes, both liquids and gases

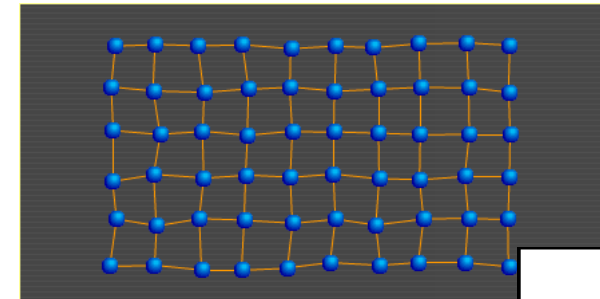
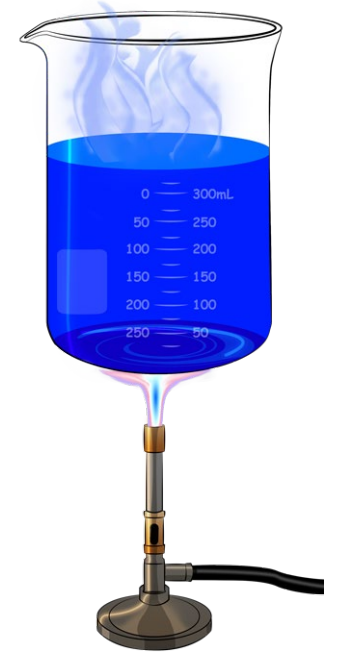
- (d) No. Fluids cannot resist normal stress

- Normal stress in a static fluid is called *pressure*

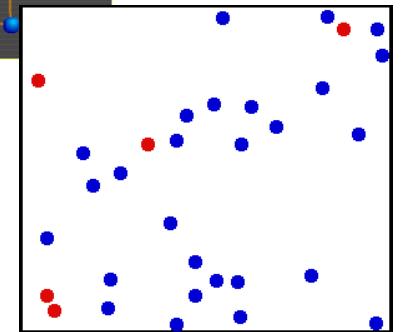


Temperature and Pressure

- If we add heat (or work) to substance, temperature increases
- But fundamentally, what is going on?
 - What is “temperature”?
- Temperature is a measure of kinetic energy of the atoms or molecules. “Jiggles”
- The “jiggling” becomes more energetic as temperature increases



Solid



Gas

Temperature and Pressure



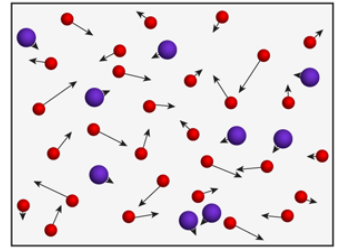
Richard Feynman

- At the atomic/molecular scale, thermal energy is kinetic energy
- In thermodynamics we call this “internal energy” (U)
- Evidence of this atomic jiggling was first seen in 1827!



“Brownian motion” of small particles, caused by random molecular motion. Add heat and the “jiggling” will get faster.

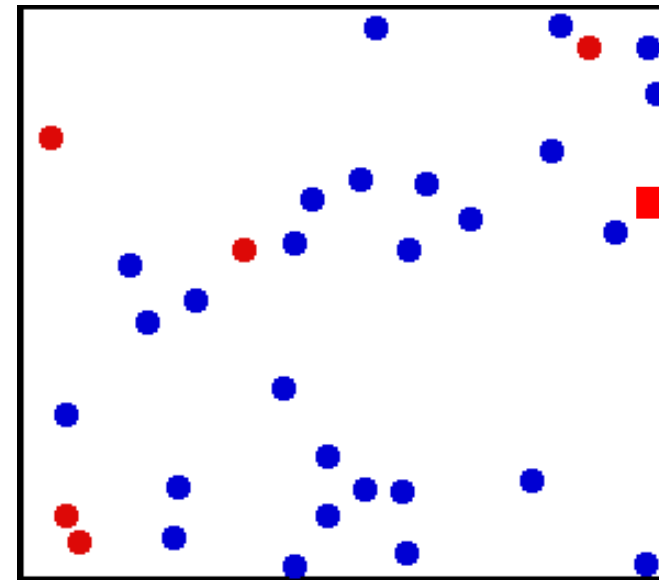
Temperature & Pressure



- Fundamentally, what is pressure? (“Force per unit area” provides no physical insight)
- When atoms bounce off the wall, they impart a force on the walls
- The change in momentum ($m \Delta \vec{v}$) causes the pressure force
- Put these two concepts together:

Why does pressure increase if you heat a sealed container?

$\Delta \vec{v}$ increases
as T increases

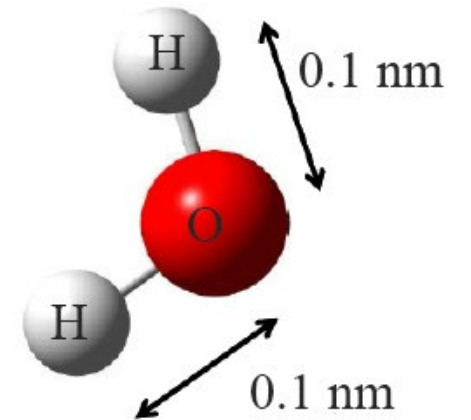
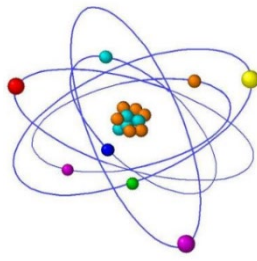
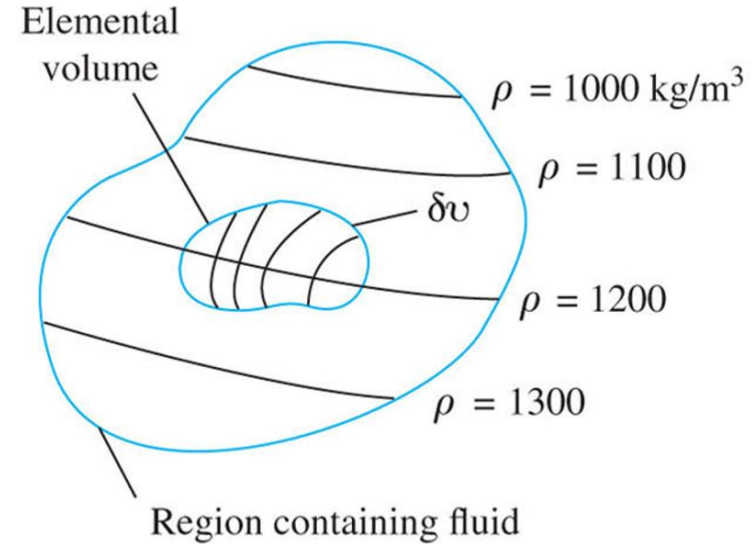


The force of atoms bouncing off the wall is pressure

Gas atoms

The Continuum Approximation

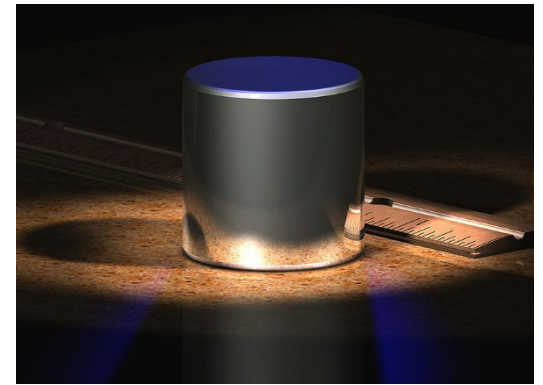
- Fluid properties are assumed to vary continuously, e.g. point density, $\rho(x, y, z)$
- We assume that $\rho(x, y, z)$ varies smoothly so we can use differential calculus
- Assumption **not** valid near the molecular/atomic scale
- Variables at a “point” actually represent an average over a small volume ($\delta V \approx 10^{-9} \text{ mm}^3$)
- Excellent approximation for most engineering purposes (breaks down at small length scales, e.g. nano-engineering)



Dimensions and Units

- In fluid mechanics we use four *Primary Dimensions*:

{ } has dimensions
 Mass {M} Length {L} Time {T} Temperature {Θ}



International Standard Kilogram (Bureau of Weights and Measures, Paris)

- A *unit* is way of attaching a number to the dimension, using an arbitrary measure

“International System” “British Gravitational System”

| Primary dimension | SI unit | BG unit | Conversion factor |
|---------------------|---------------|-------------------|---------------------|
| Mass { <i>M</i> } | Kilogram (kg) | Slug (not pounds) | 1 slug = 14.5939 kg |
| Length { <i>L</i> } | Meter (m) | Foot (ft) | 1 ft = 0.3048 m |
| Time { <i>T</i> } | Second (s) | Second (s) | 1 s = 1 s |
| Temperature {Θ} | Kelvin (K) | Rankine (°R) | 1 K = 1.8°R |

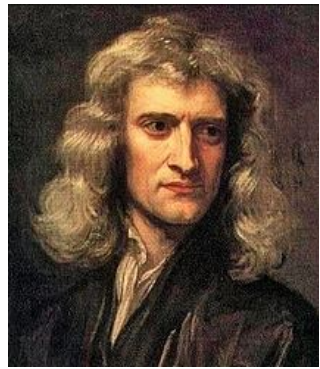
9.8 m/s²
 32.2 ft/s²

$$K = ^\circ C + 273$$

$$^\circ R = ^\circ F + 460$$

$$W = Mg, M = W/g$$

Secondary Dimensions



- The most important secondary dimension is Force {F}
- Using Newton's Second Law: $\mathbf{F = ma} \rightarrow \{F\} = \{M L T^{-2}\}$
- International System (SI): $1 \text{ N} = 1 \text{ kg} (1 \text{ m/s}^2)$
- British Gravitational System: $1 \text{ lb} = 1 \text{ slug} (1 \text{ ft/s}^2)$
- Important! "slug" is the unit of mass (not lb or lb_m) $1 \text{ slug} = 14.59 \text{ kg}$
- Conversion tables in Appendix C of textbook

Secondary Dimensions

Table 1.2

| Secondary dimension | SI unit | BG unit | Conversion factor |
|--|---------------------|-----------------------------|---|
| Area $\{L^2\}$ | m^2 | ft^2 | $1 m^2 = 10.764 ft^2$ |
| Volume $\{L^3\}$ | m^3 | ft^3 | $1 m^3 = 35.315 ft^3$ |
| Velocity $\{LT^{-1}\}$ | m/s | ft/s | $1 ft/s = 0.3048 m/s$ |
| Acceleration $\{LT^{-2}\}$ | m/s^2 | ft/s^2 | $1 ft/s^2 = 0.3048 m/s^2$ |
| Pressure or stress $\{ML^{-1}T^{-2}\}$ | $Pa = N/m^2$ | lbf/ft^2 | $1 lbf/ft^2 = 47.88 Pa$ |
| Angular velocity $\{T^{-1}\}$ | s^{-1} | s^{-1} | $1 s^{-1} = 1 s^{-1}$ |
| Energy, heat, work $\{ML^2T^{-2}\}$ | $J = N \cdot m$ | $ft \cdot lbf$ | $1 ft \cdot lbf = 1.3558 J$ |
| Power $\{ML^2T^{-3}\}$ | $W = J/s$ | $ft \cdot lbf/s$ | $1 ft \cdot lbf/s = 1.3558 W$ |
| Density $\{ML^{-3}\}$ | kg/m^3 | $slugs/ft^3$ | $1 slug/ft^3 = 515.4 kg/m^3$ |
| Viscosity $\{ML^{-1}T^{-1}\}$ | $kg/(m \cdot s)$ | $slugs/(ft \cdot s)$ | $1 slug/(ft \cdot s) = 47.88 kg/(m \cdot s)$ |
| Specific heat $\{L^2T^{-2}\Theta^{-1}\}$ | $m^2/(s^2 \cdot K)$ | $ft^2/(s^2 \cdot ^\circ R)$ | $1 m^2/(s^2 \cdot K) = 5.980 ft^2/(s^2 \cdot ^\circ R)$ |

Why does energy have dimensions $\{ML^2T^{-2}\}$?

Secondary Dimensions

- Why does energy have dimensions $\{ML^2T^{-2}\}$?

- Energy has units of Joules

$$J \equiv N m$$

- A Newton is defined by $F = ma$:

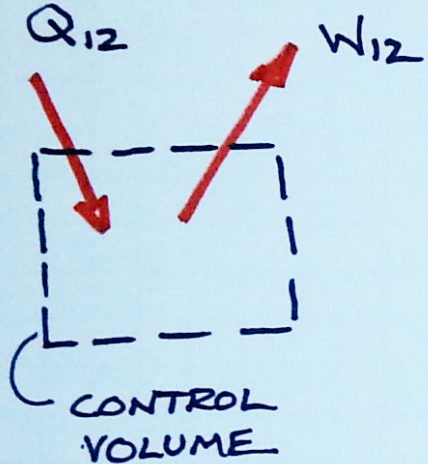
$$N \equiv \frac{kg m}{s^2}$$

$$J \equiv \frac{kg m}{s^2} m \equiv \frac{kg m^2}{s^2} \quad \text{Dimensions: } \left\{ \frac{M L^2}{T^2} \right\}$$

- Thus, energy/work/heat have dimensions $\{ML^2T^{-2}\}$

Dimensional Homogeneity

- All terms in equations must be homogeneous in both dimensions and units
- For example, the first law of thermodynamics for a closed system:



A dashed rectangular box labeled "CONTROL VOLUME" is shown. A red arrow labeled Q_{12} points downwards into the box from the top-left corner. Another red arrow labeled W_{12} points upwards and to the right, away from the box, from the top-right corner.

All terms have units of energy: $J \equiv \frac{kgm^2}{s^2}$

$$Q_{12} - W_{12} = \Delta U + \frac{1}{2} m (v_2^2 - v_1^2) + mg(z_2 - z_1)$$

| | | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| $kg \frac{m^2}{s^2}$ | $kg \frac{m^2}{s^2}$ | $kg \frac{m^2}{s^2}$ | $kg \left(\frac{m}{s}\right)^2$ | $kg \frac{m}{s^2} m$ |
| $\left\{ \frac{ML^2}{T^2} \right\}$ | $\left\{ \frac{ML^2}{T^2} \right\}$ | $\left\{ \frac{ML^2}{T^2} \right\}$ | $\left\{ \frac{ML^2}{T^2} \right\}$ | $\left\{ \frac{ML^2}{T^2} \right\}$ |

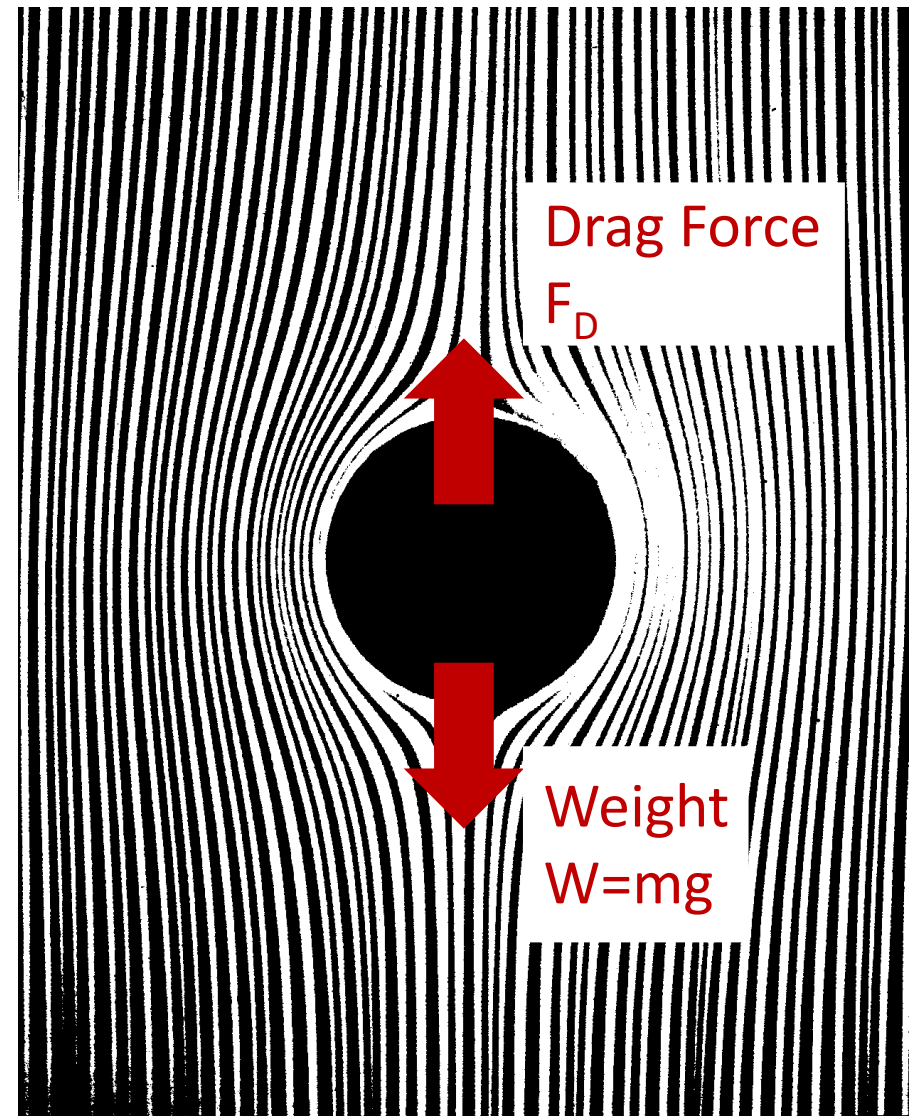
- Useful for detecting errors. Needed for *Dimensional Analysis* (Chapter 5)

Example: Dimensional Consistency

- The dynamic viscosity (μ) of an oil is calculated by measuring the terminal velocity (V) of small spheres falling under the action of gravity (g)
- For very slow flow (“Stokes Flow”):

$$\mu = \frac{D^2 g (\rho_{sphere} - \rho_{oil})}{18 V}$$

Confirm the dimensional consistency of this equation



Streamlines for Laminar Flow Over a Sphere

**What are the units of mass
in the British Gravitational
System?**

END NOTES

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