# MEC516/BME516: Fluid Mechanics I

## Chapter 1: Introduction

## Part 4: Surface Tension

RYERSON UNIVERSITY
Department of Mechanical & Industrial Engineering

# Overview

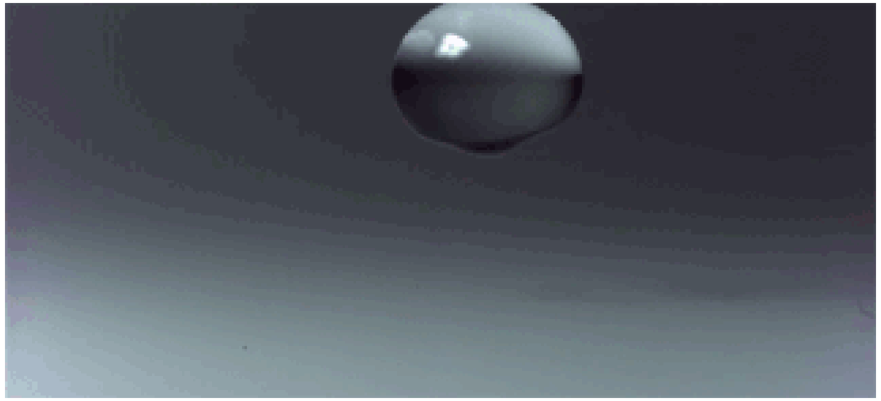
- Fluid Properties Continued

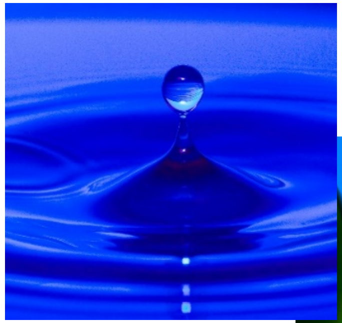
Part 4:

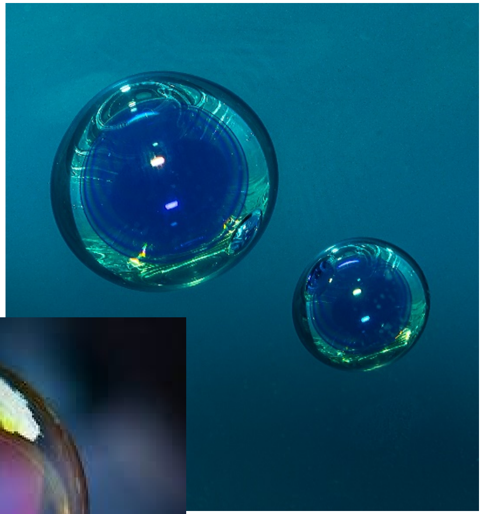
- Surface Tension
    - Wettability of Surfaces
    - Capillary Effects

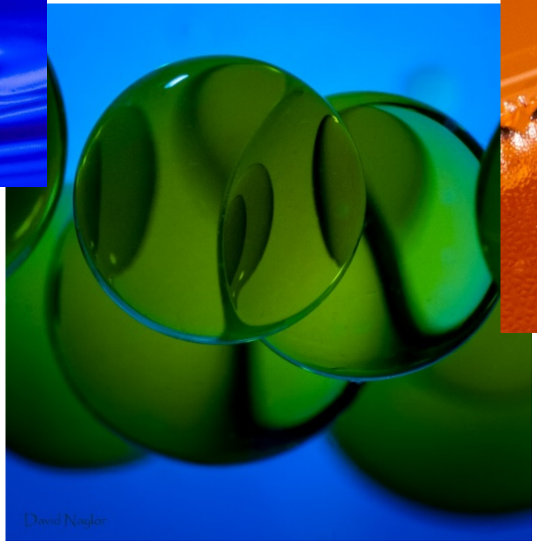
# Surface Tension, ϒ (Greek letter Upsilon)

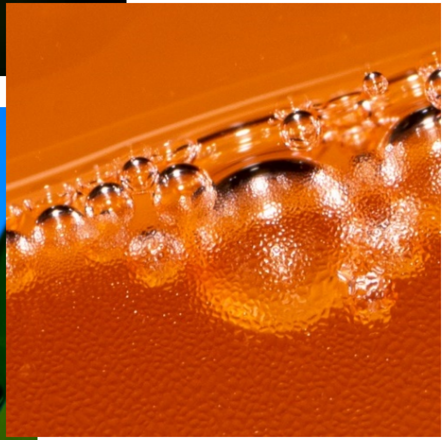
- Liquid molecules have cohesive forces. At an interface with a gas there appears to be an elastic membrane at the surface











# Surface Tension, $\Upsilon$

- Units, N/m or lb/ft
- Modelled as a membrane, where $\Upsilon$ is the force per unit length of the (fictitious) membrane
- Water-air interface, $\Upsilon$=0.073 N/m (at 20$^o$ C): Enough tension to support an insect and small metal objects at a surface

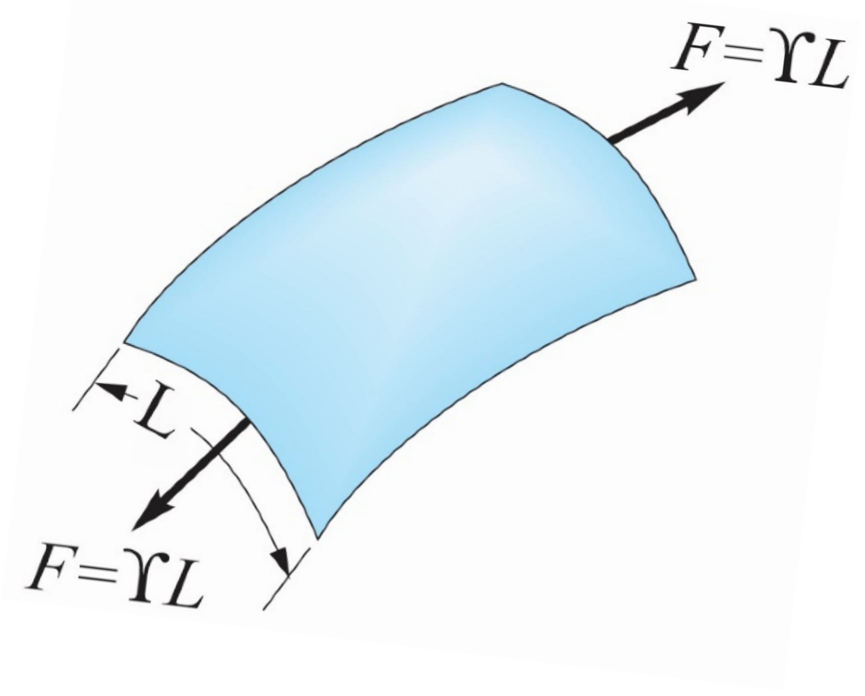
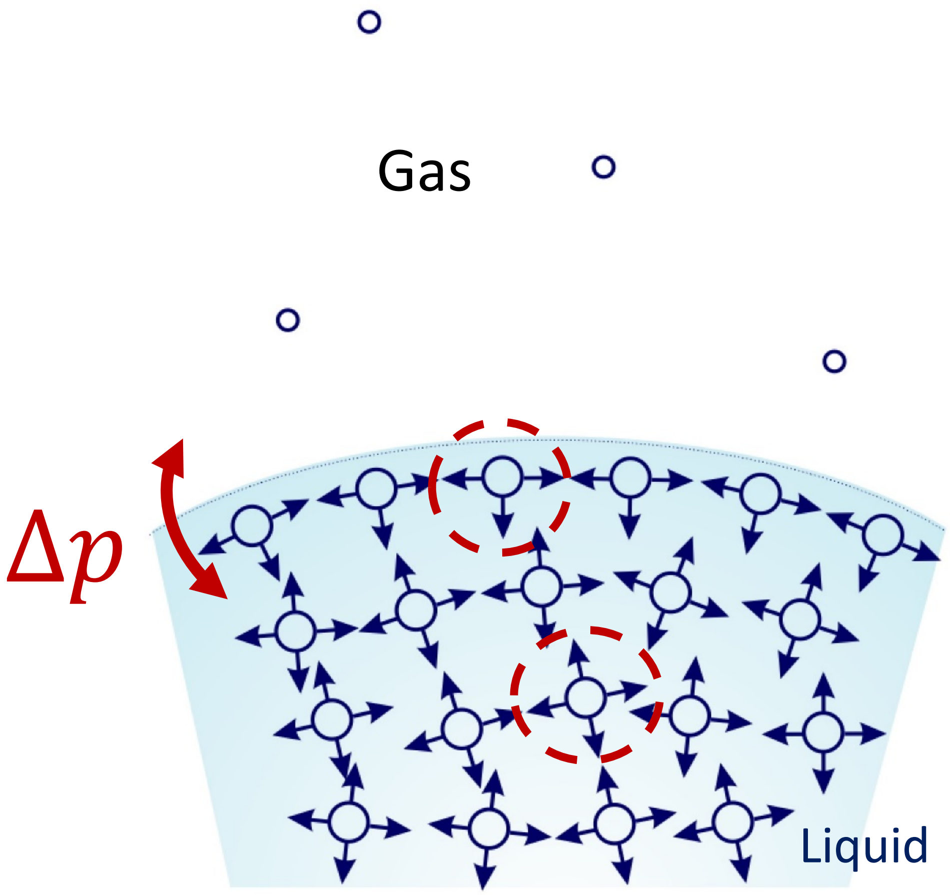
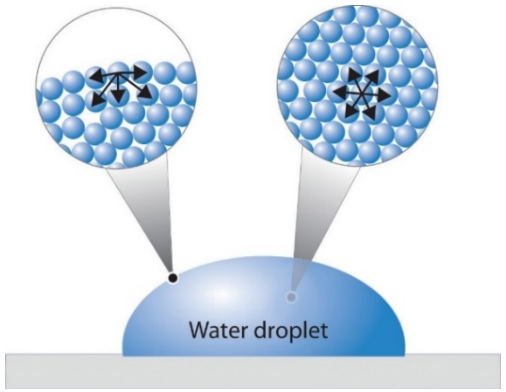



Image credit: www.freeimages.com

# Surface Tension, $\Upsilon$

- Surface tension is caused by the cohesive force between liquid molecules
- Molecules away from the interface are pulled uniformly in all direction
- Molecules at the interface are pulled sideways and inward
- At the interface the outward force is weak because the molecules in the gas are more dispersed
- This creates the tendency for interface to curve and contract under tension
- Produces pressure difference across the curved liquid-gas interface (like inside a balloon)



# Surface Tension, ϒ

- Surface tension is a function of temperature
- Decreases with temperature

Table A.5 Effect of temperature on surface tension of water in air.

| $T$, °C | ϒ, N/m |
|---|---|
| 0 | 0.0756 |
| 10 | 0.0742 |
| 20 | 0.0728 |
| 30 | 0.0712 |
| 40 | 0.0696 |
| 50 | 0.0679 |
| 60 | 0.0662 |
| 70 | 0.0644 |
| 80 | 0.0626 |
| 90 | 0.0608 |
| 100 | 0.0589 |

# Surface Tension, $\Upsilon$

- Surface tension can be strongly affected by contaminants
- Detergents reduce the surface tension

# Pressure Inside a Droplet Due to Surface Tension

- Surface tension causes the pressure inside a droplet to increase

## Example

What is the pressure inside a droplet of water with radius $R$=0.5mm at 20°C? (Ignore the effects of gravity. Local atmospheric pressure is 100.3 kPa)

# Example: Pressure Inside a Droplet

## Solution

- Consider a free body diagram of half of the droplet
  - Pressure force balances the tension in the droplet's "skin"

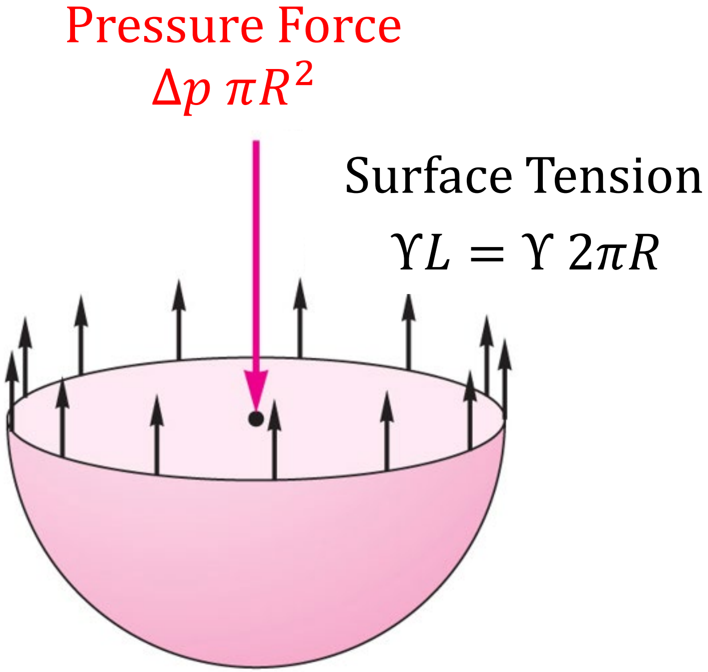
Length of interface

$$\sum F = 0 \qquad \Delta p\, A_c = \Upsilon\, L$$

$$\Delta p\, \pi R^2 = \Upsilon\, (2\pi R)$$

Droplet cross section ($\pi R^2$) — Droplet perimeter ($2\pi R$)

$$\Delta p = \frac{2\Upsilon}{R}$$




Free Body Diagram

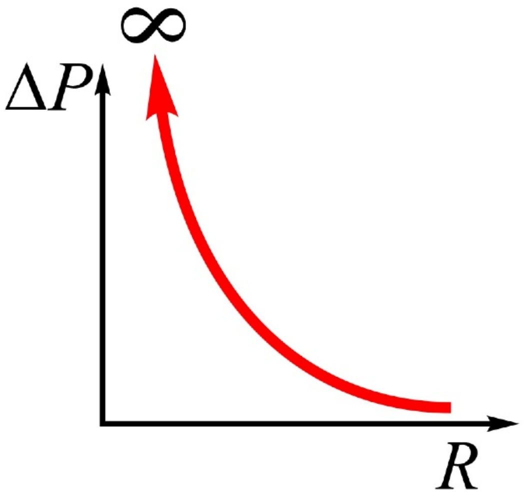
# Example: Pressure Inside a Droplet

$$\Delta p = \frac{2\Upsilon}{R},\ R = 0.5\,mm$$

Water/air at 20°C: $\Upsilon = 0.0728\,N/m$ (Table A.5)

$$\Delta p = \frac{2\,(0.0728\,N/m)}{0.0005\,m} = 291\,\frac{N}{m^2} = 0.291\,kPa$$

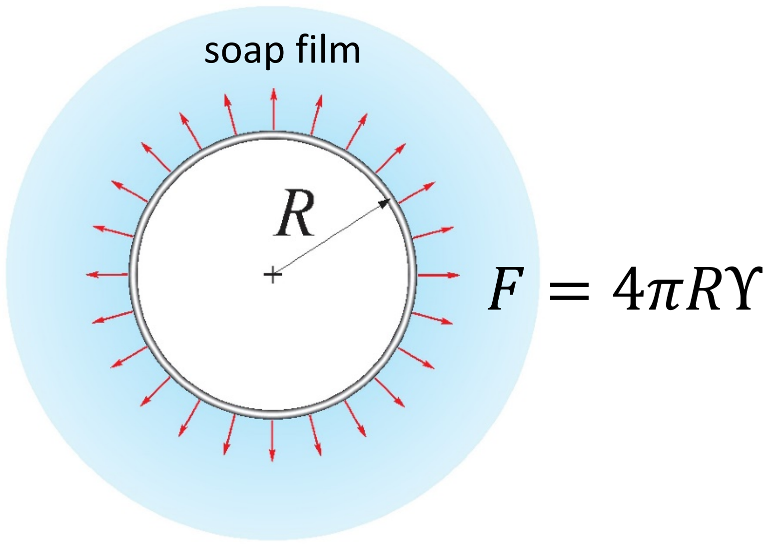
Pressure inside droplet is 291 Pa above atmospheric pressure




$p = p_{atm} + \Delta p = 100.3\,kPa + 0.291\,kPa = 100.6\,kPa$ Ans.

Aside: $R \to 0,\ \Delta p \to \infty$ (In reality?)

# Surface Tension in a Soap Film




- Total radial force in the loop of thread:

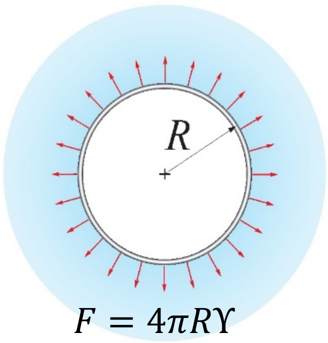
$$F = 4\pi R\Upsilon$$

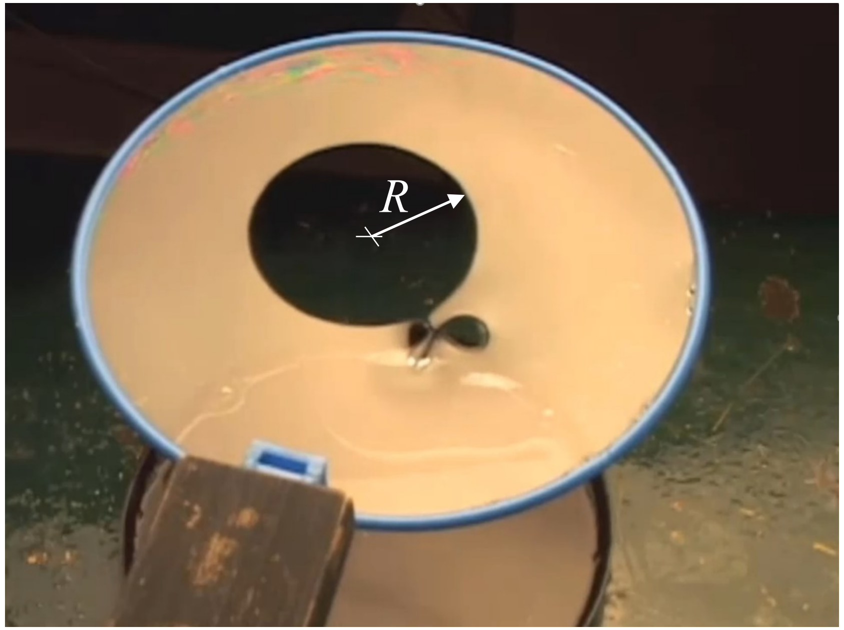
- Why not $2\pi R\Upsilon$?

Video Credit: Harvard Natural Sciences https://youtu.be/e0fhh1830Kc

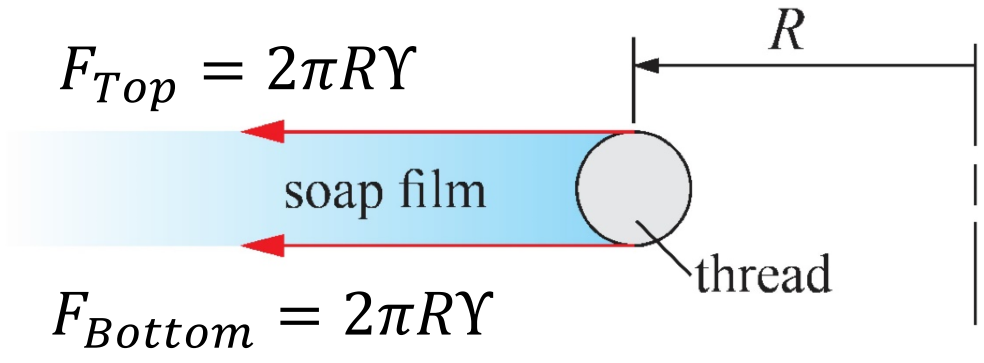
# Surface Tension in a Soap Film

- Consider a cross section of the thread



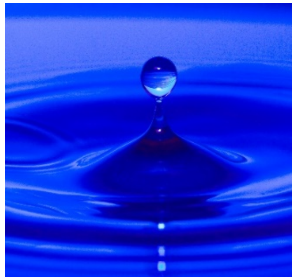

- Soap film has **two** gas/liquid interfaces:

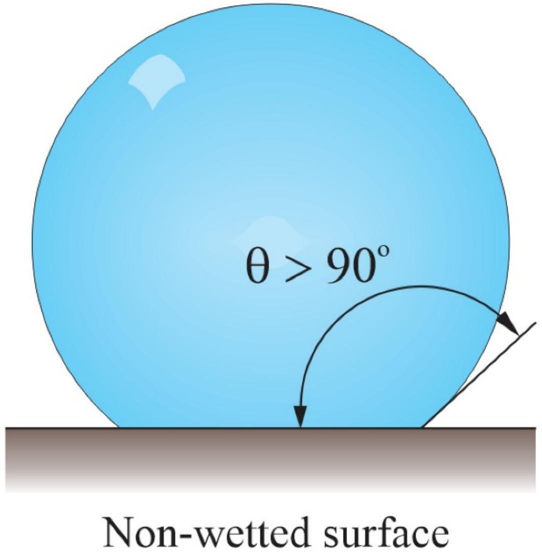


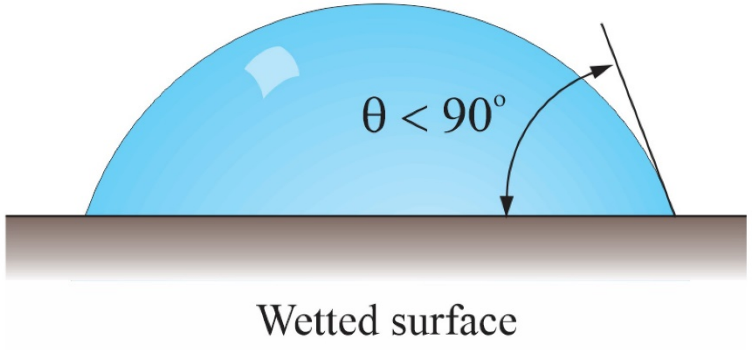

- So, the total radial force is: $F = 4\pi R\Upsilon$

$$\therefore\ \Delta p_{bubble} = 2\Delta p_{droplet} = \frac{4\Upsilon}{R}$$

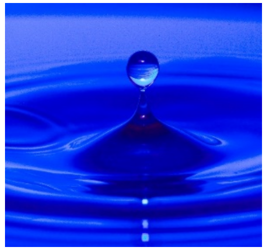
# Surface Wettability



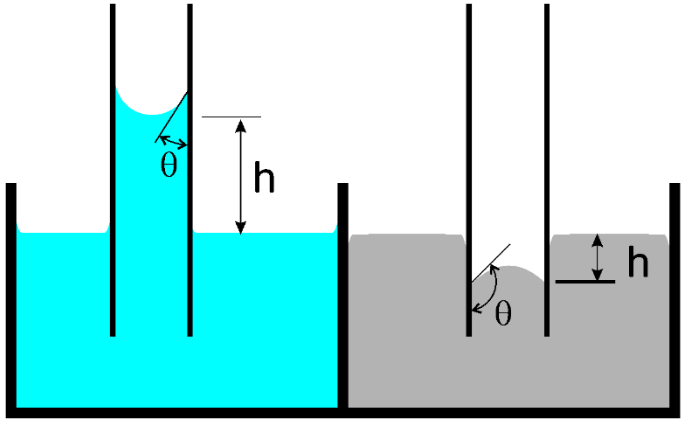



# Surface Tension, $\Upsilon$

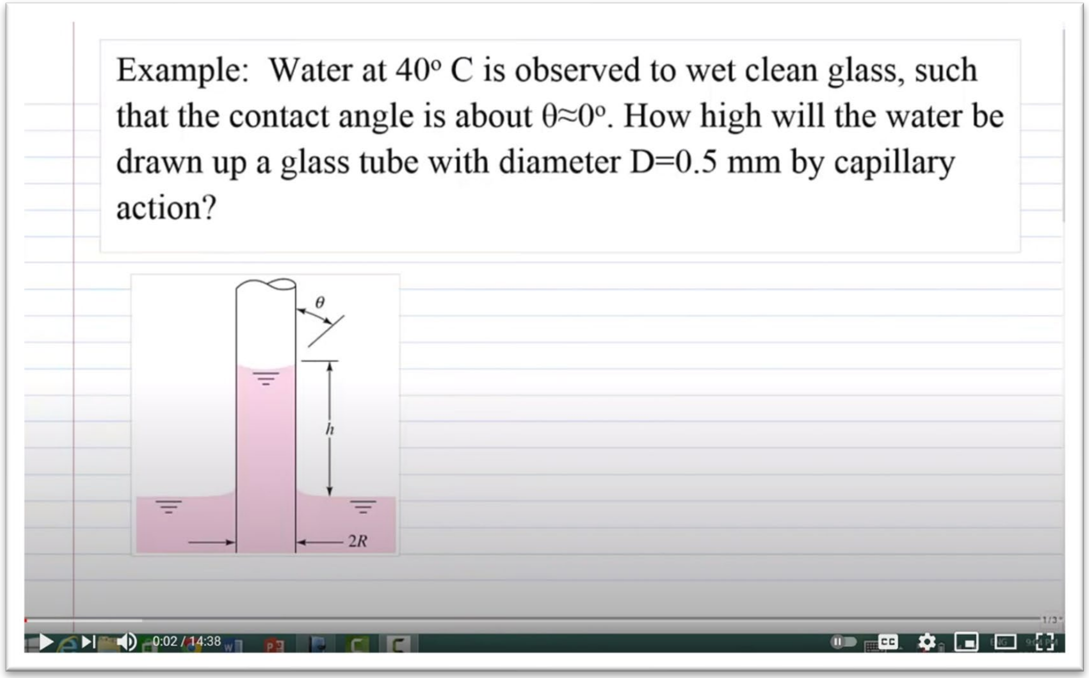


- Surface tension is the cause of *capillary action*

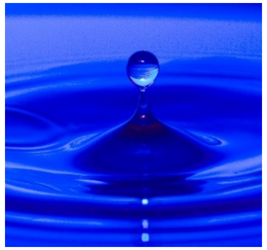



- Wetting liquid/solid pair $\theta<90^o$
  Liquid drawn up into a small tube
- Non-wetting liquid/solid pair $\theta>90^o$
  Liquid drawn down into a small tube
- Watch Video:
  Solved Example Problem: Capillary Tube

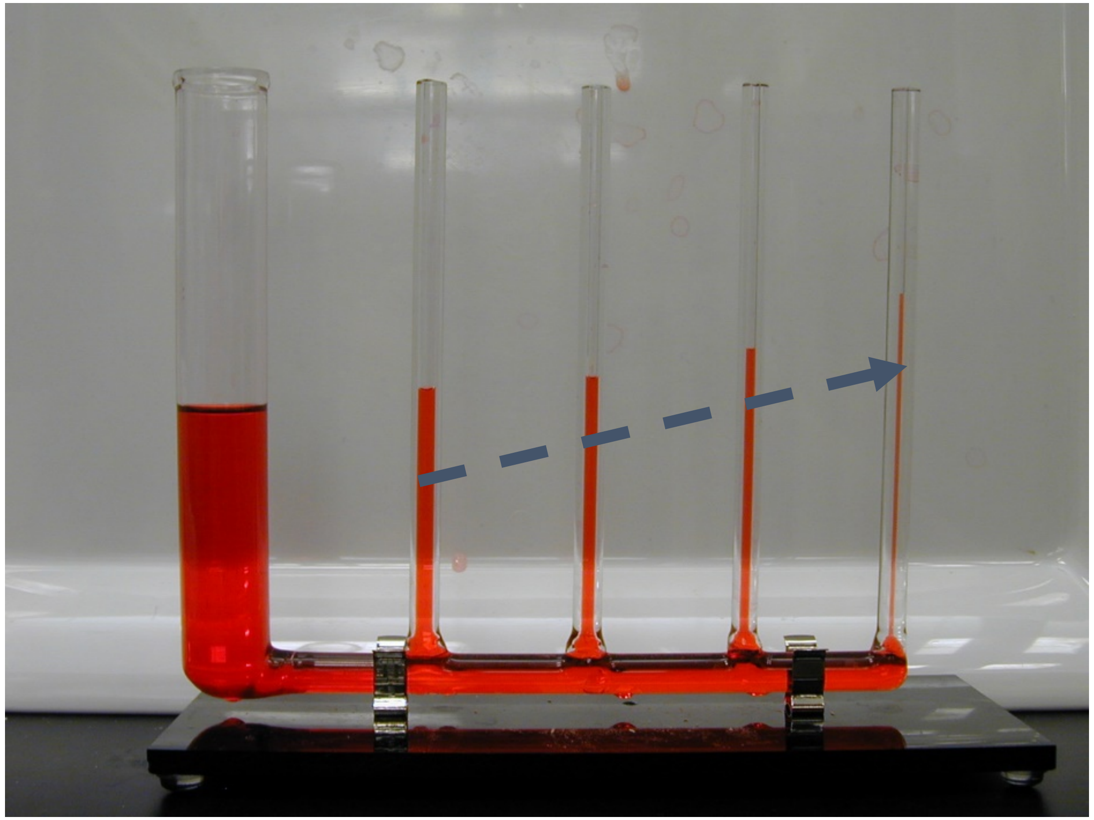
Solved Example Problem: Capillary Tube



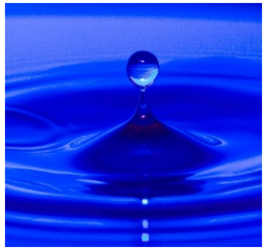
# Capillary Action



- Liquids are said to “find their own level”
- **Not true** for small diameter tubes

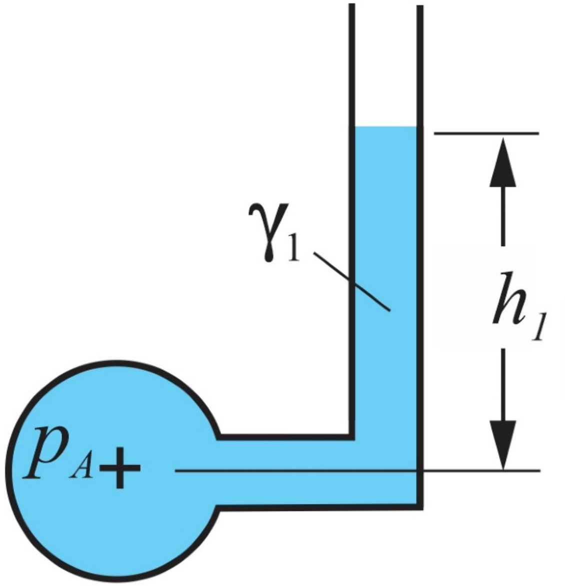


# Pressure Measurement Error



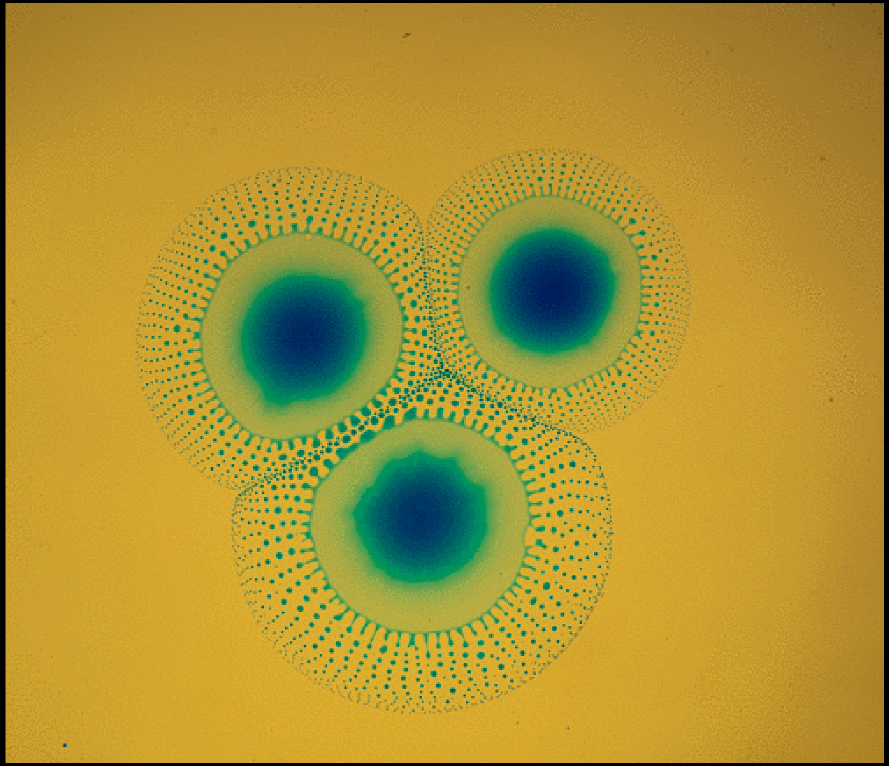
- Height of fluid in a tube used to measure pressure (Chapter 2)
- Need capillary effects to be negligible
- Tube diameter must be > 5mm (approx.)


Commercial inclined manometer

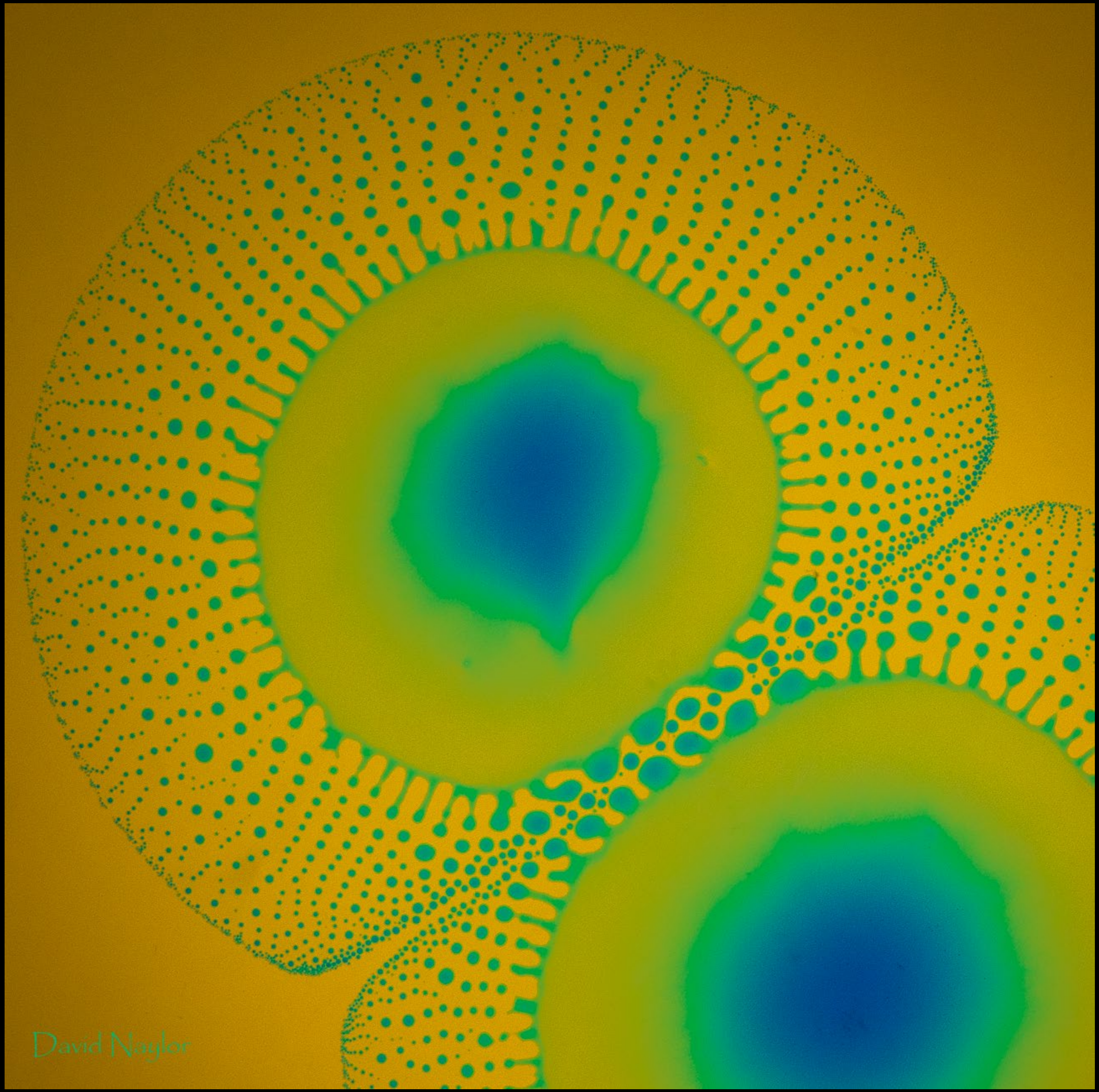



Piezometer Tube

$$p_A = \gamma h$$



Marangoni droplet bursting (Surface tension-driven flow)




# END NOTES

Presentation prepared and delivered by Dr. David Naylor