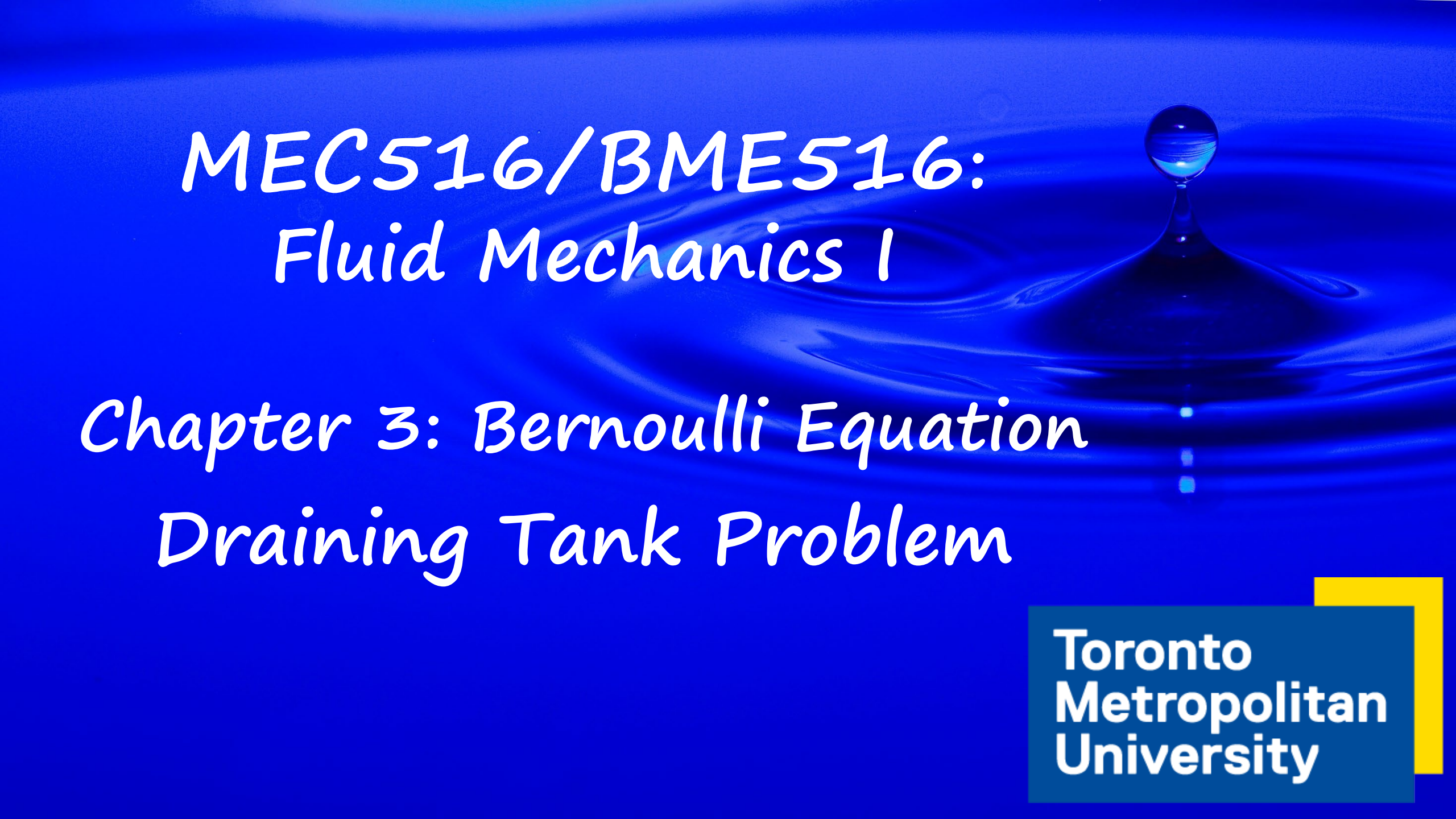


MEC516/BME516: Fluid Mechanics I

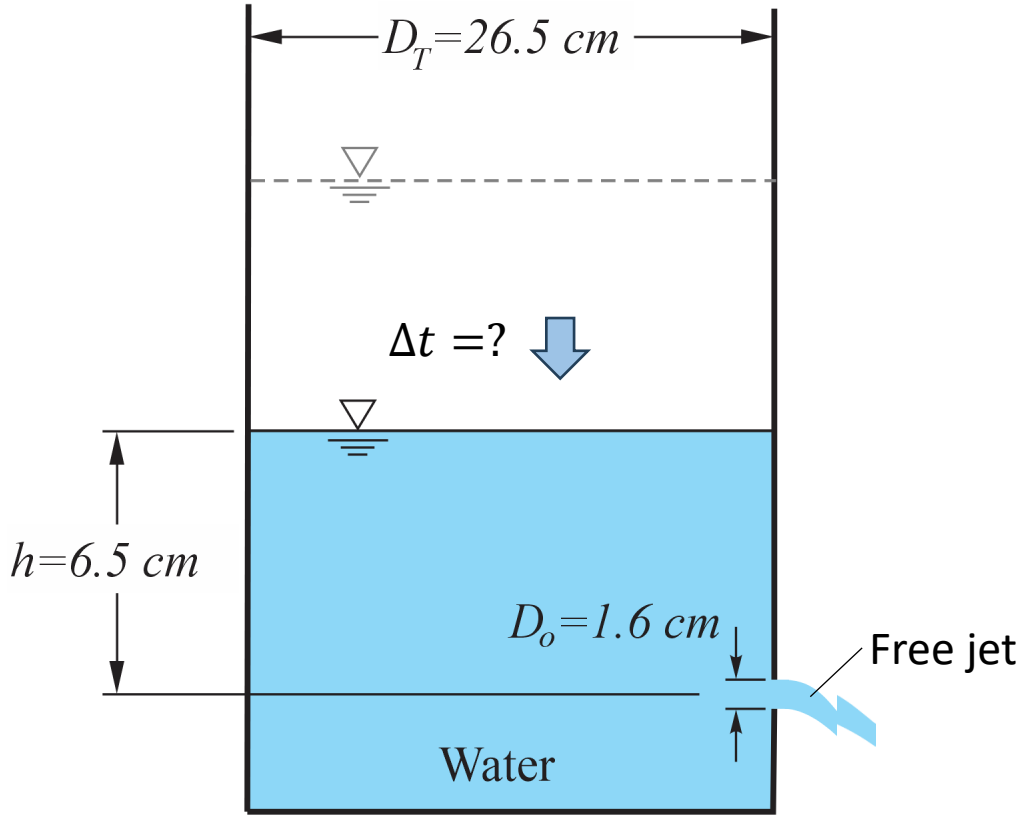
Chapter 3: Bernoulli Equation

Draining Tank Problem

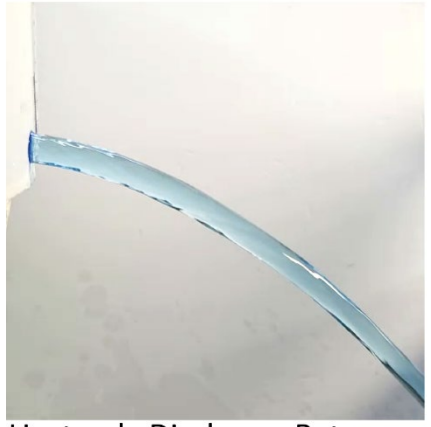
Toronto Metropolitan University

Draining Tank (Bernoulli & Continuity Problem)

Water drains from round tank ($D_T = 26.5\ cm$) by gravity through a round hole with diameter $D_o = 1.6\ cm$. The initial height of the water above the drain hole is $H = 13.0\ cm$. The tank drains slowly such that the instantaneous flow can be approximated as quasi-steady inviscid flow. ***Estimate the time for the tank level to decrease by one half*** $(\boldsymbol{h = 6.5\ cm})$.



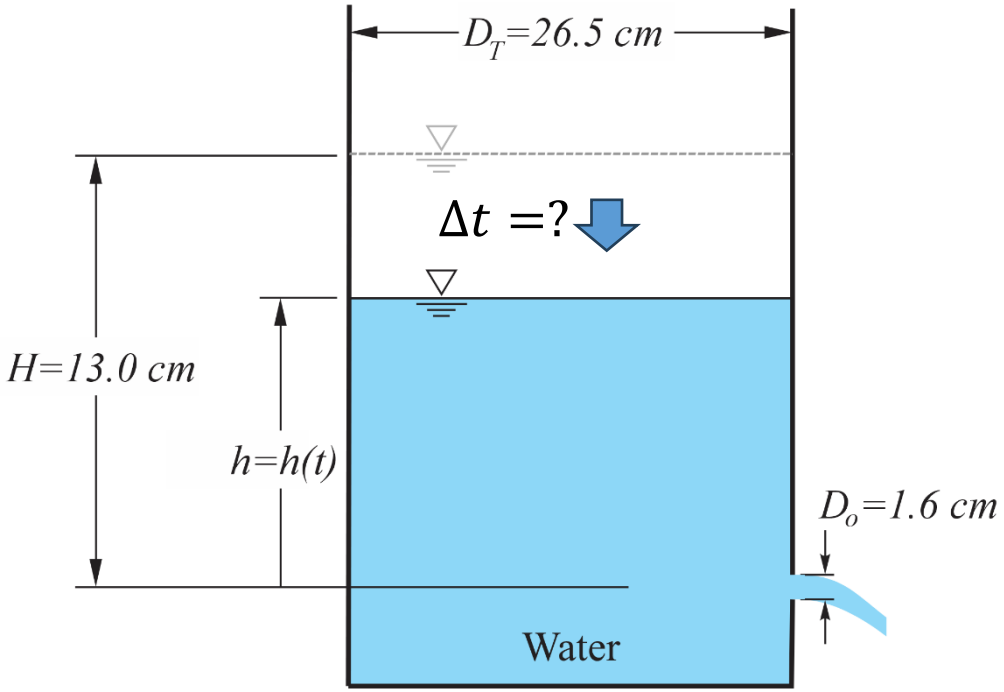
PDF available at
www.drdavidnaylor.net



Unsteady Discharge Rate

Draining Tank (Bernoulli Problem)

- End of video: Experimental test of the drain time!

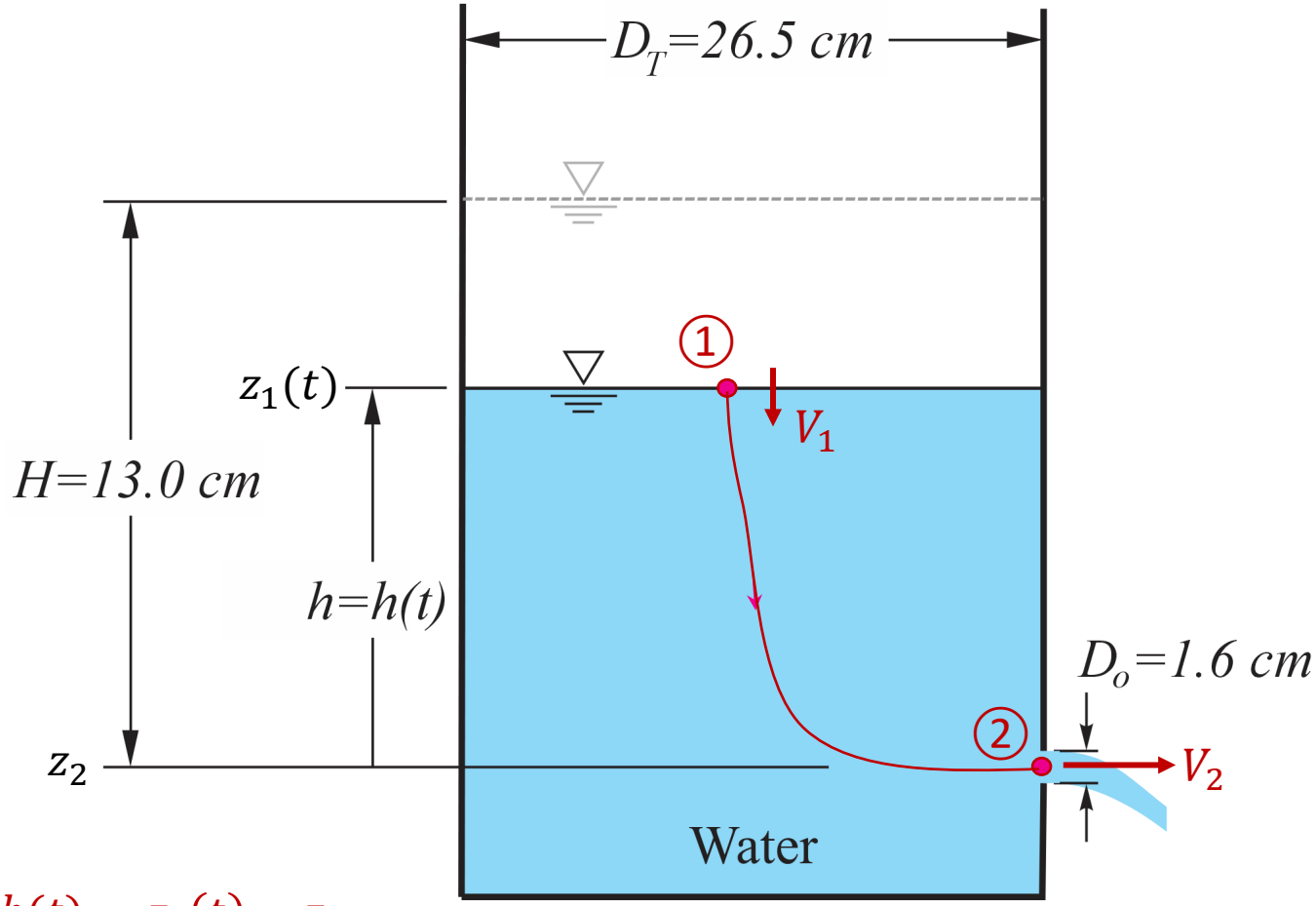


Solution: Draining Tank Problem

- Apply the Bernoulli equation
  - Streamline from ① to ②

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

negligible



- $p_1 = p_2 = p_{atm}$
- Tank drains slowly, $D_T \gg D_o$

$h(t) = z_1(t) - z_2$

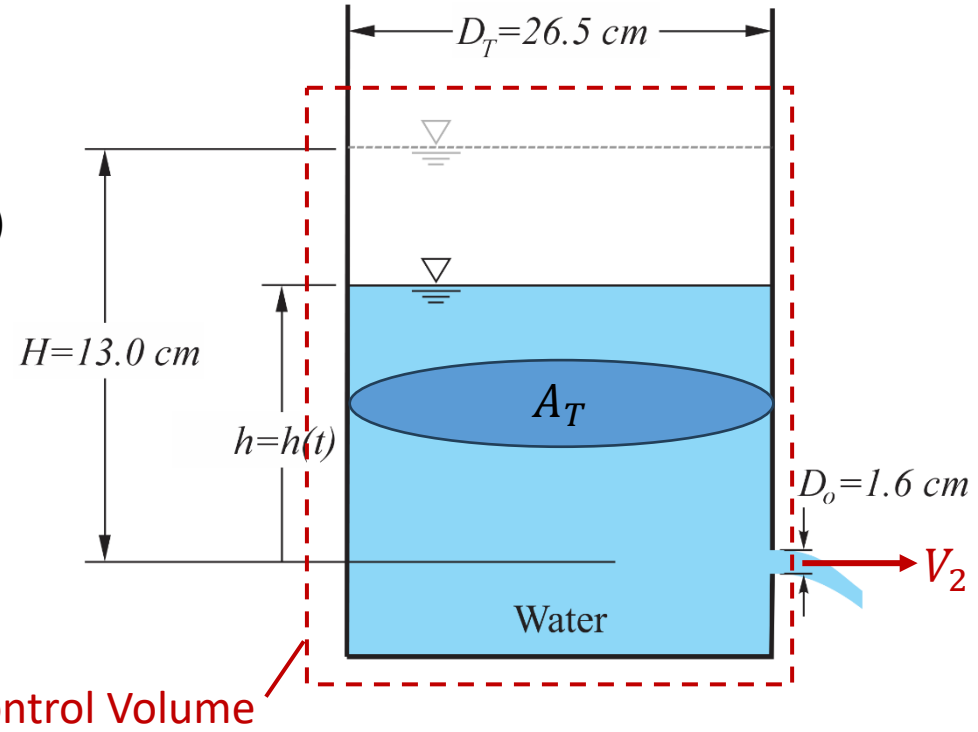
- Ideal discharge velocity: $V_2 = \sqrt{2gh}$ (Torricelli's law, 1644)

- Average with viscous effects: $\overline{V_2} = C_D \sqrt{2gh}$ $\quad 0.6 < C_D < 1.0$ (White & Xu, Chapter 6)

Discharge coefficient

Solution: Draining Tank Problem

- Discharge velocity: $V_2 = C_D\sqrt{2gh}$ ($C_D = 1$, 1-D inviscid flow)



- Continuity equation applied to CV:

$$\frac{d}{dt}(m_{cv}) = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$\sum \dot{m}_{in} \rightarrow 0$

Instantaneous water volume

- Instantaneous mass of water: $m_{cv} = \rho A_T h$
- Outlet mass flow rate: $\dot{m}_{out} = \rho V_2 A_o$

$$\frac{A_0}{A_T} = \frac{\pi D_0^2/4}{\pi D_T^2/4} = \left(\frac{D_o}{D_T}\right)^2$$

$$\frac{d}{dt}(\rho A_T h) = -\rho V_2 A_o \rightarrow \frac{dh}{dt} = -\frac{A_o}{A_T} C_D\sqrt{2gh} \rightarrow \frac{dh}{dt} = -\left(\frac{D_o}{D_T}\right)^2 C_D\sqrt{2gh}$$

A_T is constant

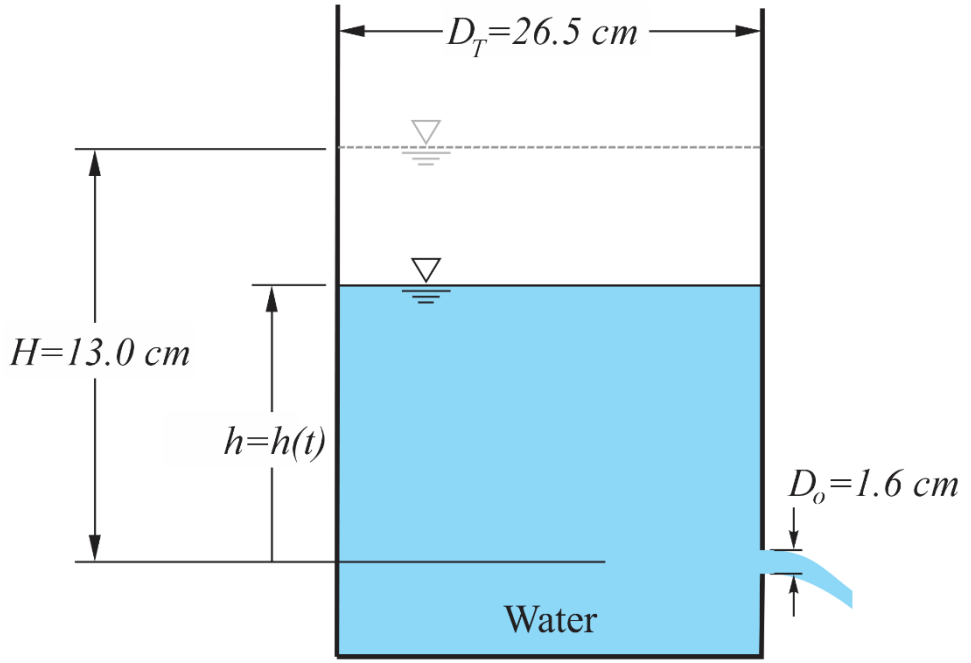
Solution: Draining Tank Problem

Differential equation: $\frac{dh}{dt} = -\left(\frac{D_o}{D_T}\right)^2 C_D\sqrt{2gh}$

Separating variables:

$$\int_0^t dt = -\underbrace{\frac{1}{C_D\sqrt{2g}}\left(\frac{D_T}{D_o}\right)^2}_{\text{Constants}} \int_H^h h^{-1/2}\, dh$$

$$t = -\frac{1}{C_D\sqrt{2g}}\left(\frac{D_T}{D_o}\right)^2 \left[2h^{1/2}\right]_H^h$$



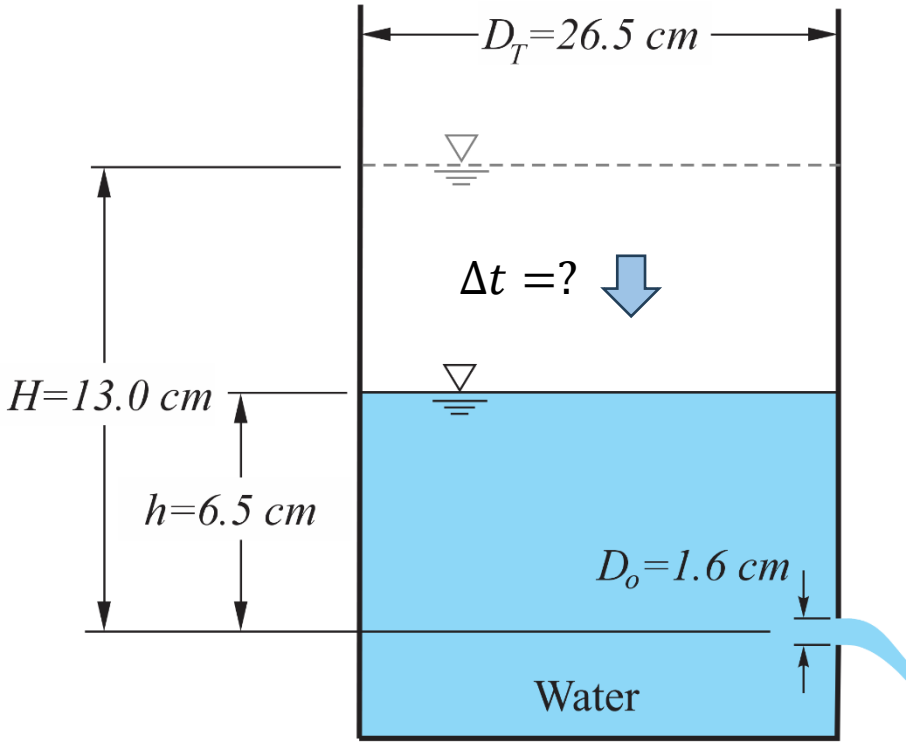
$$t = \frac{2}{C_D\sqrt{2g}}\left(\frac{D_T}{D_o}\right)^2 \left(H^{\frac{1}{2}} - h^{\frac{1}{2}}\right) \quad \text{Eq. (1)}$$

Solve for $h(t)$

$$h = \left(H^{1/2} - \left(\frac{D_o}{D_T}\right)^2 \frac{C_D\sqrt{2g}}{2}\, t\right)^2 \quad \text{Eq. (2)}$$

Solution: Draining Tank Problem

$$t = \frac{2}{C_D\sqrt{2g}}\left(\frac{D_T}{D_o}\right)^2 (H^{\frac{1}{2}} - h^{\frac{1}{2}}) \quad \text{Eq. (1)}$$



- Inviscid flow, $C_D = 1$
- The time to drain to $h = \frac{13cm}{2} = 6.5\ cm$:

$$t = \frac{2}{1\sqrt{2(9.81\frac{m}{s^2})}}\left(\frac{26.5\ cm}{1.6\ cm}\right)^2 \left((0.13\ m)^{\frac{1}{2}} - (0.065\ m)^{\frac{1}{2}}\right)$$

$$t_{inviscid} = 13.1\ s \quad \text{Answer}$$

- Neglects fluid viscosity – the actual time will be longer. How much?

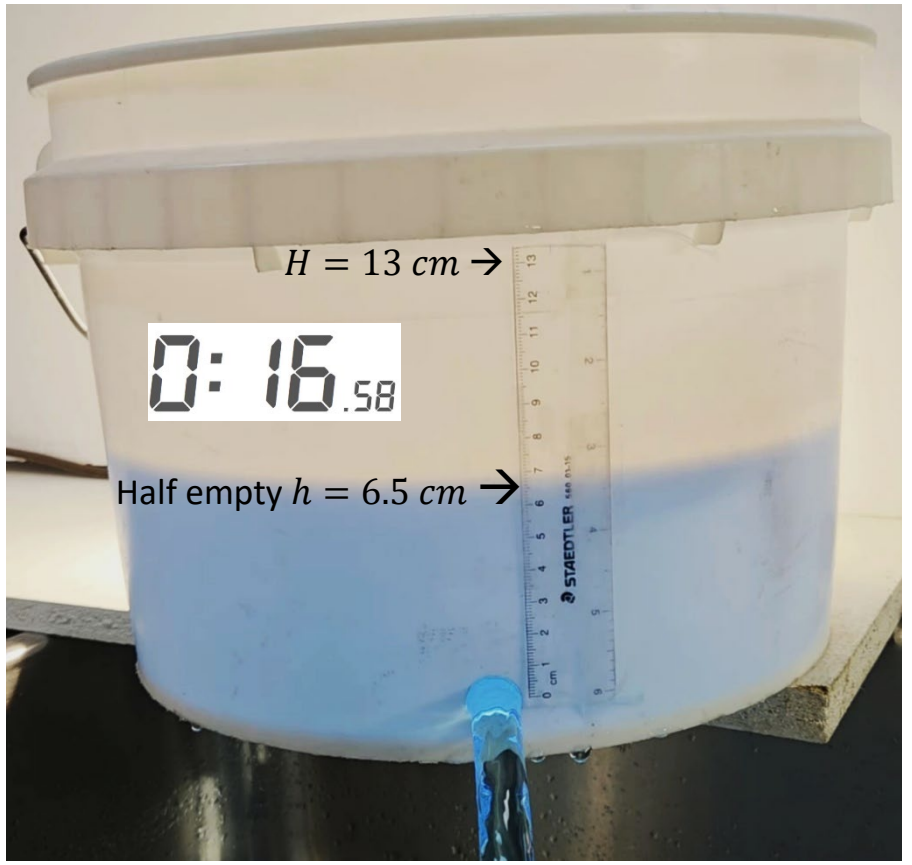
Solution: Draining Tank Problem

$$t_{water} = \frac{2}{C_D\sqrt{2g}}\left(\frac{D_T}{D_o}\right)^2\left(H^{\frac{1}{2}} - h^{\frac{1}{2}}\right) = \frac{t_{inviscid}}{C_D} \qquad \text{Eq. (1)}$$

- For inviscid flow, $C_D = 1$: $t_{inviscid} = 13.1\ s$
- For real fluid: $t_{water} = 16.6\ s$
- From Eq. (1), discharge coefficient for water flow is:

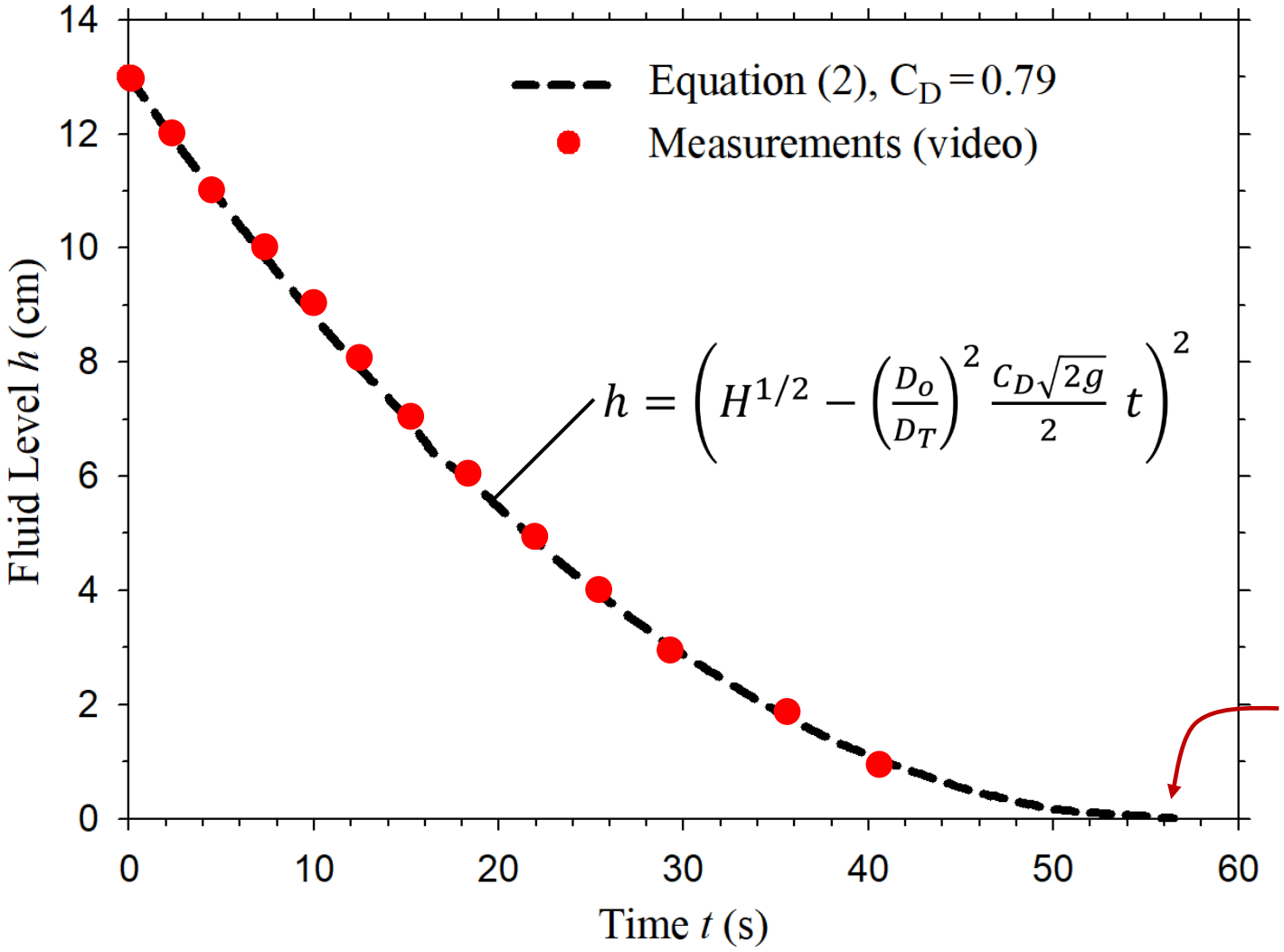
$$C_D = \frac{t_{inviscid}}{t_{water}} = \frac{13.1\ s}{16.6\ s} \approx 0.79$$

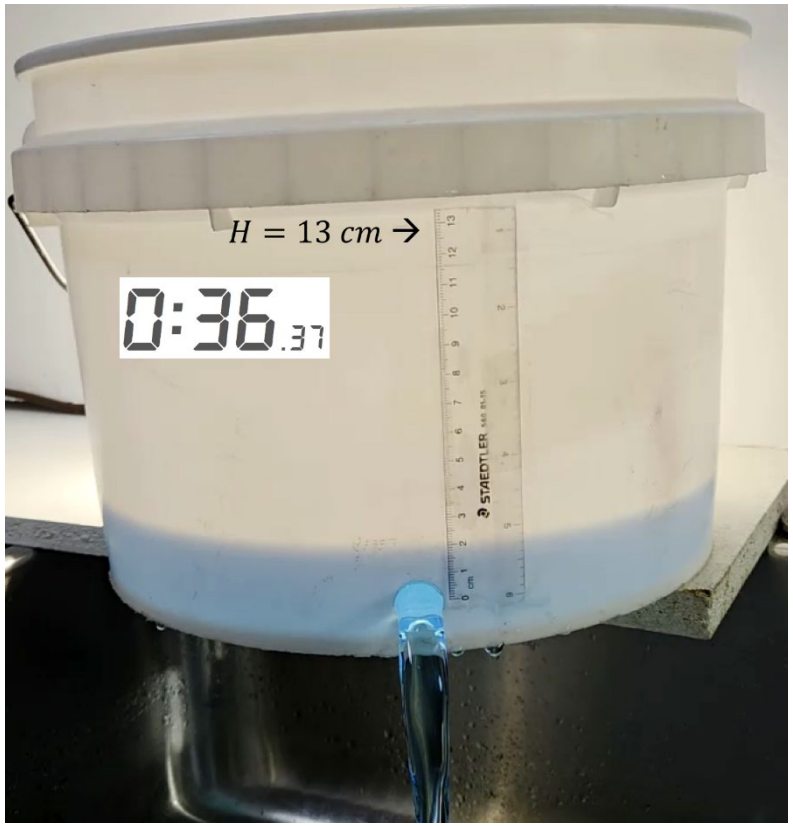
- Water flow rate is 79% of that predicted by Bernoulli's equation



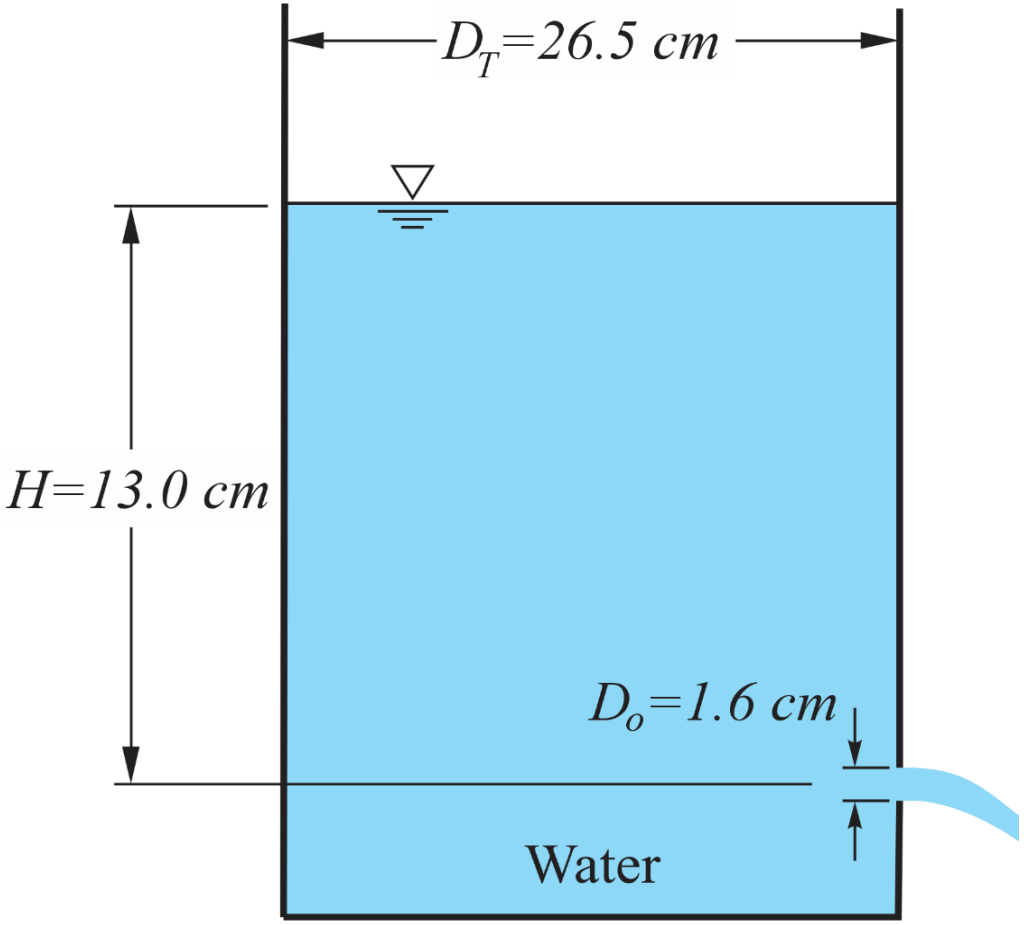
Solution: Draining Tank Problem

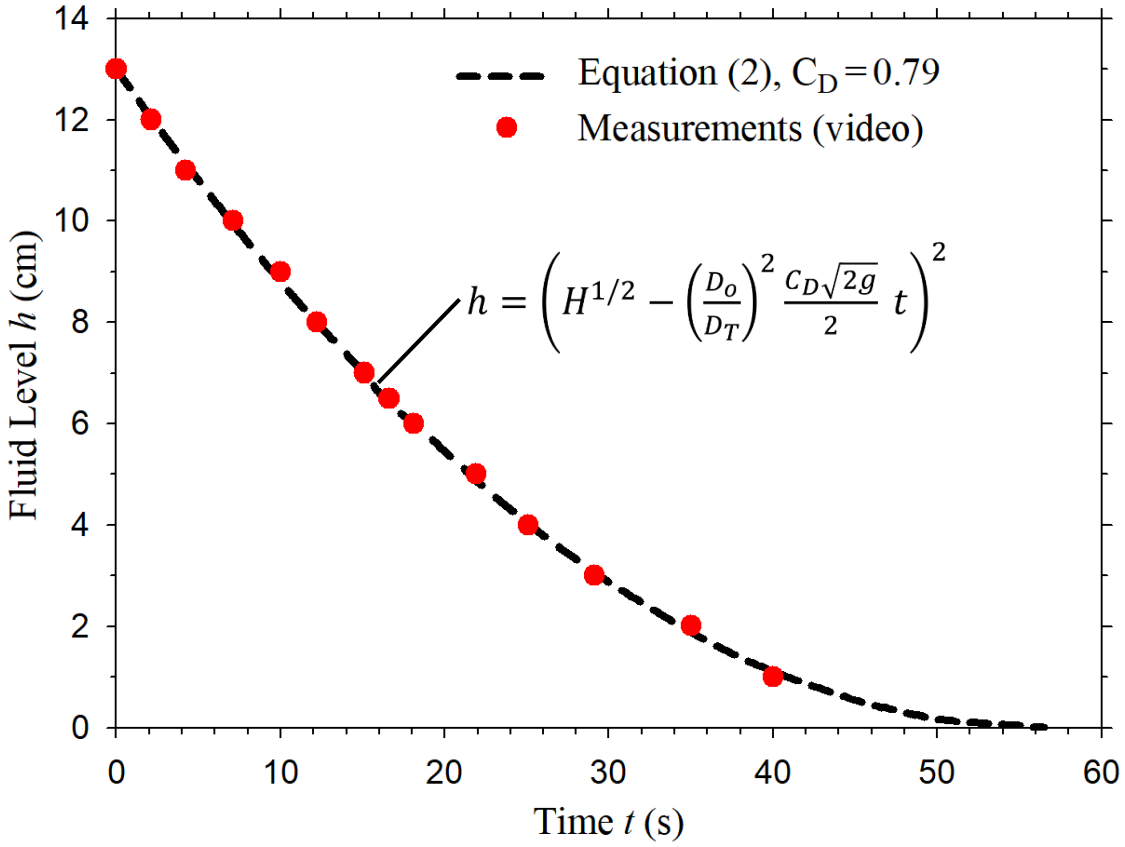
- $C_D = 0.79$ How well does this work?





$t_{h=0} \approx 57\ seconds$





END NOTES

Presentation prepared and delivered by Professor David Naylor

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